Utilisation of Photometric Moments in Visual Servoing

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Context | Relevance

SPARC
The Partnership for Robotics in Europe

euRobotics aisbl

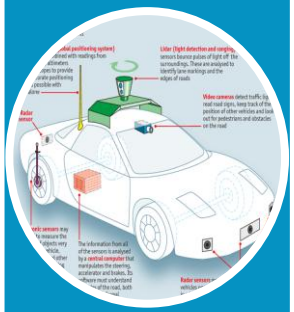
2014-2020
Robotics2020 Roadmap
for Robotics in Europe

Manufacturing

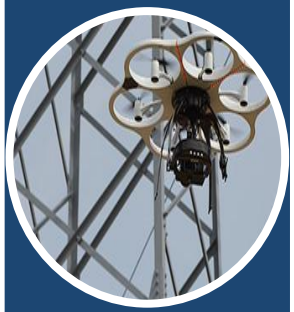
Healthcare

Agriculture

Social



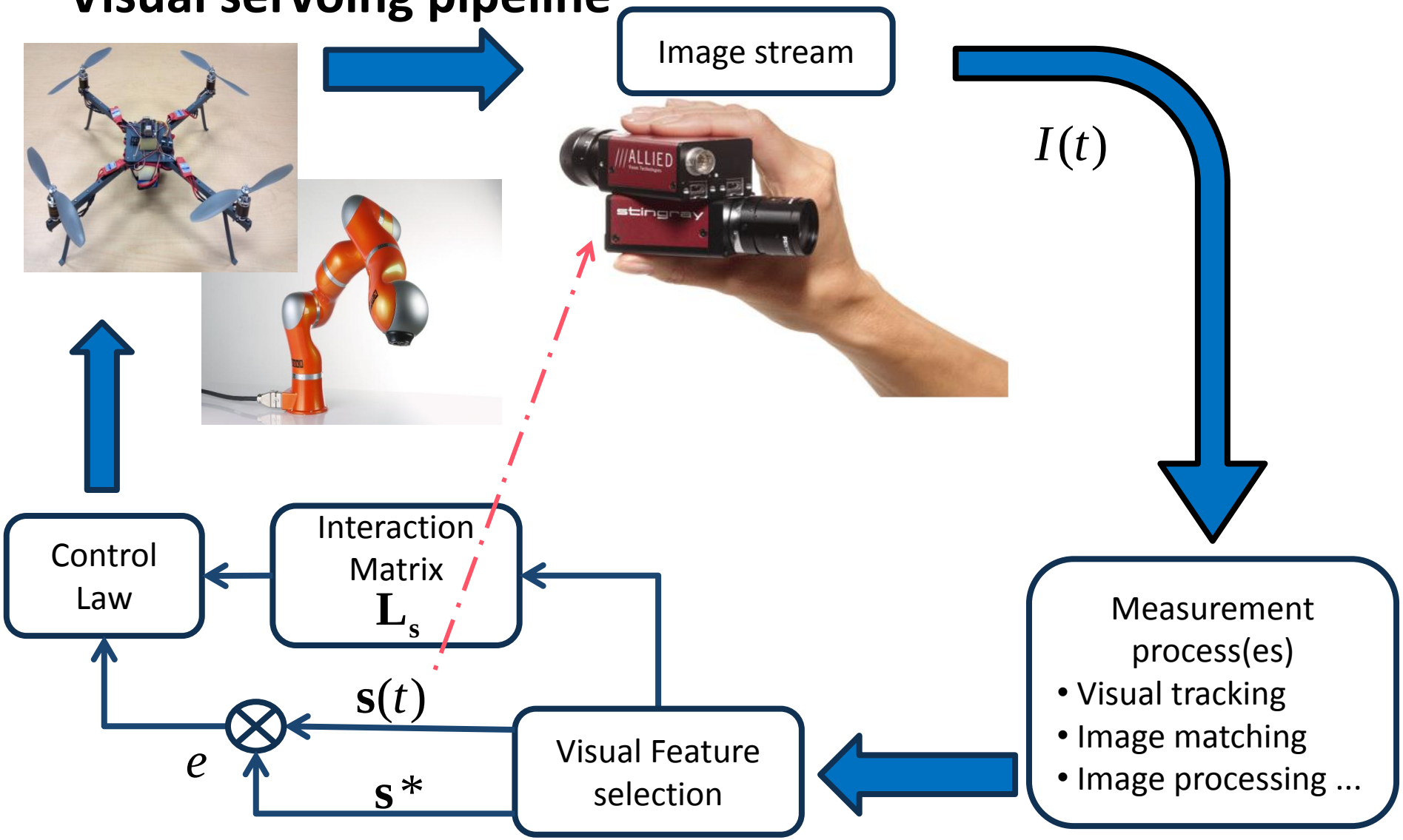
Transport



Civil
Commercial
Military

«Perception, Action, Cognition»

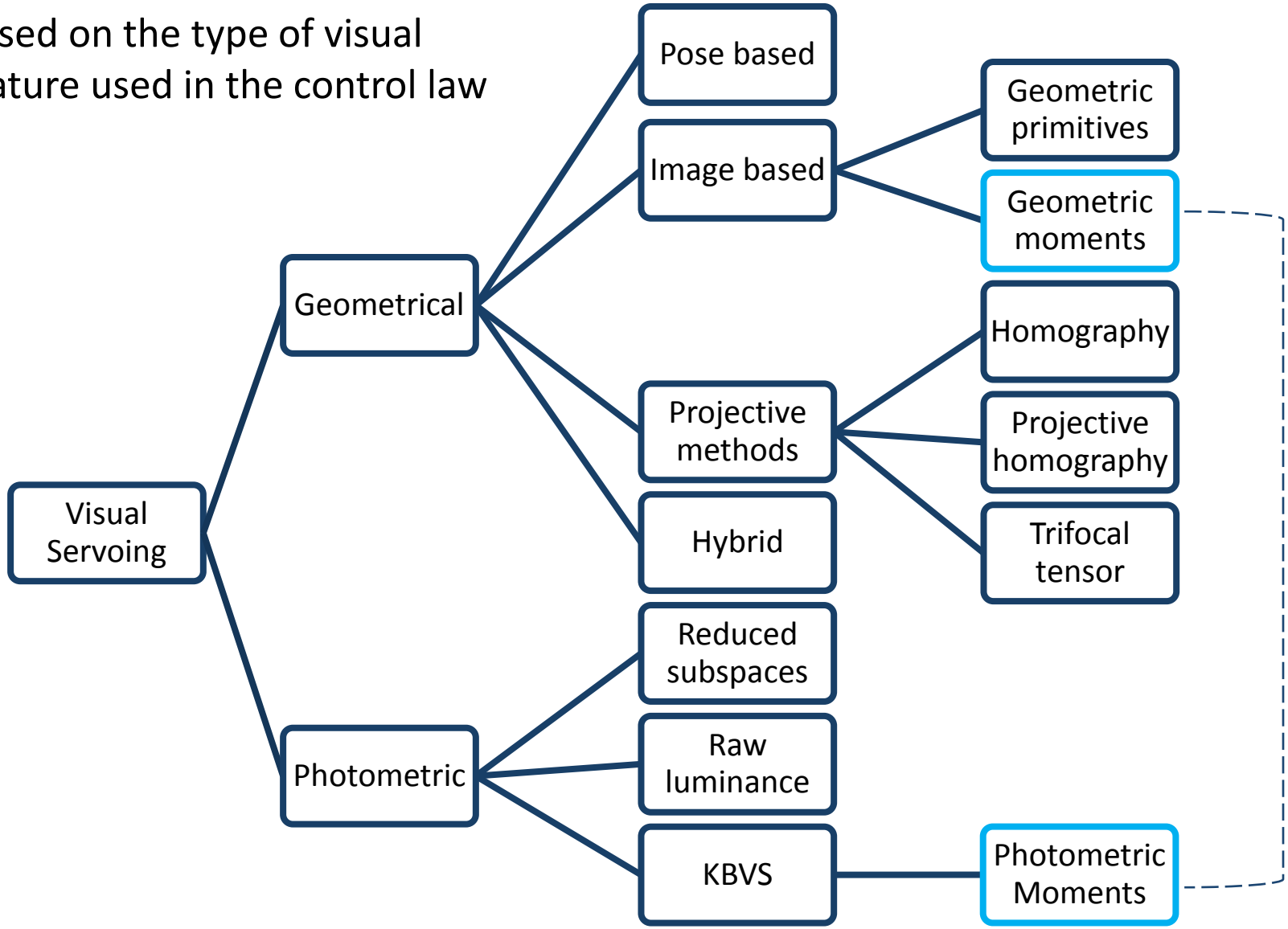
Visual servoing pipeline



$$e = \mathrm{s}\,(\mathrm{m}(I(t)) - \mathrm{s}\,(\mathrm{m}(I^*))$$

Classification of VS approaches

Based on the type of visual feature used in the control law



Classification of VS approaches

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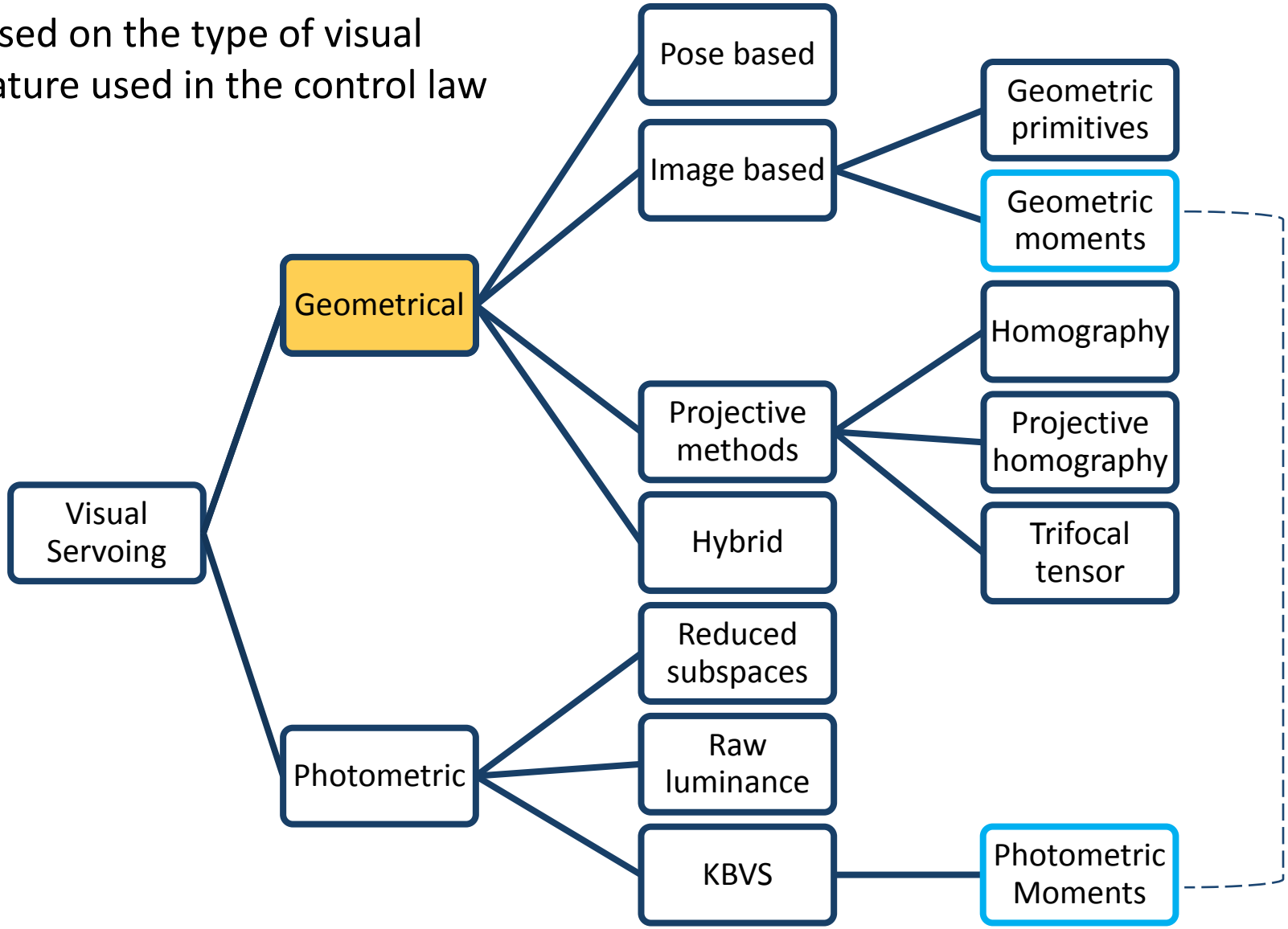
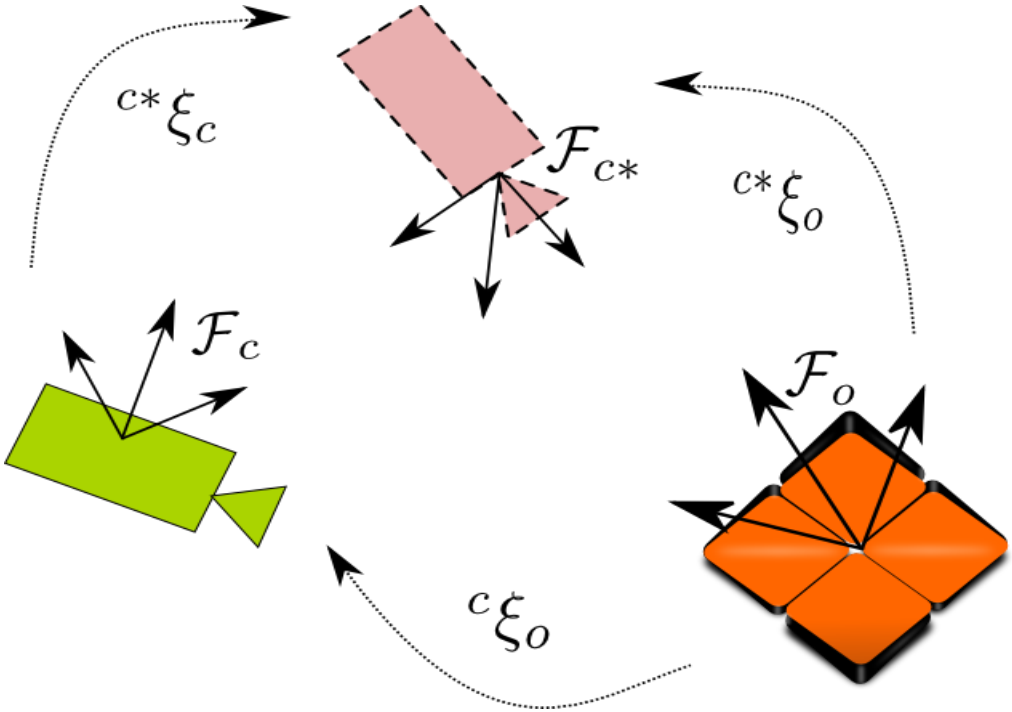
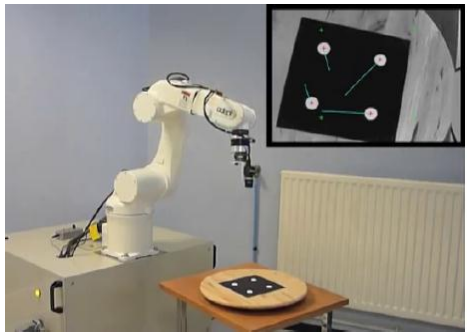


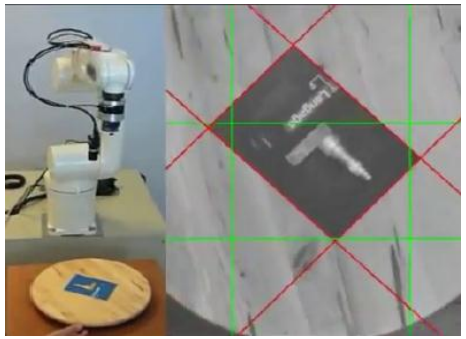
Geometrical Approaches | Pose-based Visual Servoing



- Error is defined in the cartesian space
- ➕ Control over spatial trajectories
- ➖ Reconstruction of camera pose required + error sensitivity
- ➖ For the reconstruction, features have to be visually tracked

Geometrical Approaches | Image-based Visual Servoing





$$\mathbf{v}_c = -\lambda \hat{\mathbf{L}}_{\mathbf{s}}^{+} \begin{bmatrix} \mathbf{x}_1 - \mathbf{x}_1^* \\ \mathbf{x}_2 - \mathbf{x}_2^* \\ \vdots \\ \mathbf{x}_n - \mathbf{x}_n^* \end{bmatrix}$$

Error defined directly in the image space

Geometric primitives

- ➕ No pose reconstruction
- ➕ Robust to modelling errors
- ➖ Relies on presence of geometric or volumetric primitives in the scene
- ➖ Visual tracking necessary

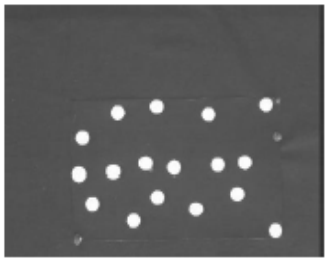
- Visual feature $s(\mathrm{I}) \rightarrow \square$
- An ideal set of visual features $\mathrm{s} = \{s_i(\mathrm{I})\}_{i=1\ldots n}$
  - each feature varies with motions along 1 dof
  - does not change with motions along the remaining (n-1) dof

$$\mathbf{L}_{\mathbf{s}} = \mathbf{I}_n$$

Image-based Visual Servoing | Geometric Moments

Introduced for VS in [Chaumette IROS'02, T-RO'04]

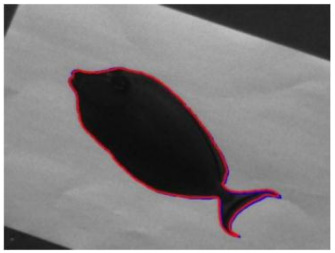
$$m_{pq} = \sum_{i=1}^{N} x_i^p y_i^q \qquad m_{pq} = \iint_{\mathcal{R}} x^p y^q \mathrm{d}x\mathrm{d}y$$



- Compactly represent the essential characteristics of a shape

− Discrete points or well-segmented regions need to be available

+ Simple, geometrically intuitive features to control specific dof



Control of translational dof

$$\mathrm{s}_t = (x_n, y_n, a_n)$$ [Tahri et al., T-RO'05]

$$x_n = x_g\, a_n \qquad y_n = y_g\, a_n \qquad a_n = Z^* \frac{\sqrt{a^*}}{a}$$

$$\mathrm{L}^{\parallel}_{x_n} = \begin{bmatrix} -1 & 0 & 0 & a_n\varepsilon_{11} & -a_n(1+\varepsilon_{12}) & y_n \end{bmatrix}$$

$$\mathrm{L}^{\parallel}_{y_n} = \begin{bmatrix} 0 & -1 & 0 & a_n(1+\varepsilon_{21}) & -a_n\varepsilon_{22} & -x_n \end{bmatrix}$$

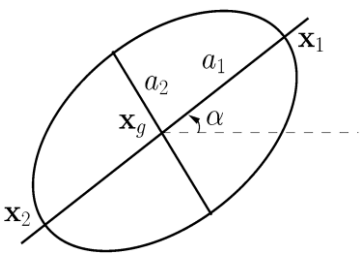
$$\mathrm{L}^{\parallel}_{a_n} = \begin{bmatrix} 0 & 0 & -1 & -a_n\varepsilon_{31} & -a_n\varepsilon_{32} & 0 \end{bmatrix}$$

Image-based Visual Servoing | Geometric Moments

Control of rotational dof

- For rotations around the optic axis $\alpha = \frac{1}{2}\arctan\left(\frac{2\mu_{11}}{\mu_{20}-\mu_{02}}\right)$

$$\mathbf{L}^{\parallel}_{\alpha} = \begin{bmatrix} 0 & 0 & 0 & \mathbf{L}^{\omega_x}_{\alpha} & \mathbf{L}^{\omega_y}_{\alpha} & \boxed{-1} \end{bmatrix}$$



  - from centred moments $\mu_{pq} = \iint (x-x_g)^p (y-y_g)^q \, dx\, dy$

$$\mathbf{L}^{\parallel}_{\mu_{pq}} = \begin{bmatrix} 0 & 0 & \mathbf{L}^{v_z}_{\mu_{pq}} & \mathbf{L}^{\omega_x}_{\mu_{pq}} & \mathbf{L}^{\omega_y}_{\mu_{pq}} & \mathbf{L}^{\omega_z}_{\mu_{pq}} \end{bmatrix}$$

T

- For rotations around the image axes
  - 2+2 visual features based on Hu invariants [Chaumette ,T-RO'04] [Hu'62]
  - From the centred moments, 15 rotation invariant moment polynomials based on theory of moment invariants [Tahri et al., T-RO'05]
    - An appropriately scaled pair can be used as a visual feature

R

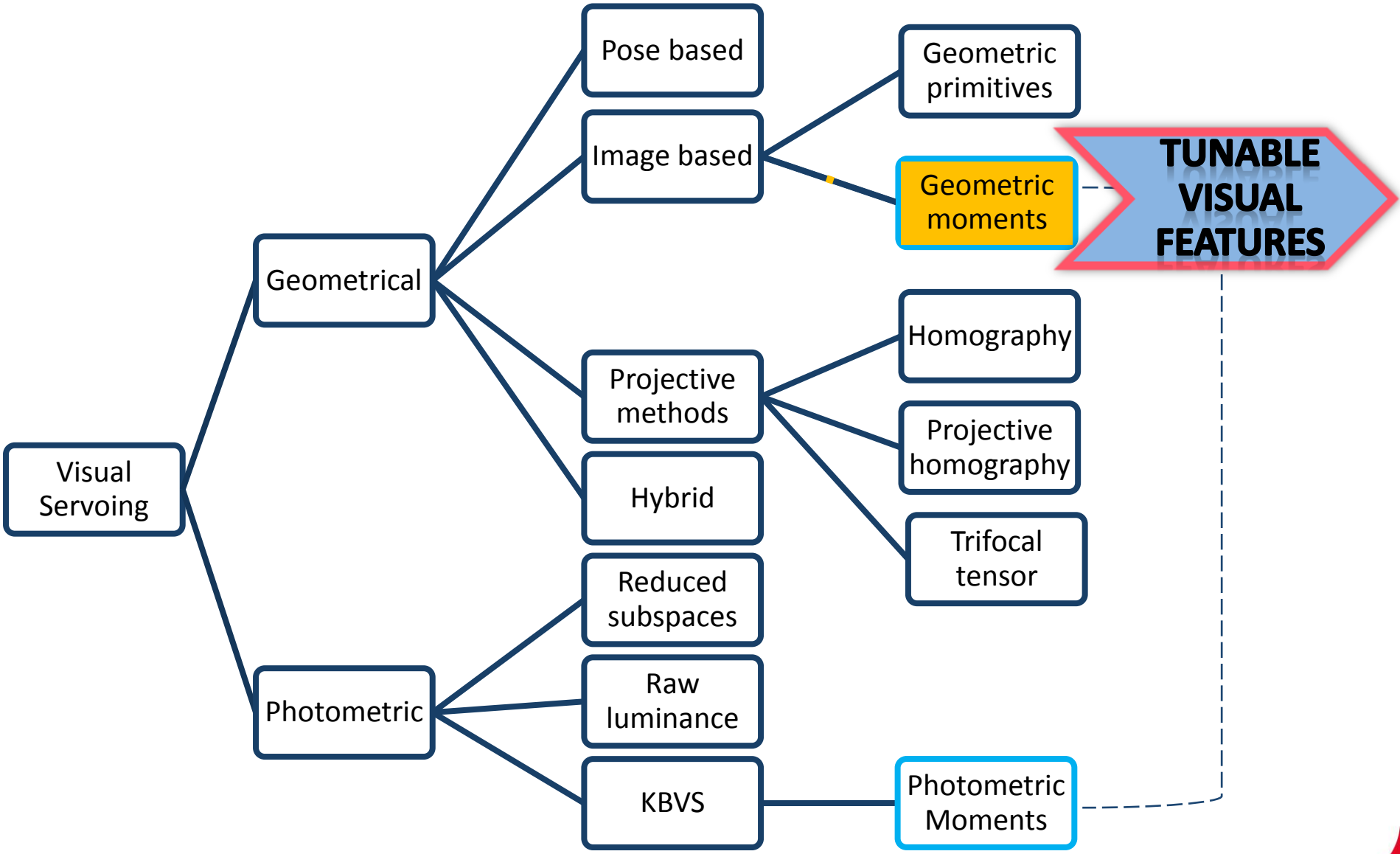
$$\mathbf{L}^{\parallel}_{\phi_i} = \begin{bmatrix} 0 & 0 & 0 & \mathbf{L}^{\omega_x}_{\phi_i} & \mathbf{L}^{\omega_y}_{\phi_i} & 0 \end{bmatrix}$$

T+R+s

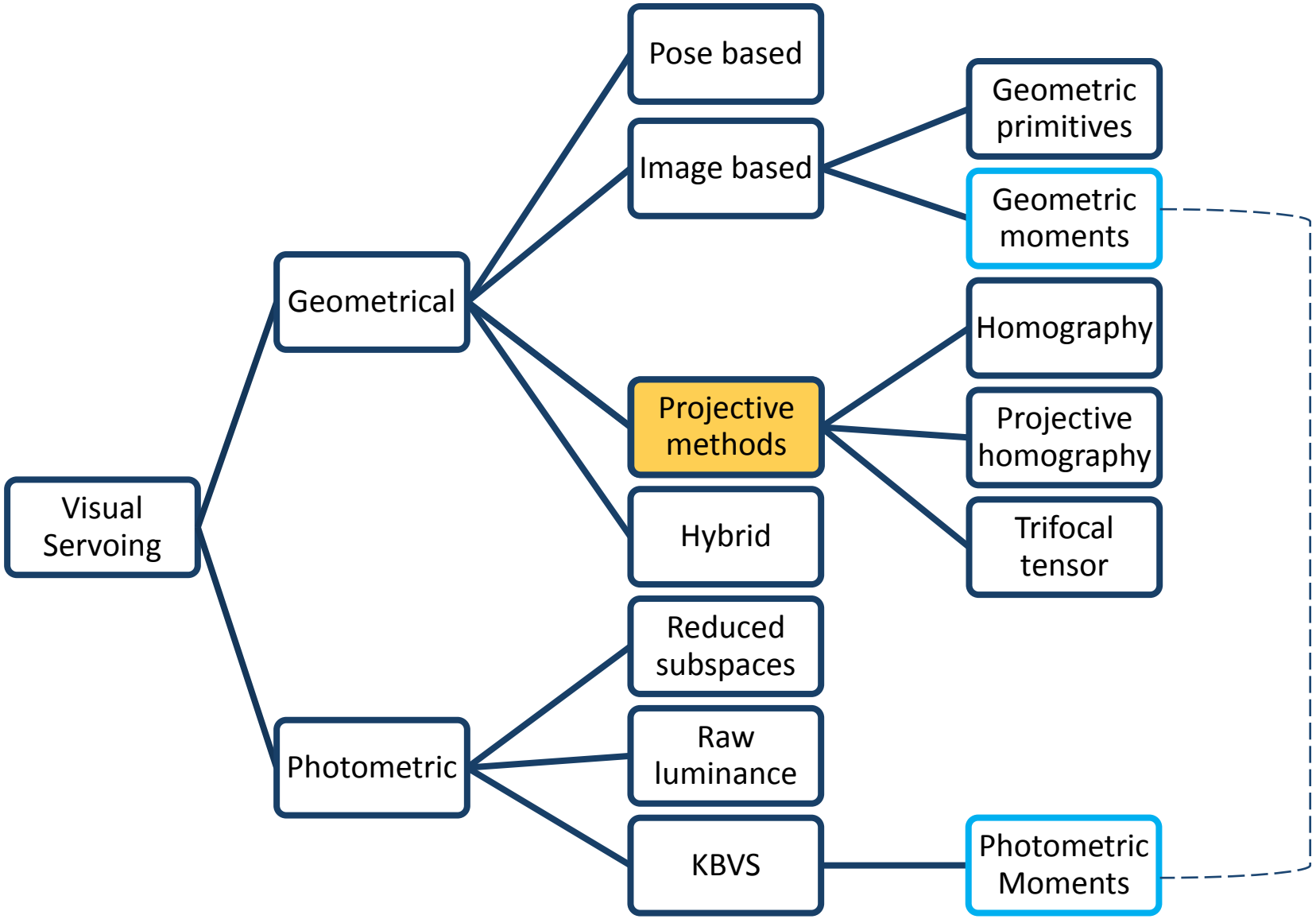
- Difficult selection of 4 polynomials from 15+ choices available
- Need separate features for symmetrical and non-symmetrical shapes

Active : Simultaneous control of rotations around image axes (along with other dof)

Classification of VS approaches



Classification of VS approaches



Geometrical Approaches

Projective methods

- Homography based on ESM widely used [Benhimane ICRA'06, 07]
  - ➖ Manual specification and tracking of region of interest
- Projective homography [Silviera T-RO12, 14]
  - ➖ Implicit tracking image registration
  - ➕ Non-planar scenes

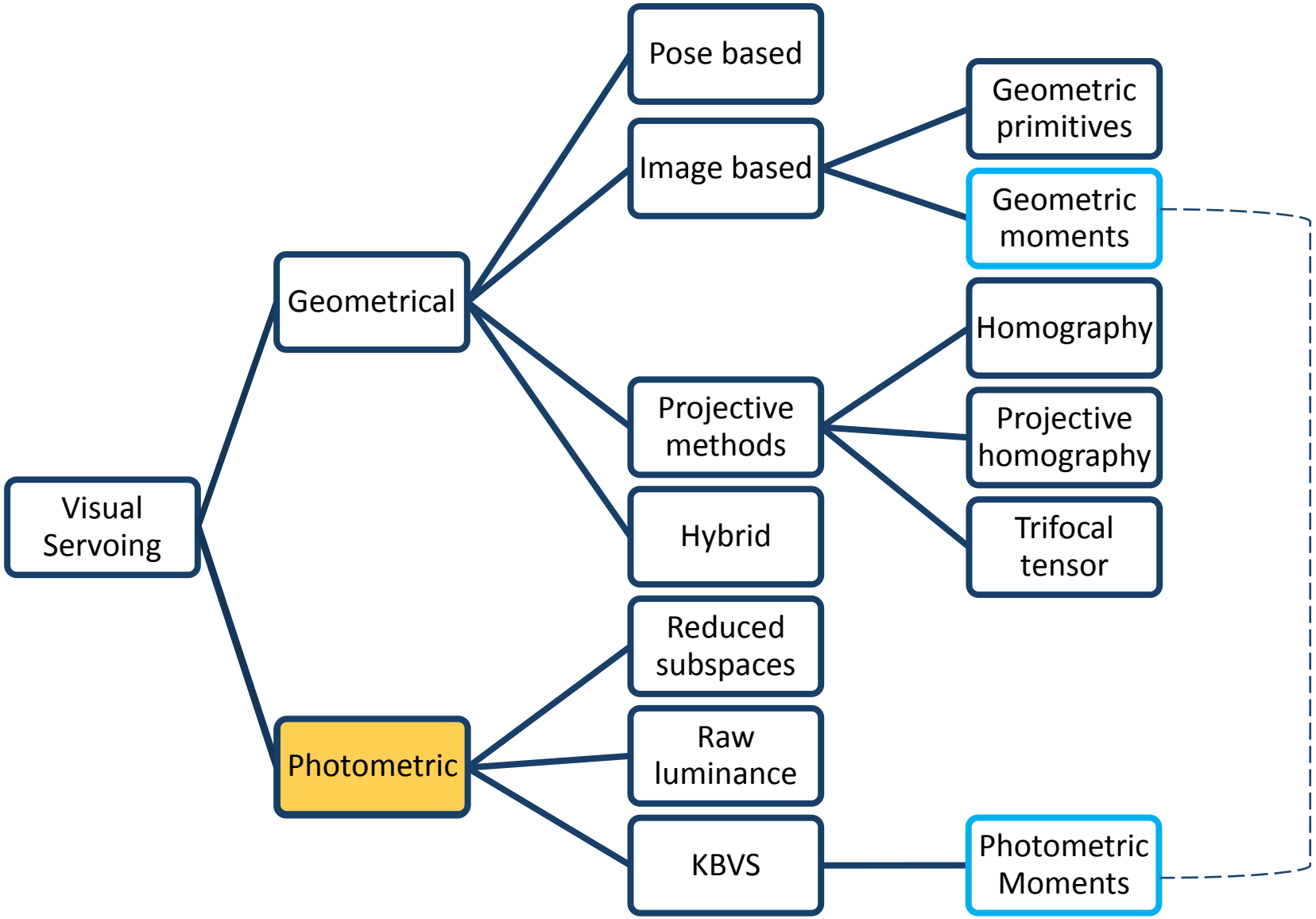
$$\min_{\mathbf{g},\mathbf{h}} \frac{1}{2} \sum_{\mathbf{p}_i^* \in \mathcal{R}^*} \mathbf{I}'(\mathbf{g},\mathbf{h},\mathbf{p}_i^*) - \mathbf{I}^*(\mathbf{p}_i^*)$$

- Hybrid approaches
  - 2.5D combine IBVS and PBVS and inherit their properties [Malis T-R0'00]
  - Switch between IBVS and PBVS based on L [Gans T-R0'07]

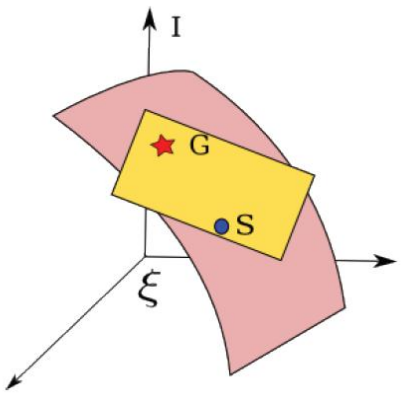
Challenges in VS

- ➖ Tracking is a bottleneck to the expansion of VS [Collewet ICRA'08]
- ➖ Hindrance for adoption of VS in real-world applications [Gridseth RSS'13]
- Rich intensity information is lost or under utilized

Classification of VS approaches



Photometric Approaches



Reduced dimensional subspaces [Nayar 96, Deguchi 97, 2000]

- Transform to less dimensions using PCA
- ➖ Learning-based approach
- ➖ Difficulty to scale to multiple dof

Raw luminance [Collewet, Marchand T-RO 2011]

- ➕ Bypassed tracking procedures in the pipeline
- ➕ Excellent accuracies due to redundancy
- ➖ Convergence limited by its non-linearity

Kernel-based VS [Kallem,Dewan IROS'07]

$$\nu(t) = \iint_{\mathcal{I}} K(\mathbf{w}) \; s(\mathbf{w}, t) \; \mathrm{d}\mathbf{w}$$

- Features obtained by projection of the signal onto a kernel
- ➕ Abstract formulation
- ➖ *Kernel and signal combinations could control only 4dof*
- ➖ No analytical model (Theoretical analyses are very difficult)

Contributions

Moments-based VS | Shifted Moments [Tamtsia et al, ICRA'13]

- The basis functions were computed with respect to two fixed shift points

$$\tilde{\mu}_{pq} = \iint (x - x_g + x_{sh})^p (y - y_g + y_{sh})^q \, \mathrm{d}x \, \mathrm{d}y$$

Shift point $\mathbf{p}_1$

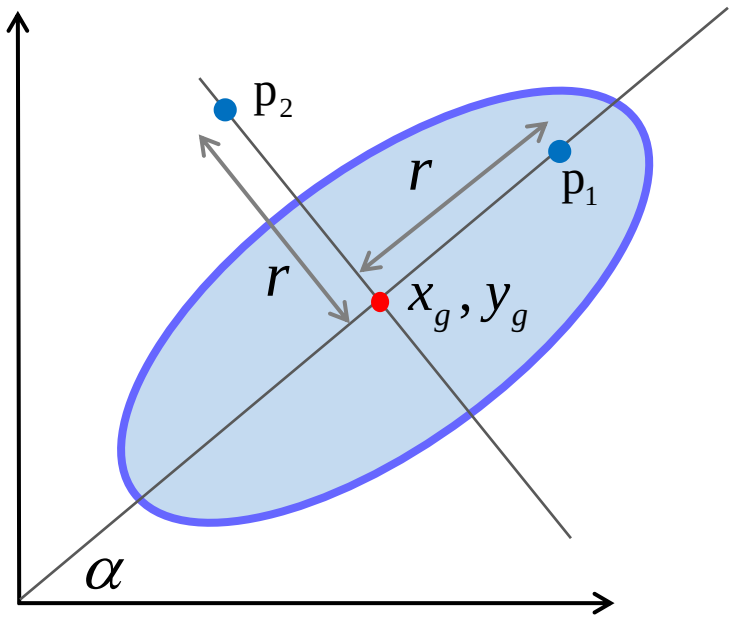
$$x_{sh1} = r\cos(\alpha)$$

$$y_{sh1} = r\sin(\alpha)$$

Shift point $\mathbf{p}_2$

$$x_{sh2} = r\cos(\alpha + \frac{\pi}{2})$$

$$y_{sh2} = r\sin(\alpha + \frac{\pi}{2})$$



- r chosen such that the shift points change consistently with scale and rotation variations of the object, for instance $r = (\mu_{20} + \mu_{02})^{\frac{1}{4}}$
- These points are virtual and not visually tracked in the image
- Shifted moments exist even when the centred moments vanish due to symmetry

Moments-based VS | Shifted Moments [Tamtsia et al, ICRA'13]

- With the shifted moments, applying the theory of moment invariants, a set of moment polynomials were developed as in [Tahri et al., T-RO'05]

$$I_{s1} = \tilde{\mu}_{20}\,\tilde{\mu}_{02} - \tilde{\mu}_{11}^2$$

$$I_{s2} = -\tilde{\mu}_{30}\,\tilde{\mu}_{12} + \tilde{\mu}_{21}^2 - \tilde{\mu}_{03}\,\tilde{\mu}_{21} + \tilde{\mu}_{12}^2$$

$$I_{s3} = 3\tilde{\mu}_{30}\,\tilde{\mu}_{12} + \tilde{\mu}_{30}^2 + 3\tilde{\mu}_{03}\tilde{\mu}_{21} + \tilde{\mu}_{03}^2$$

- Visual feature obtained again by suitably scaling 2 polynomials

$$rs_1 = \frac{Is_2}{Is_1^{5/4}} \qquad rs_2 = \frac{Is_3}{Is_1^{5/4}} \qquad rs_3 = \frac{Is_3}{Is_2}$$

$$\mathrm{L}_{rs_i}^{\parallel} = \begin{bmatrix} 0 & 0 & 0 & \mathrm{L}_{rs_i}^{\omega_x} & \mathrm{L}_{rs_i}^{\omega_y} & 0 \end{bmatrix}$$

T+R+s

- Thanks to this original approach visual features independent of object shape could be selected
- Strive for a large convergence domain in moments-based VS
- We propose next a generalized version of the shifted moments and we will adopt the above visual features for our tests

Moments-based VS | Tunable visual features

$$\tilde{\mu}_{pq} = \iint (x - x_g + x_{sh})^p (y - y_g + y_{sh})^q \, \mathrm{d}x \, \mathrm{d}y$$

Tunable shift point $\mathbf{p}_1$

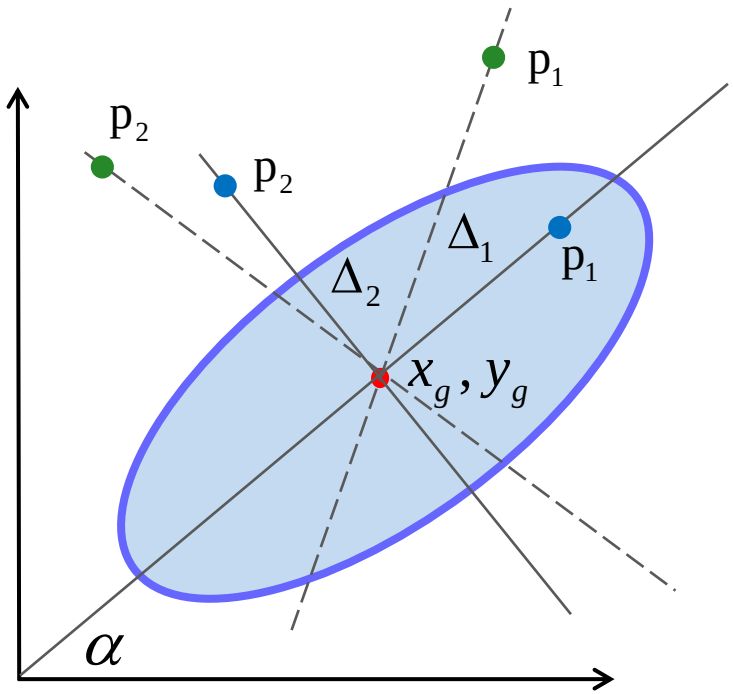
$$x_{s1} = \sqrt{m_{00}} \cos(\alpha + \Delta_1)$$

$$y_{s1} = \sqrt{m_{00}} \sin(\alpha + \Delta_1)$$

Tunable shift point $\mathbf{p}_2$

$$x_{s2} = \sqrt{m_{00}} \cos(\alpha + \Delta_2)$$

$$y_{s2} = \sqrt{m_{00}} \sin(\alpha + \Delta_2)$$



- Instead of computing the moments wrt points along fixed orientations, use points whose coordinates are defined as a function of the orientation
- This *variable* definition introduces a freedom which is exploited to optimally select them from interaction matrix based criteria
- We have chosen for *r* the square root of the area moment

Tunable visual features | Selection criteria

- The interaction matrix related to rotations around the image axes is given by

$$\mathbf{L}_{\phi} = \begin{bmatrix} \mathbf{L}^{\omega}_{p_1} \\ \mathbf{L}^{\omega}_{p_2} \end{bmatrix} = \begin{bmatrix} L^{\omega_x}_{p_1} & L^{\omega_y}_{p_1} \\ L^{\omega_x}_{p2} & L^{\omega_y}_{p_2} \end{bmatrix}$$

- Responsiveness to motion along dof

$$\Delta_1^* = \max_{\Delta_1} \quad \mathcal{F}_1(\Delta_1)$$

$$\mathcal{F}_1(\Delta_1) = \left\| \mathbf{L}^{\omega}_{\mathrm{p}_1} \right\| = \left\| \begin{bmatrix} L^{\omega_x}_{\mathrm{p}_1} & L^{\omega_y}_{\mathrm{p}_1} \end{bmatrix} \right\|$$

- Orthogonality between components of p_1 and p_2
  - To facilitate maximal decoupling

$$\Delta_2^* \in \mathbb{R} = \{d : \forall d \in \mathcal{D},\ \mathcal{F}_2(d) \simeq 0\}$$

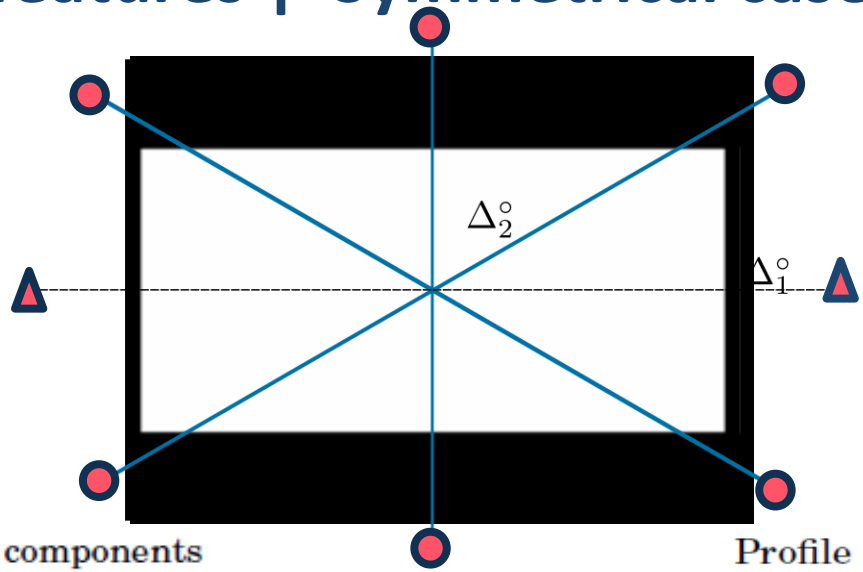
$$\mathcal{F}_2(\Delta_2) = \kappa_{dn}(\mathbf{L}_{\omega_{x,y}}) = \frac{\mathbf{L}^{\top}_{\mathrm{p}_1}.\mathbf{L}_{\mathrm{p}_2}}{\|\mathbf{L}_{\mathrm{p}_2}\|}$$

- Joint selection can be performed such that the system has an optimal condition number
  - Important from stability point of view

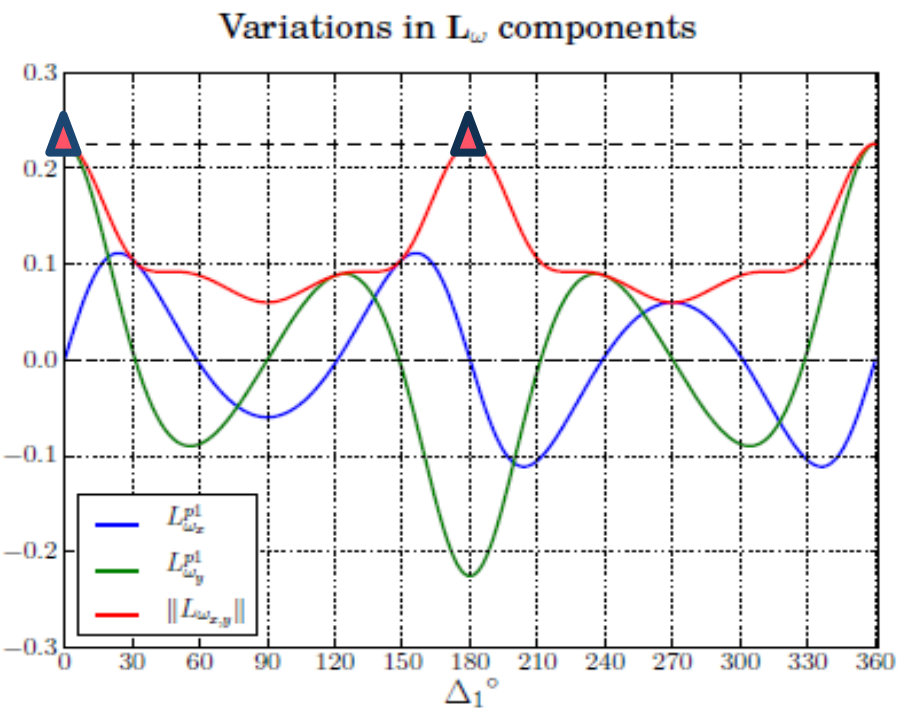
$$\mathcal{J}(\Delta_1, \Delta_2) = \frac{1}{\kappa\left(\mathbf{L}_s(\Delta_1, \Delta_2)\right)}$$

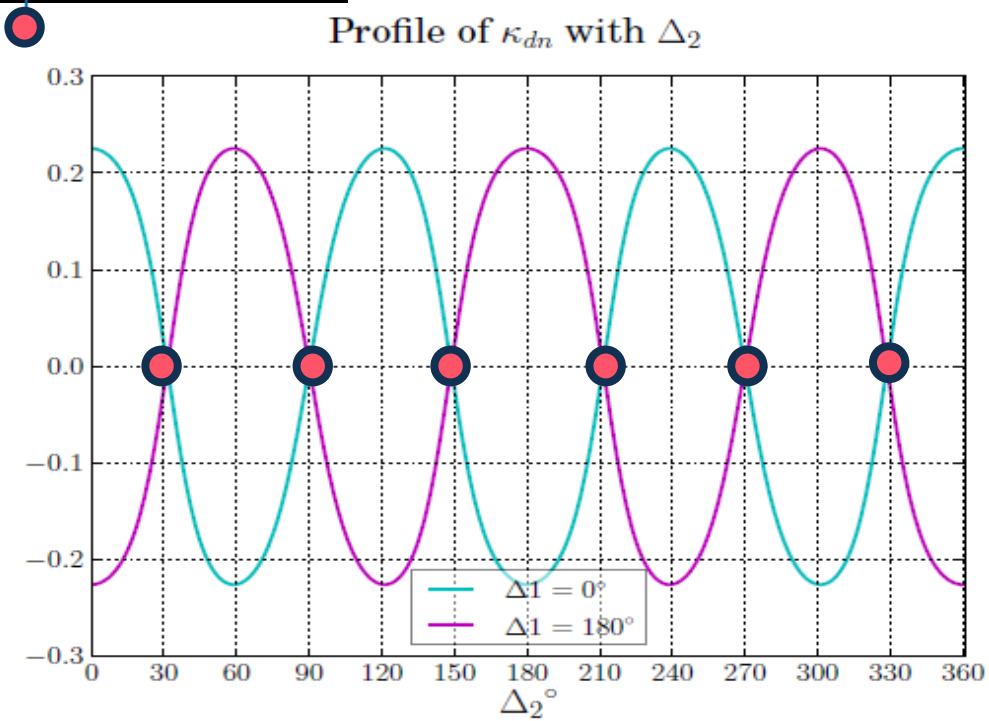
Case studies

Tunable visual features | Symmetrical case

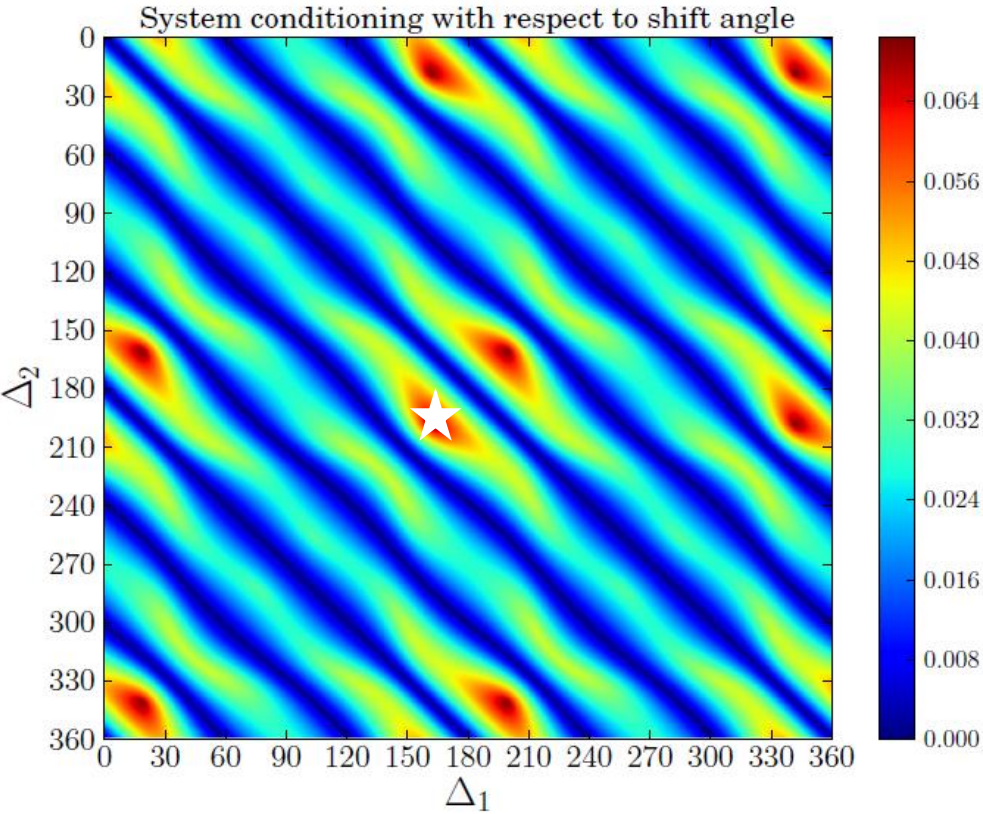


Reference image

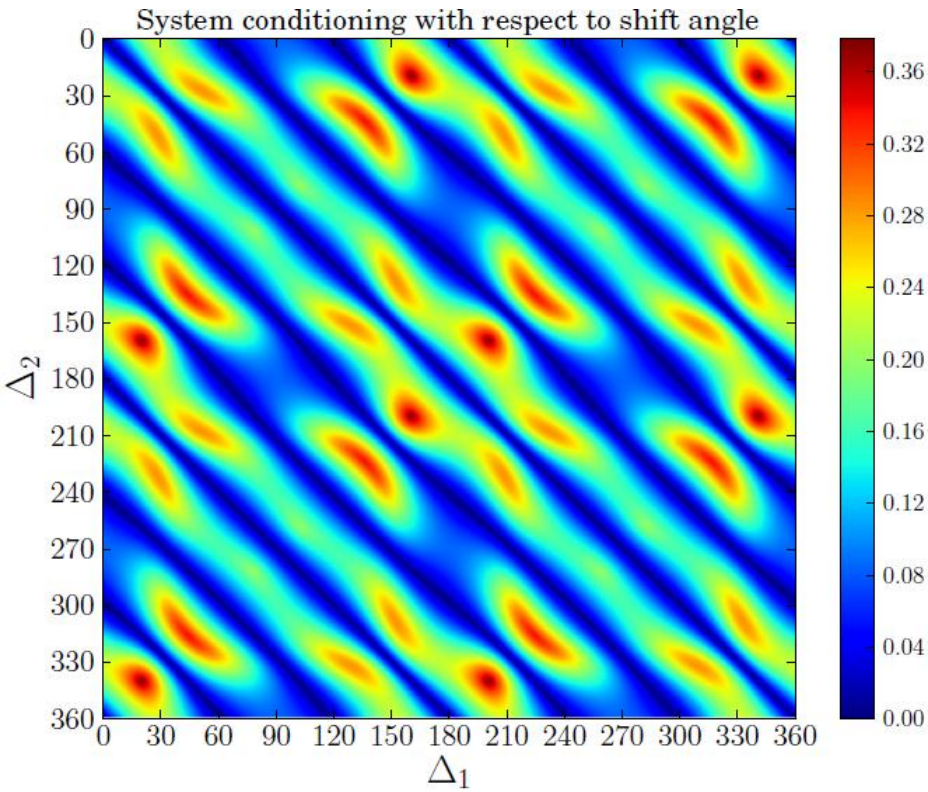




Tunable visual features | Symmetrical case

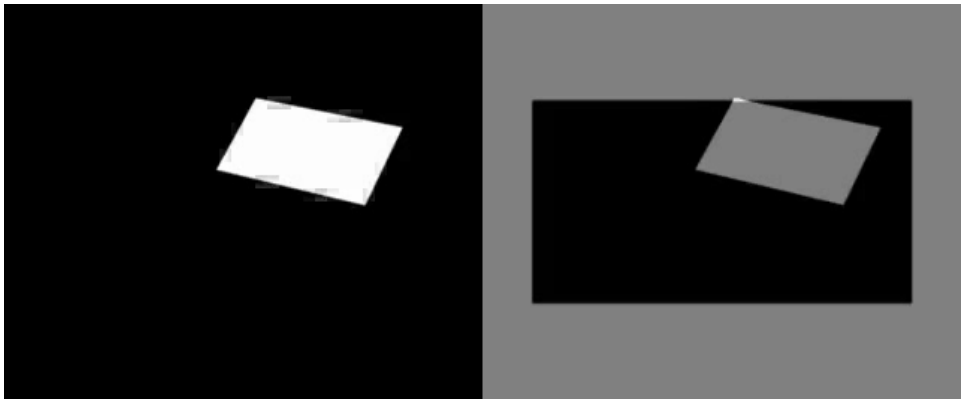


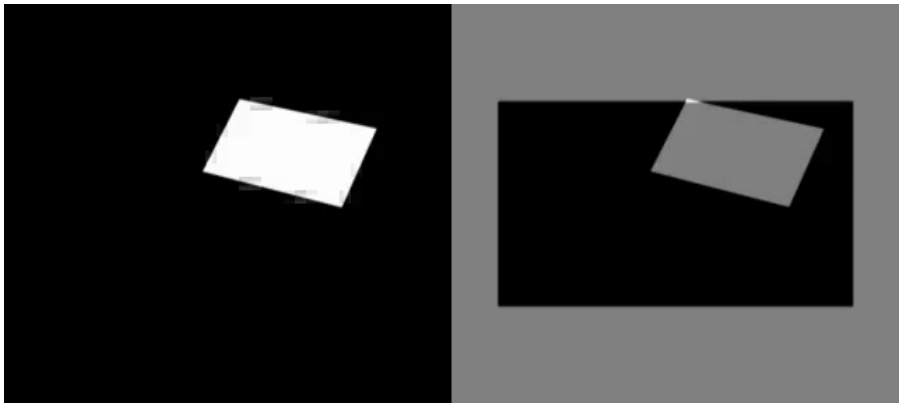
Joint selection with rs_1

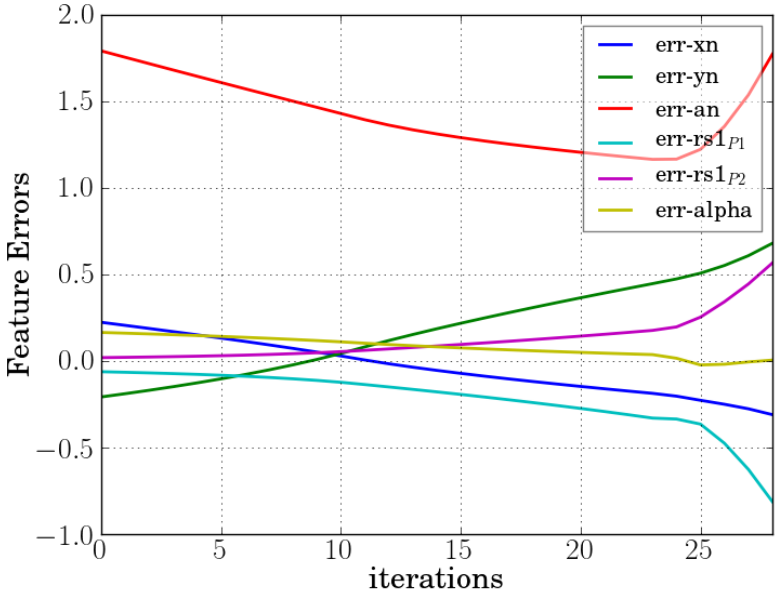


Joint selection with rs_3

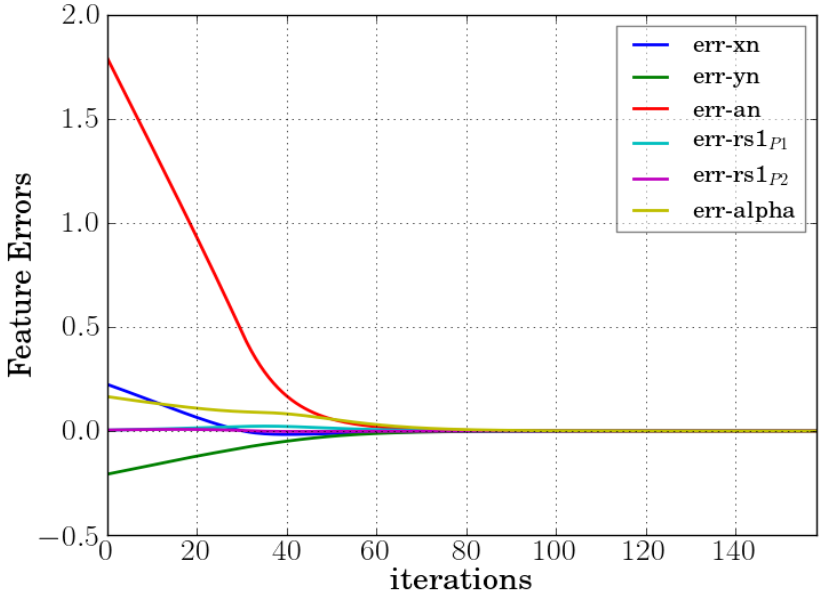
Tunable visual features | Simulation : example case





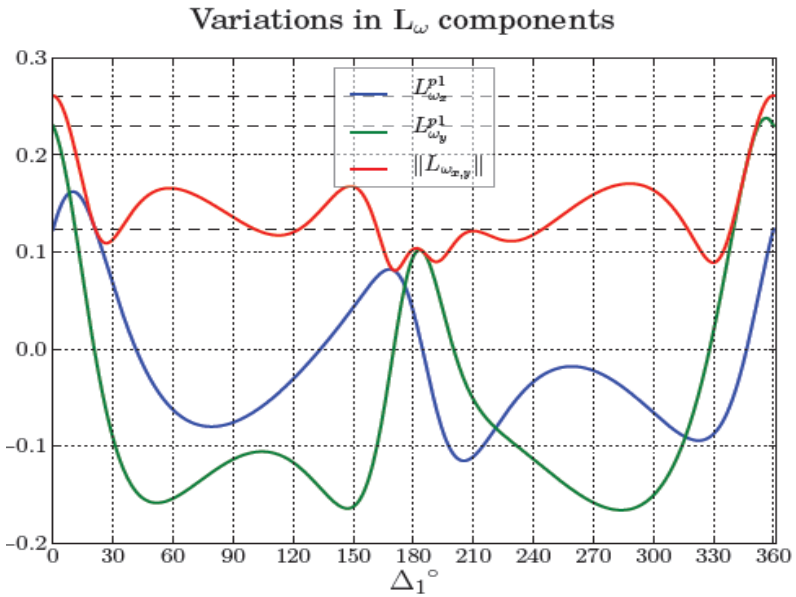


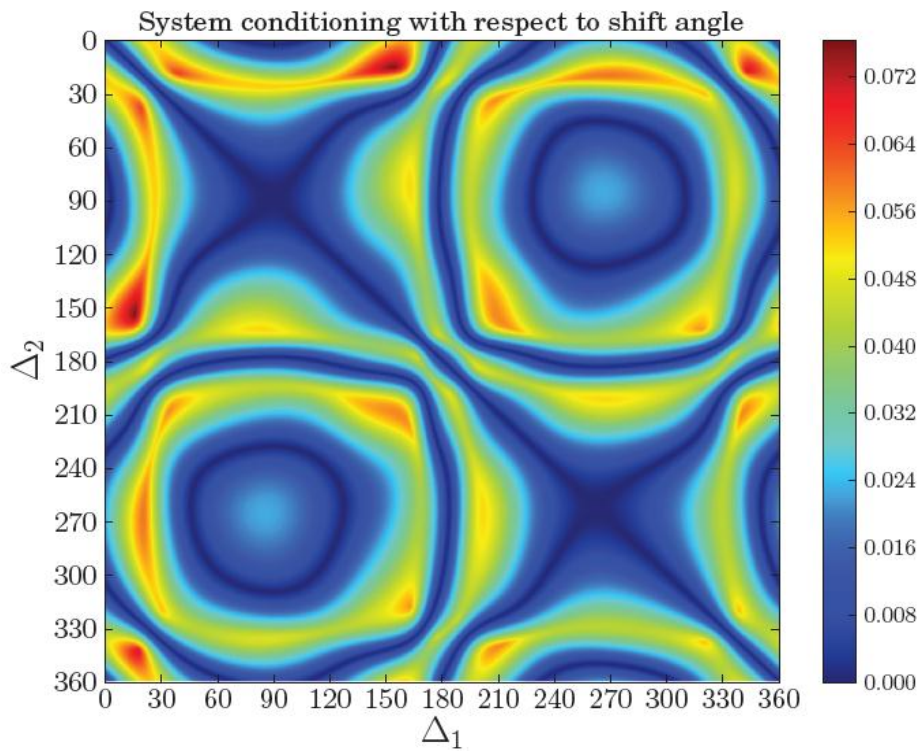
Without selection

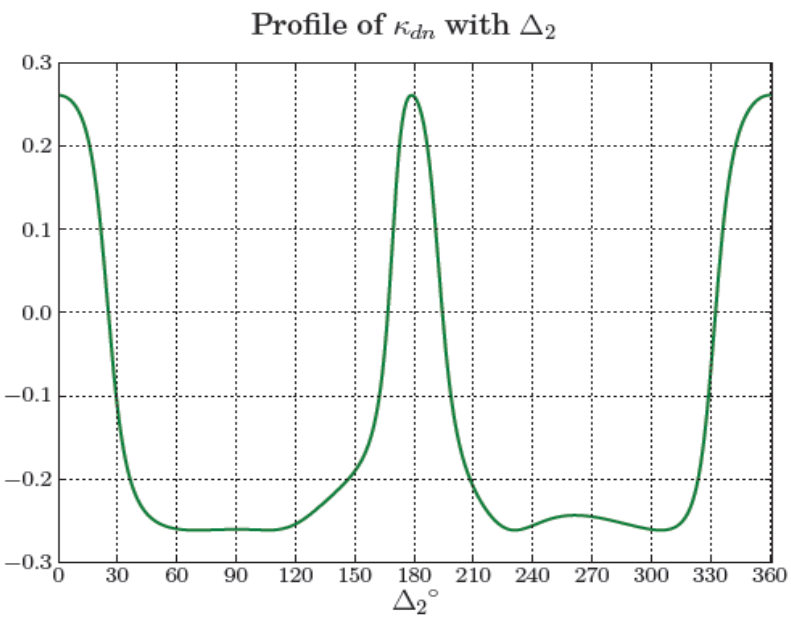


With joint selection

Tunable visual features | Non-symmetrical case

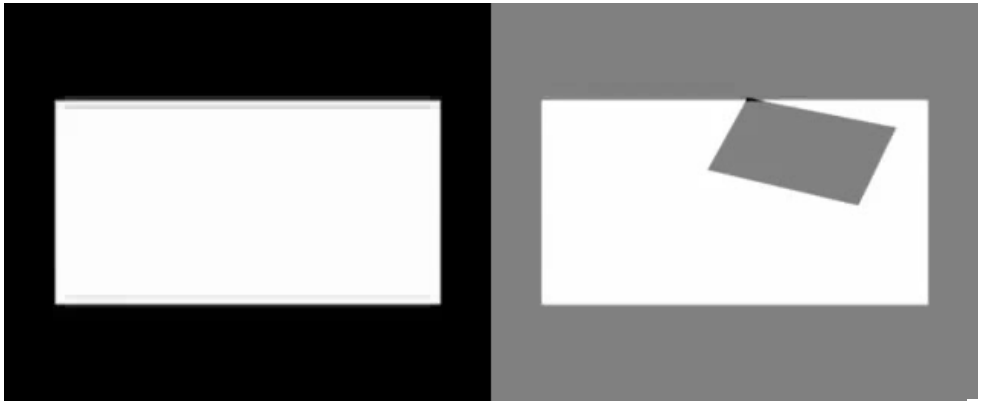


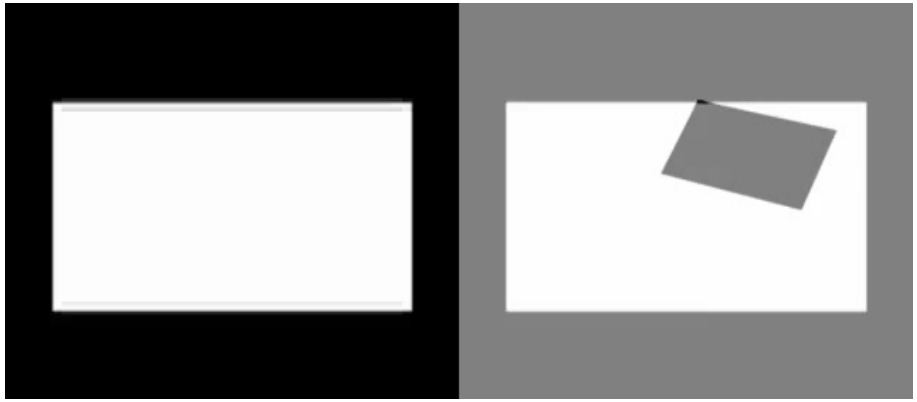


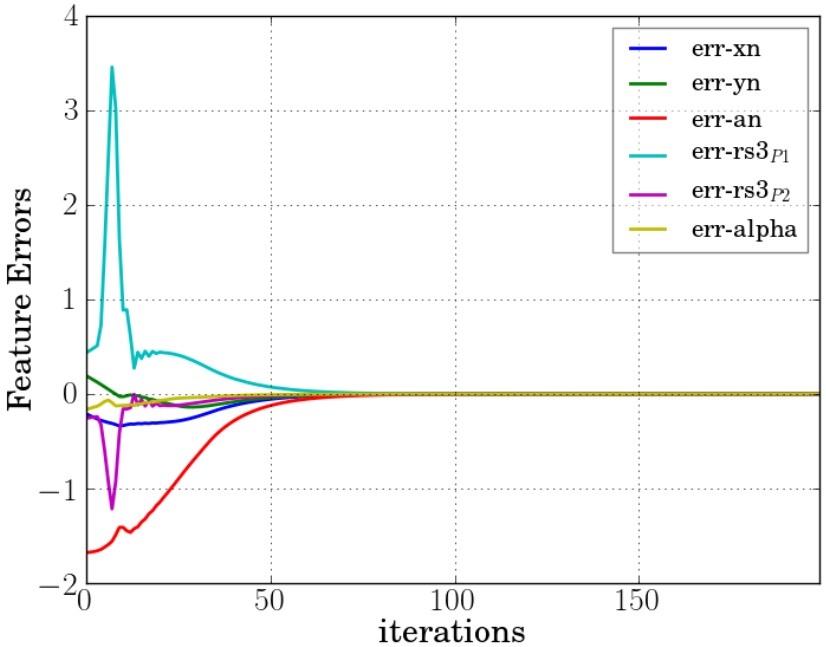


- This case occurs when a non-symmetrical object is observed by the camera or when a symmetrical object is observed from a skewed angle
- No specific symmetries but all criteria are again applicable

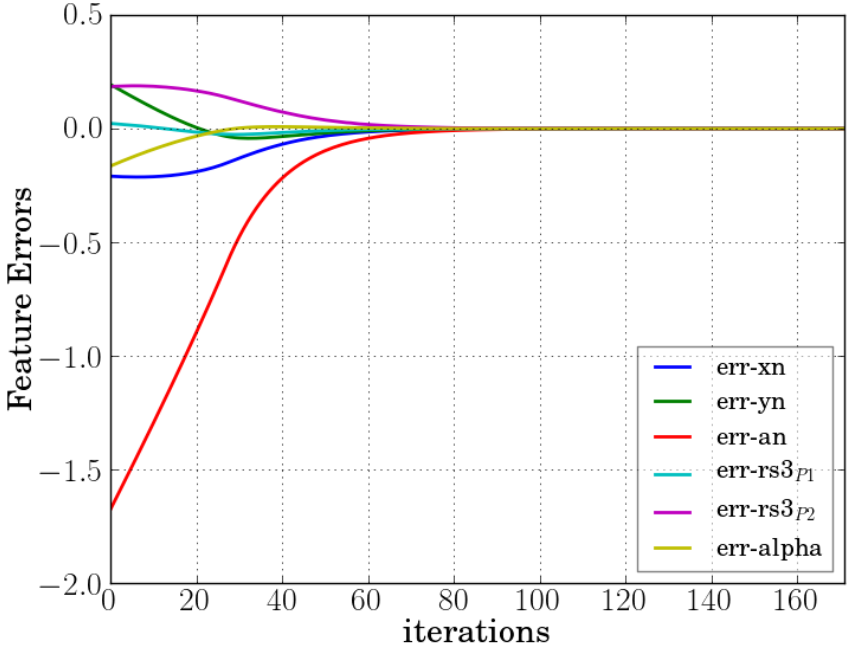
Tunable visual features | Simulation : Non-parallel reference





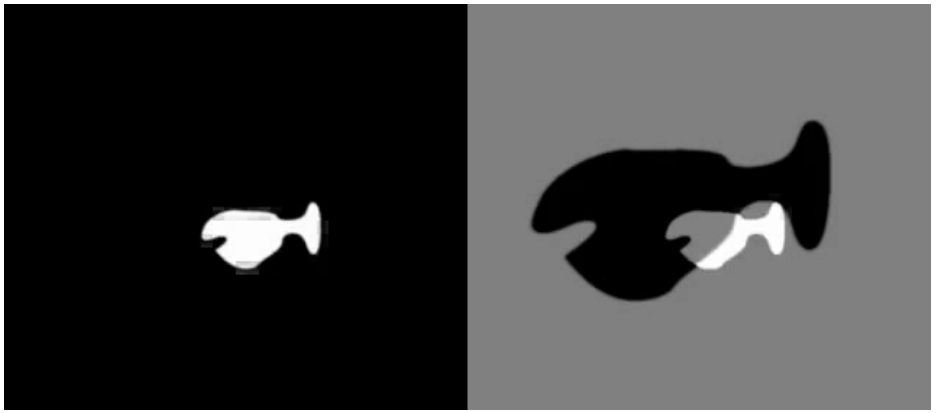


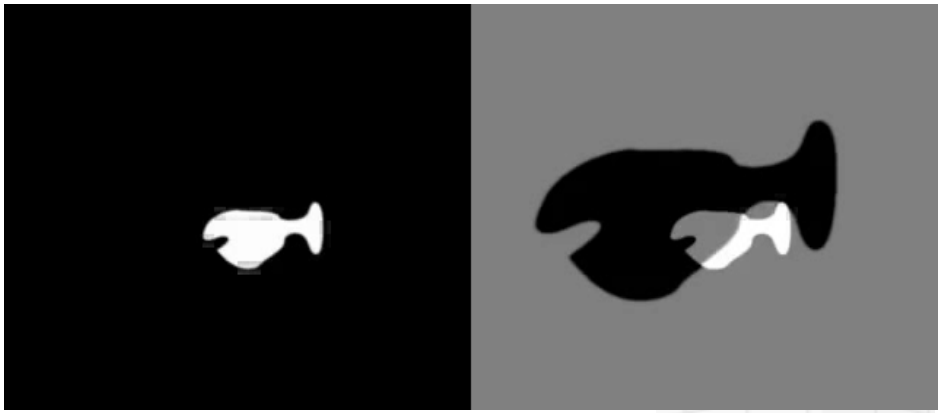
Without selection

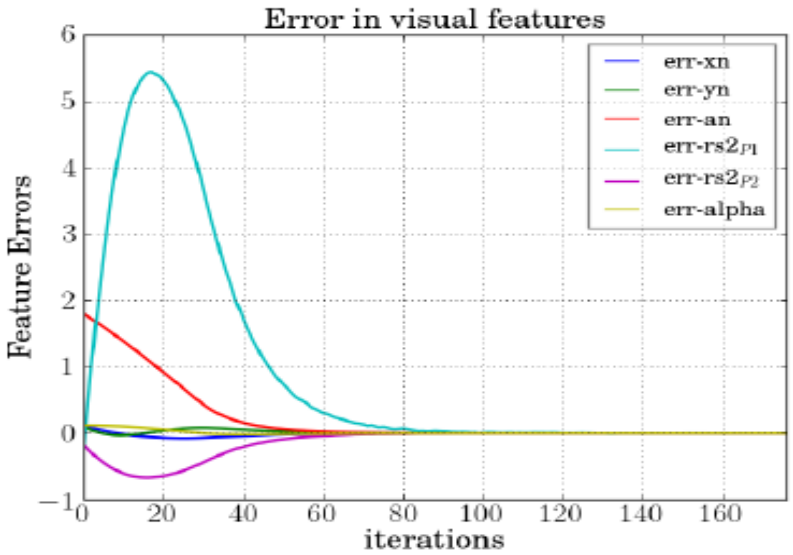


With joint selection

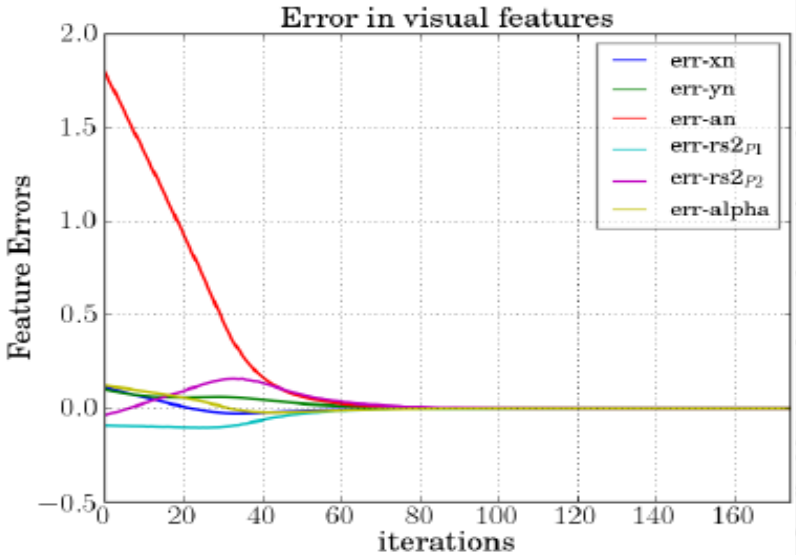
Tunable visual features | Simulation : Non-symmetrical case



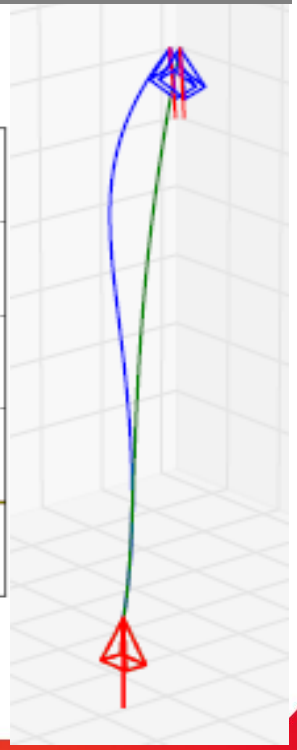




Without selection



With joint selection



Tunable visual features

- While we demonstrated that these selection criteria are useful, it is difficult to pick out a single criteria or a visual feature as the best one for all types of scenes.

- Based on several simulation tests for convergence from different initial conditions and from a comparison of their conditioning with differently shaped objects, rs_3 is favored over the others

Comparison of system conditioning			
Shape	rs_1	rs_2	rs_3
Symmetrical	291.44	125.49	29.04
Non-symmetrical	56.34	373.3	14.407

- Applicable to existing moment-based methods for selecting optimal visual features

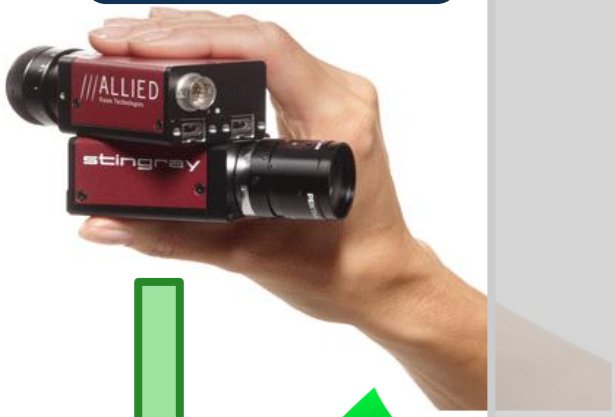
TUNABLE VISUAL FEATURES

PHOTOMETRIC MOMENTS

EXTRANEOUS IMAGE REGIONS

Visual servoing pipeline

Image stream



$I(t)$

Can we avoid these steps?

Measurement process(es)
- Visual tracking
- Image matching
- Image processing ..

Control Law

Interaction Matrix $\mathbf{L}_s$

$\mathbf{s}(t)$

e

$\mathbf{s}^*$

Visual Feature selection

$$e = \mathrm{s}\,(\mathrm{m}(I(t)) - \mathrm{s}\,(\mathrm{m}(I^*))$$

Modelling the variation of Photometric Moments

$$m_{pq} = \iint_{\pi} x^p y^q \, w(\mathbf{x}) \; I(\mathbf{x}, t) \; \mathrm{d}x \, \mathrm{d}y$$

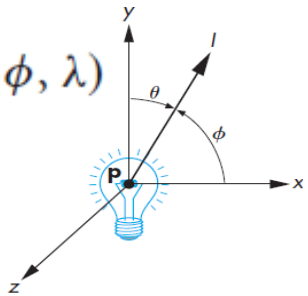
No more restrictions on I

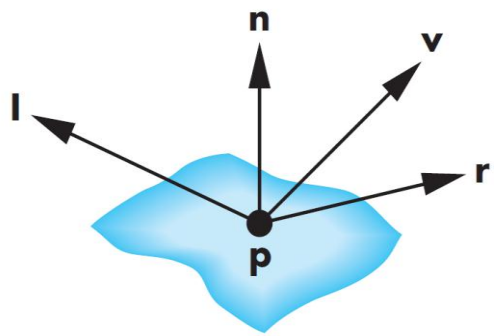
$$\dot{m}_{pq} = \iint_{\pi} x^p y^q \, w(\mathbf{x}) \boxed{\dot{I}(x, y)} \mathrm{d}x \, \mathrm{d}y$$

$$\dot{m}_{pq} = \mathbf{L}_{m_{pq}} \, \mathbf{v}_c$$

- Model the intensity variations $I = \frac{1}{a + bd + cd^2}(k_\mathrm{d} L_\mathrm{d} \max(\mathbf{l} \cdot \mathbf{n}, 0) + k_\mathrm{s} L_\mathrm{s} \max((\mathbf{r} \cdot \mathbf{v})^\alpha, 0)) + k_\mathrm{a} L_\mathrm{a}$

$I(x, y, z, \theta, \phi, \lambda)$





[Engel, Shreiner 2011]

- Intensity is a result of light and its interaction with surfaces in the scene
  - Diffuse, specular and ambient terms [Collewet, Marchand ICRA'09]
  - Light sources, direction and attentuation models
  - Models grows complex and cumbersome measurements for variables

Modelling the variation of Photometric Moments

- Use the classic brightness constancy hypothesis [Horn, Schunck 1980]

$$I(\mathbf{x}+\delta\mathbf{x}, t+\delta t) = I(\mathbf{x}, t)$$

$$\nabla\mathbf{I}^\top \dot{\mathbf{x}} + \dot{I} = 0$$

$$\dot{I}(x,y) = -\nabla\mathbf{I}^\top \dot{\mathbf{x}} = -\mathbf{L}_I^\top \dot{\mathbf{x}}$$

- Substituting this relation into $\mathbf{L}_{m_{pq}} = \iint_\pi x^p y^q \, w(\mathbf{x}) \, \mathbf{L}_I \, \mathrm{d}x \, \mathrm{d}y$

$$\mathbf{L}_{m_{pq}}^\top = \begin{bmatrix} \boxed{\iint_\pi x^p y^q \, w(\mathbf{x}) \frac{\partial I}{\partial x}} (Ax+By+C) \, \mathrm{d}x \, \mathrm{d}y \\ \iint_\pi x^p y^q \, w(\mathbf{x}) \frac{\partial I}{\partial y} (Ax+By+C) \, \mathrm{d}x \, \mathrm{d}y \\ \iint_\pi x^p y^q \, w(\mathbf{x}) \, (-x\frac{\partial I}{\partial x} - y\frac{\partial I}{\partial y})(Ax+By+C) \, \mathrm{d}x \, \mathrm{d}y \\ \iint_\pi x^p y^q \, w(\mathbf{x}) \, (-xy\frac{\partial I}{\partial x} - (1+y^2)\frac{\partial I}{\partial y}) \, \mathrm{d}x \, \mathrm{d}y \\ \iint_\pi x^p y^q \, w(\mathbf{x}) \, ((1+x^2)\frac{\partial I}{\partial x} + xy\frac{\partial I}{\partial y}) \, \mathrm{d}x \, \mathrm{d}y \\ \iint_\pi x^p y^q \, w(\mathbf{x}) \, (x\frac{\partial I}{\partial y} - y\frac{\partial I}{\partial x}) \, \mathrm{d}x \, \mathrm{d}y \end{bmatrix}$$

Modelling the variation of Photometric Moments

- Pick the common terms for simplification and compact notation

$$m_{pq}^{\nabla x} = \iint_{\pi} x^p y^q \, w(\mathbf{x}) \, \frac{\partial I}{\partial x} \, \mathrm{d}x \, \mathrm{d}y$$

$$m_{pq}^{\nabla y} = \iint_{\pi} x^p y^q \, w(\mathbf{x}) \, \frac{\partial I}{\partial y} \, \mathrm{d}x \, \mathrm{d}y$$

- If we let $Q = x^p \, y^q \, w(\mathbf{x}) \, I(\mathbf{x})$ and $P = 0$ in Green's Theorem,

$$\boxed{\iint_{\pi}} \left[p \, x^{p-1} \, y^q \, w(\mathbf{x}) I(\mathbf{x}) + x^p y^q \, \frac{\partial w(x,y)}{\partial x} \, I(\mathbf{x}) + x^p y^q \, w(\mathbf{x}) \, \frac{\partial I(x,y)}{\partial x} \right] \mathrm{d}x\mathrm{d}y = \boxed{\oint_{\partial\pi}} x^p \, y^q \, w(\mathbf{x}) \, I(\mathbf{x}) \mathrm{d}y$$

$$m_{pq}^{\nabla x} = - \iint_{\pi} \left(p \, x^{p-1} \, y^q \, w(\mathbf{x}) I(\mathbf{x}) + x^p \, y^q \, \frac{\partial w(x,y)}{\partial x} I(\mathbf{x}) \right) \mathrm{d}x \, \mathrm{d}y + \oint_{\partial\pi} x^p y^q w(\mathbf{x}) I(\mathbf{x}) \mathrm{d}y$$

$$m_{pq}^{\nabla x} = -p \, m_{p-1,q} - \iint_{\pi} x^p \, y^q \, \frac{\partial w(x,y)}{\partial x} I(\mathbf{x}) \, \mathrm{d}x \, \mathrm{d}y + \oint_{\partial\pi} x^p \, y^q \, w(x,y) \, I(x,y) \, \mathrm{d}y$$

$$m_{pq}^{\nabla y} = -q \, m_{p,q-1} - \iint_{\pi} x^p \, y^q \, \frac{\partial w(x,y)}{\partial y} I(\mathbf{x}) \, \mathrm{d}x \, \mathrm{d}y - \oint_{\partial\pi} x^p \, y^q \, w(x,y) \, I(x,y) \, \mathrm{d}x$$

- The only image processing step in raw luminance method is eliminated

Uniformly weighted photometric moments

- Equally weight all the measured intensities on the image plane

w1 $$w(\mathbf{x}, t) = 1 \, \forall \mathbf{x} \, \forall t$$

- The spatial gradients of $w(\mathbf{x}, t)$ vanish and we get

$$m_{pq}^{\nabla x} = -p \, m_{p-1,q} + \oint_{\partial\pi} x^p \, y^q \, I(x, y) \, \mathrm{d}y$$

$$m_{pq}^{\nabla y} = -q \, m_{p,q-1} - \oint_{\partial\pi} x^p \, y^q \, I(x, y) \, \mathrm{d}x$$

- Introducing zero image border assumption
  - facilitated by having images with a black background
- Mild violations of the ZBA are tolerable, will be removed subsequently

Uniformly weighted photometric moments

- We obtain the interaction matrix of the uniformly weighted moments

$$
\begin{aligned}
L^{v_x}_{\mathtt{m_{pq}}} &= -A(p+1)m_{pq} - Bpm_{p-1,q+1} - Cpm_{p-1,q} \\
L^{v_y}_{\mathtt{m_{pq}}} &= -Aqm_{p+1,q-1} - B(q+1)m_{p,q} - Cqm_{p,q-1} \\
L^{v_z}_{\mathtt{m_{pq}}} &= A\,(p+q+3)\,m_{p+1,q} \\
&\quad + B(p+q+3)\,m_{p,q+1} + C(p+q+2)\,m_{pq} \\
L^{\omega_x}_{\mathtt{m_{pq}}} &= q\,m_{p,q-1} + (p+q+3)\,m_{p,q+1} \\
L^{\omega_y}_{\mathtt{m_{pq}}} &= -p\,m_{p-1,q} - (p+q+3)\,m_{p+1,q} \\
L^{\omega_z}_{\mathtt{m_{pq}}} &= p\,m_{p-1,q+1} - q\,m_{p+1,q-1}
\end{aligned}
$$

$\boxed{\mathbf{L}_1}$

- Computation of the interaction matrix requires order+1 moments
- Different methodology but consistent with results in state-of-the-art
  - Binary moments (widely adopted) [Chaumette, T-RO'04]
  - Gâteaux derivatives method [Tamtsia, PhD'14]
- Previously established results based on invariance properties are valid

Experimental Methodology

- We use the classical control law $\mathbf{v}_c = -\lambda \widehat{\mathbf{L}}_{\mathbf{s}}^{-1}(\mathbf{s} - \mathbf{s}^*)$

- For 4dof, we adopt the below visual features [Tahri et al., T-RO'05]

$$\mathrm{s} = (x_n, y_n, a_n, \alpha) \;\Big|\; x_n = x_g\, a_n \;\Big|\; y_n = y_g\, a_n \;\Big|\; a_n = Z^* \frac{\sqrt{a^*}}{a} \;\Big|\; \alpha = \frac{1}{2}\arctan\left(\frac{2\mu_{11}}{\mu_{20} - \mu_{02}}\right)$$

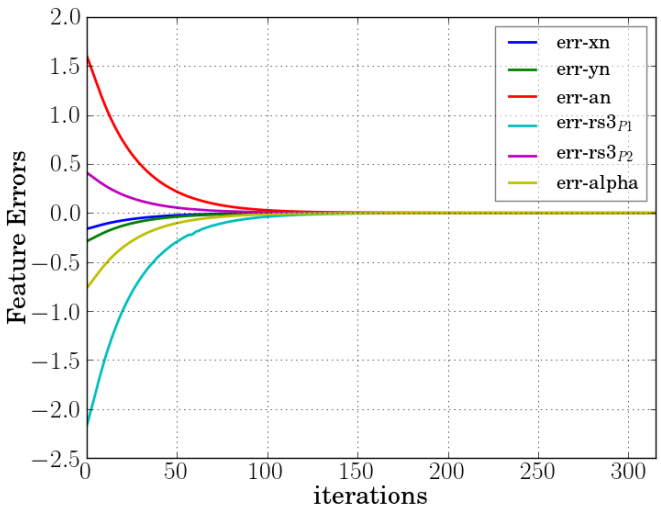
  - The same motion subset treated by KBVS
  - There are industrial-grade tasks which require this subset of motions
  - Simplified step to test the control schemes and assumptions before 6dof

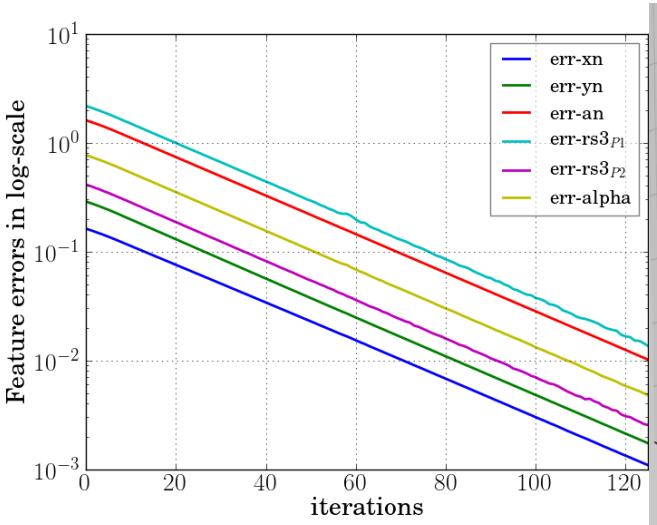
- For 6dof, for the control of rotations around image axes, $rs_3 = \dfrac{Is_3}{Is_2}$

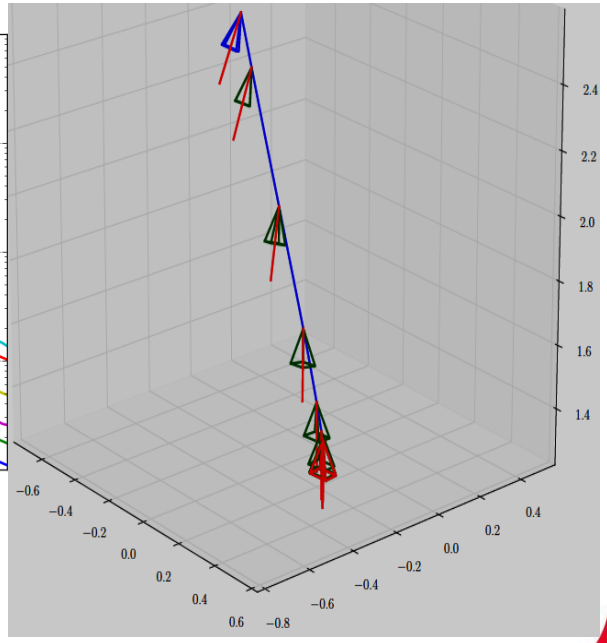
  - computed with respect to 2 shift points as in [Tamtsia et al, ICRA'13]

- For comparison with pure luminance, we use $\mathbf{v}_c = -\lambda \widehat{\mathbf{L}}_{\mathbf{I}}^{+}(\mathbf{I} - \mathbf{I}^*)$
- No image processing, filtering or tracking but only a simple computation of moments on the image plane.
- Experiments with different planar targets. Representative ones are shown
- No GPU

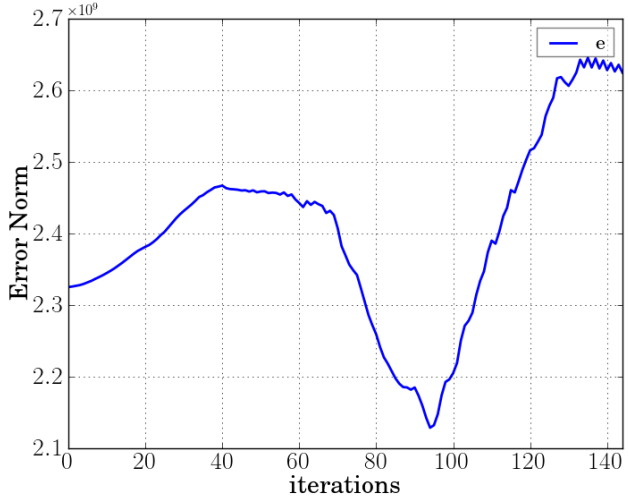
Simulation Results | Modelling Validation 6dof

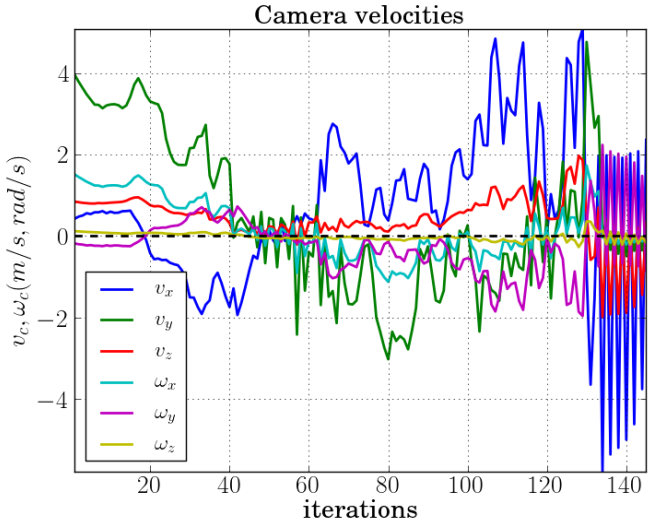


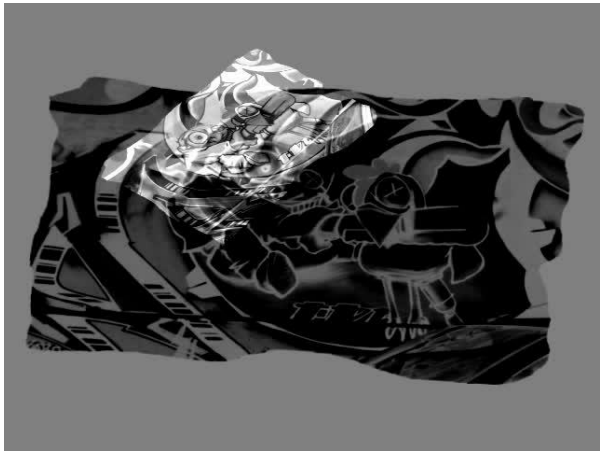


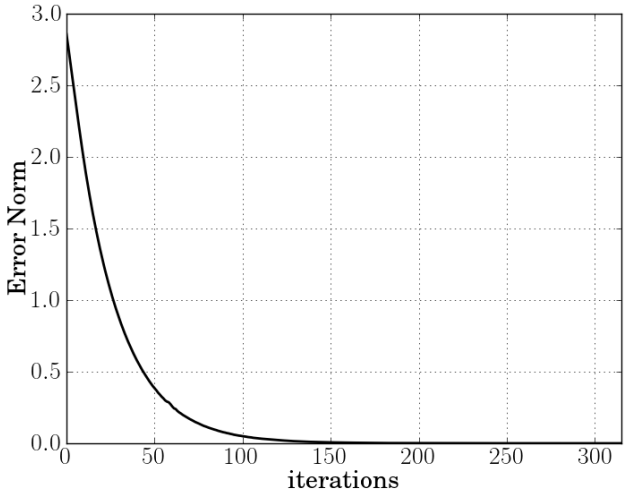


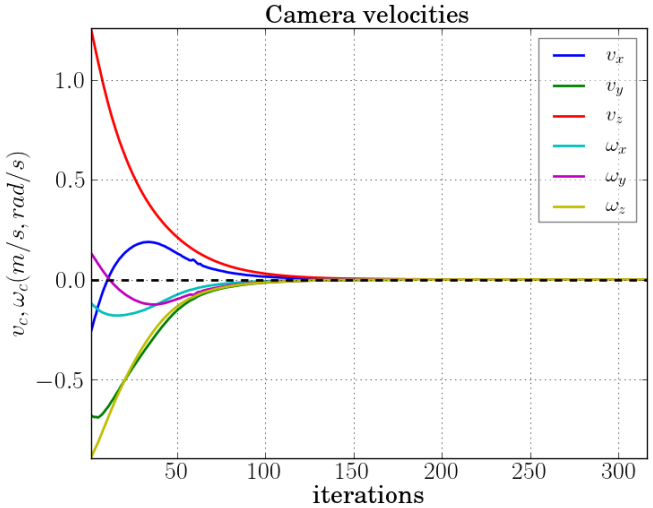
Comparison to raw luminance | 6dof

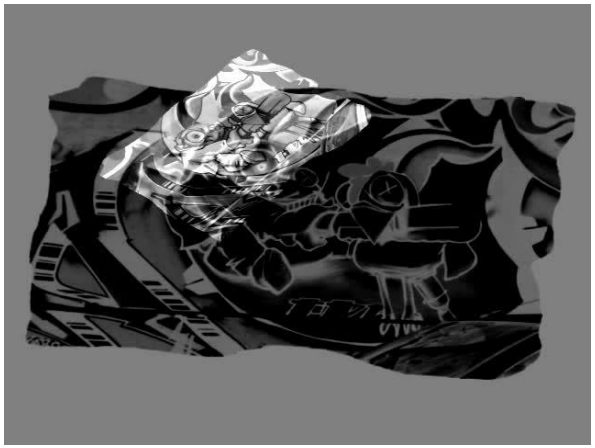




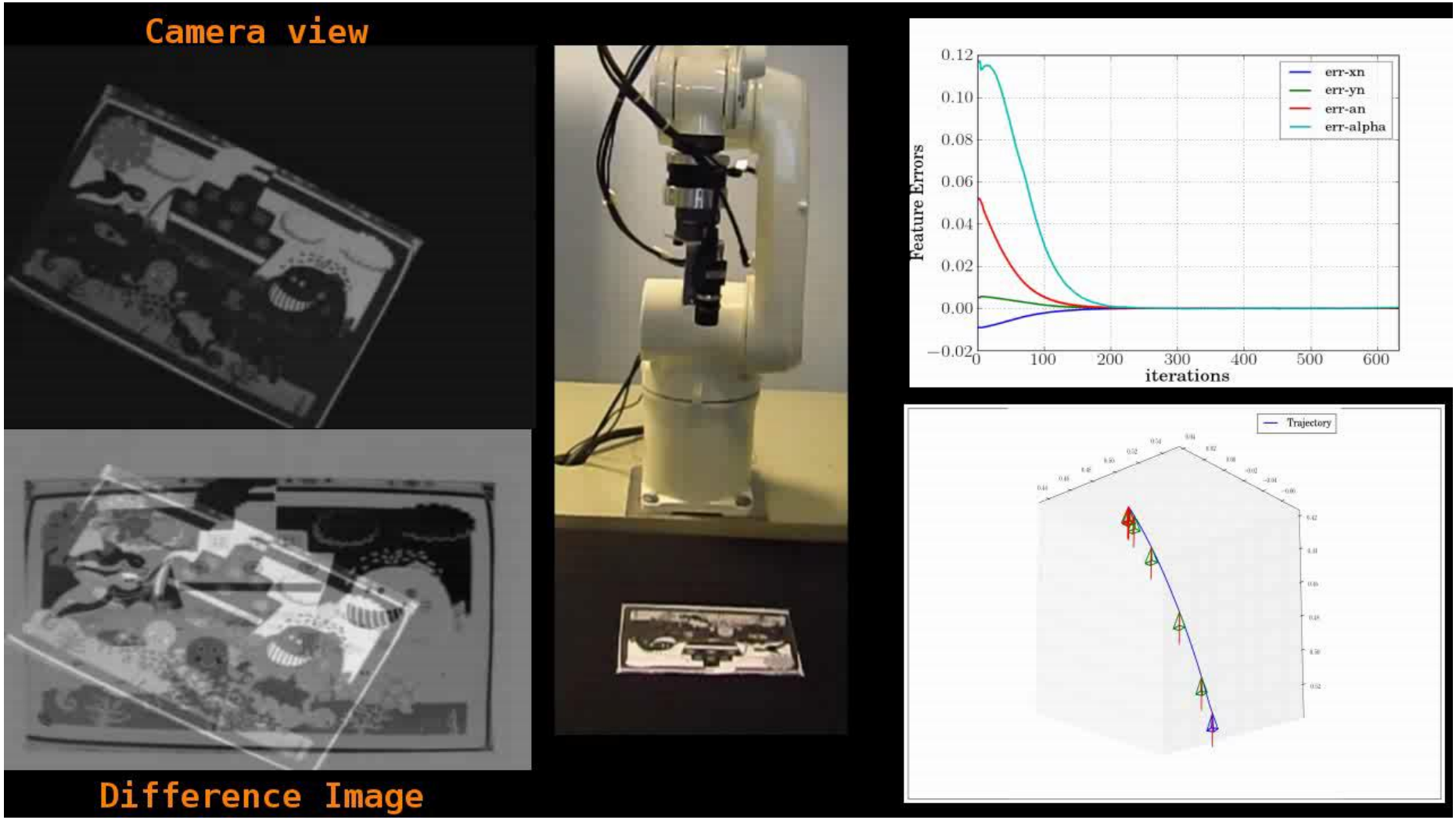




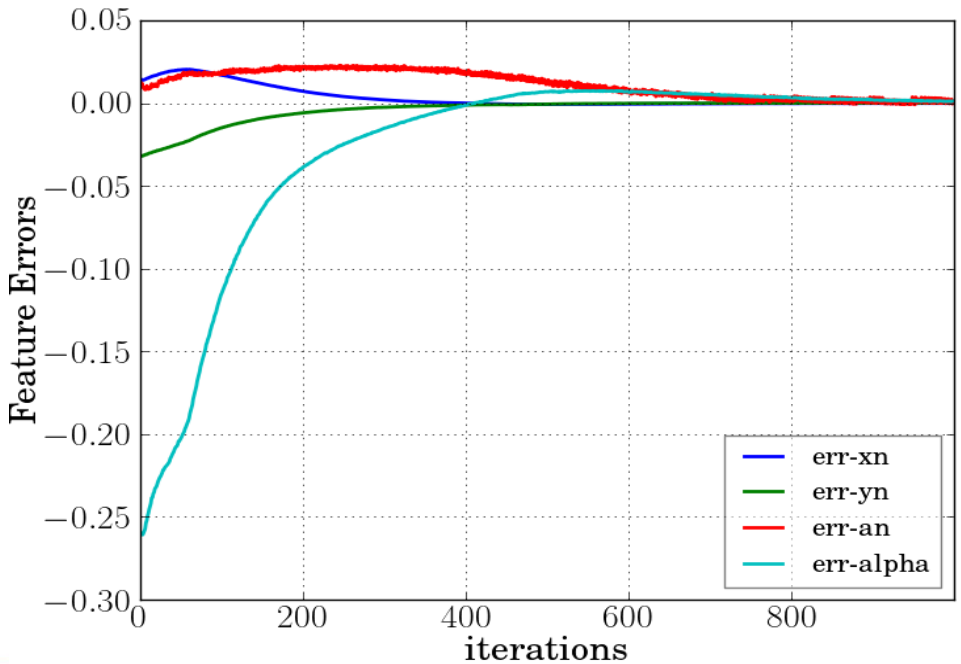


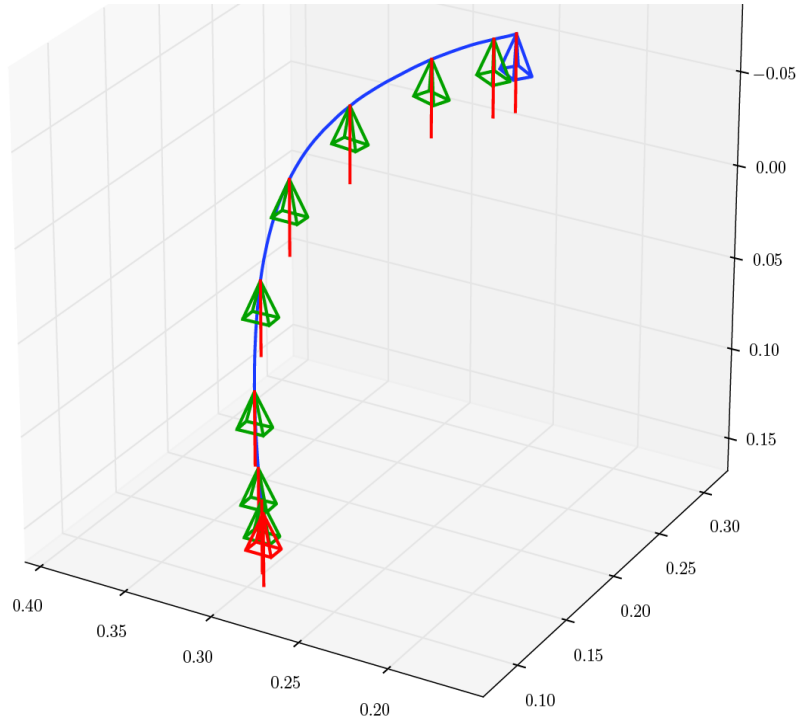


Experimental Results | SCARA motion



Experimental results | SCARA motion with violations





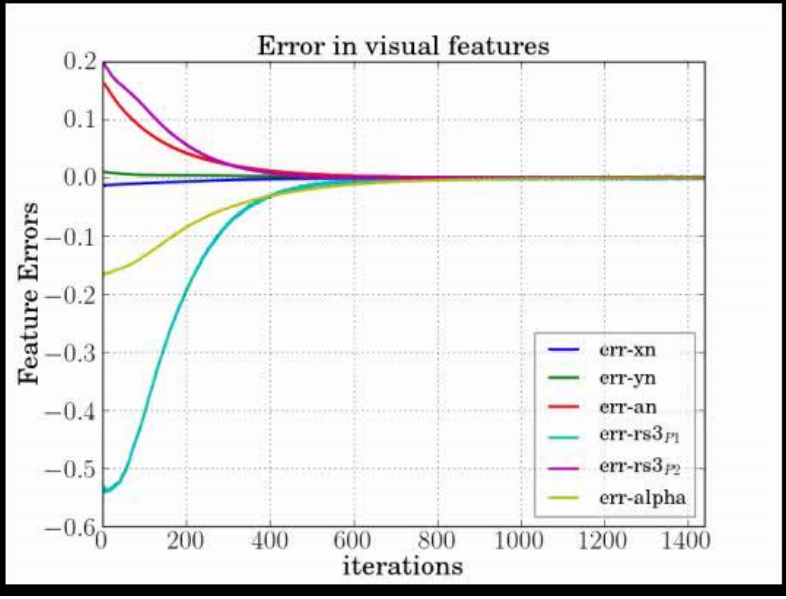
Experimental Results | 6dof

- No image processing allowing violation in the ZBA assumption

- Assumptions in modelling allowed us to exploit existing results based on invariance and the following decoupled interaction matrix is obtained

$$L_s(s=s^*) = \begin{bmatrix} -1 & 0 & 0 & 0.0037 & -0.9234 & -0.0202 \\ 0 & -1 & 0 & 0.8677 & -0.0037 & -0.0098 \\ 0 & 0 & -1 & 0.0303 & 0.0148 & 0 \\ 0 & 0 & 0 & -0.9580 & -3.0864 & 0 \\ 0 & 0 & 0 & 0.4925 & 0.1484 & 0 \\ 0 & 0 & 0 & 0.0552 & -0.0071 & -1 \end{bmatrix} \qquad \kappa(L_s) = 10.84$$

Experimental Results | 6dof



$[-0.56mm, -0.08mm, 0.14mm]$ $[-0.01°, 0.04°, -0.03°]$

Contributions

Problem of extraneous image regions

I

I*

- In dense methods, VS is affected by the appearance and disappearance of portions of the scene from the camera field of view [Chaumette, T-RO 2004]

$$\dot{\mathbf{s}} = \mathbf{L_s}\mathbf{v}_c \quad \dot{\mathbf{s}} = (\mathbf{L_s} + \epsilon)\mathbf{v}_c$$

This unmodelled disturbance affects the control and leads to failure

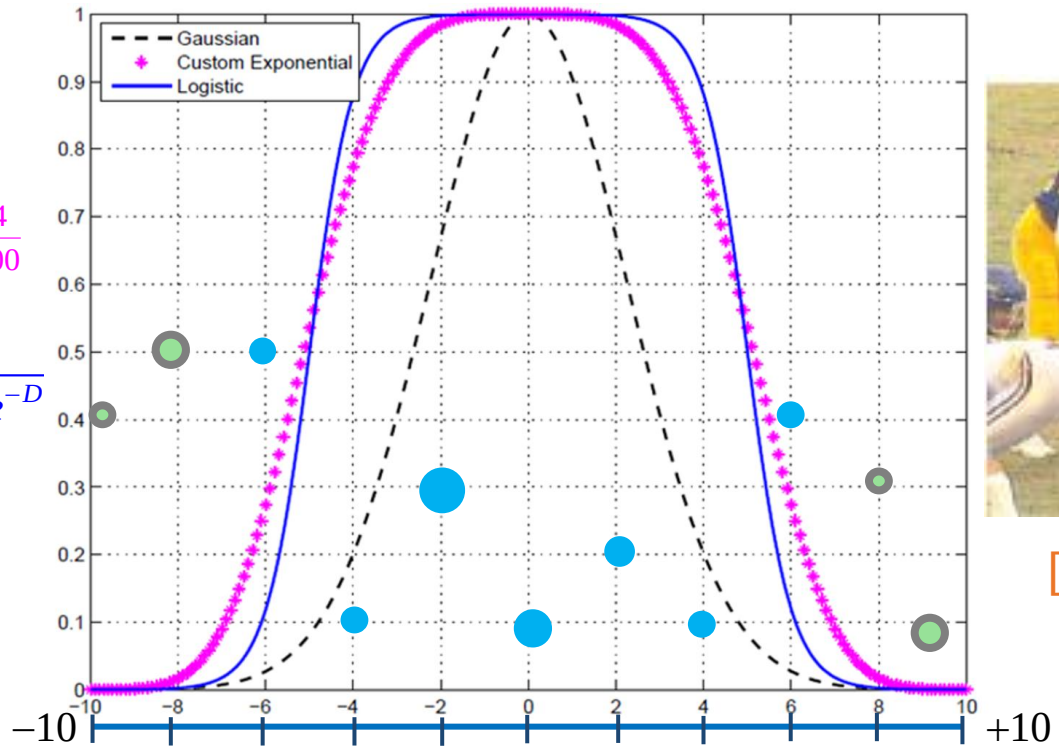
- What is sought is immunity to the effects of the uncommon image parts between $I(t)$ and $I(t+n)$

An enhanced modelling scheme

$$g(x) = e^{-\frac{x2}{10}}$$

$$c(x) = e^{-\frac{x4}{1000}}$$

$$l(x) = \frac{A}{B + C\,e^{-D}}$$



[Comaniciu, T-PAMI'03]

w2

The key idea is to assign weights such that the highest priority is central portions of the image, smoothly decreasing to null at the periphery

$$m_{pq} = \iint_{\pi} x^p y^q \boxed{w(\mathbf{x})} I(\mathbf{x}, t)\, \mathrm{d}x\, \mathrm{d}y$$

An enhanced modelling scheme | Weighting strategy

- If a smooth decrease to 0 can be obtained, $w(x,y)=0\forall(x,y)\in\partial\pi$

$$m_{pq}^{\nabla x} = -p\, m_{p-1,q} - \iint_{\pi} x^p\, y^q\, \frac{\partial w(x,y)}{\partial x} I(\mathbf{x})\, \mathrm{d}x\, \mathrm{d}y + \oint_{\partial\pi} x^p\, y^q\, w(x,y)\, I(x,y)\, \mathrm{d}y$$

$$m_{pq}^{\nabla y} = -q\, m_{p,q-1} - \iint_{\pi} x^p\, y^q\, \frac{\partial w(x,y)}{\partial y} I(\mathbf{x})\, \mathrm{d}x\, \mathrm{d}y - \oint_{\partial\pi} x^p\, y^q\, w(x,y)\, I(x,y)\, \mathrm{d}x$$

  - With this strategy, the earlier zero border assumption is eliminated

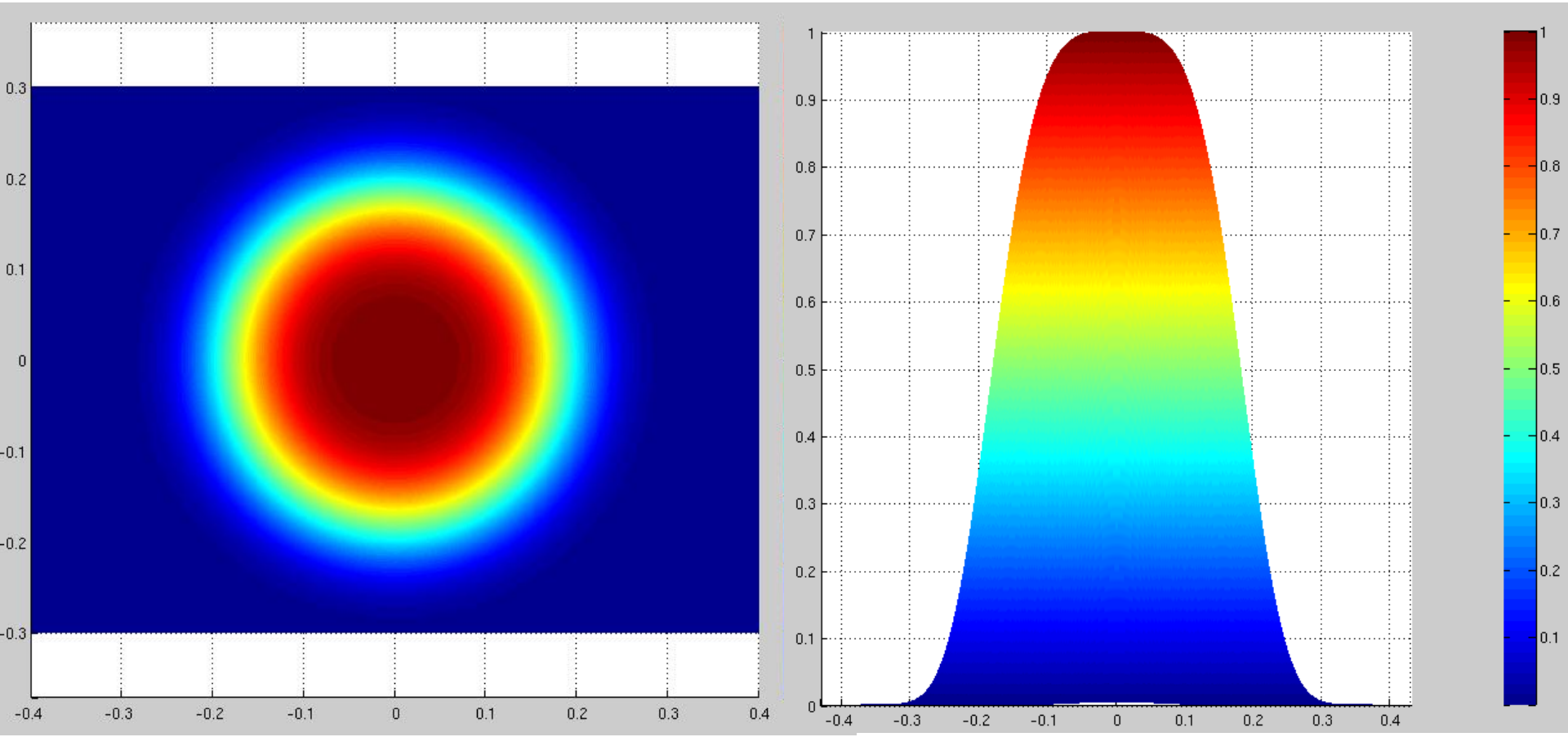
- The weighting function can be chosen such that the interaction matrix can be obtained in closed-form as functions of the weighted moments [ICRA'15]
  - For this, the following general family of functions was proposed

$$\mathcal{F}(x) = \exp^{-p(x)} \text{ with}$$
$$p(x) = a_0 + a_1 x + \frac{1}{2}a_2 x^2 + \frac{1}{3}a_3 x^3 + \ldots + \frac{1}{n}a_n x^n$$

$$\mathcal{F}'(x) = \exp^{-p(x)}\, p'(x) \quad \text{with}$$
$$p'(x) = a_1 + a_2 x + a_3 x^2 + \ldots + a_n x^{n-1}$$

An enhanced modelling scheme | Weighting strategy

$$w(x,y) = K \exp^{-a\,(x^2+y^2)^2} \qquad K = 1,\ a = 650$$



An enhanced modelling scheme | Interaction Matrix

- For this choice of the weighting function

$$w(x,y) = K \exp^{-a(x^2+y^2)^2}$$

- The spatial derivatives are straight-forward to obtain

$$\frac{\partial w}{\partial x} = -4ax(x^2+y^2)\, w(\mathbf{x})$$
$$\frac{\partial w}{\partial y} = -4ay(x^2+y^2)\, w(\mathbf{x})$$

- By plugging them into the earlier term, we have

$$m_{pq}^{\nabla x} = -p\; m_{p-1,q} + 4a\left(m_{p+3,q} + m_{p+1,q+2}\right)$$
$$m_{pq}^{\nabla y} = -q\; m_{p,q-1} + 4a\left(m_{p,q+3} + m_{p+2,q+1}\right)$$

  - These terms are composed of only the weighted moments

An enhanced modelling scheme | Interaction Matrix

- The interaction matrix is obtained in closed-form in terms of weighted moments

$$
\begin{aligned}
L^{v_x}_{\mathtt{m}_{\mathtt{pq}}} &= L^{v_x}_{1\mathtt{m}_{\mathtt{pq}}} + A\left(4\,a\left(m_{p+4,q} + m_{p+2,q+2}\right)\right) + B\left(4\,a\left(m_{p+3,q+1} + m_{p+1,q+3}\right)\right) \\
&\quad + C\left(4\,a\left(m_{p+3,q} + m_{p+1,q+2}\right)\right) \\
L^{v_y}_{\mathtt{m}_{\mathtt{pq}}} &= L^{v_y}_{1\mathtt{m}_{\mathtt{pq}}} + A\left(4\,a\left(m_{p+3,q+1} + m_{p+1,q+3}\right)\right) + B\left(4\,a\left(m_{p,q+4} + m_{p+2,q+2}\right)\right) \\
&\quad + C\left(4\,a\left(m_{p,q+3} + m_{p+2,q+1}\right)\right) \\
L^{v_z}_{\mathtt{m}_{\mathtt{pq}}} &= L^{v_z}_{1\mathtt{m}_{\mathtt{pq}}} + A[-4\,a\left(m_{p+5,q} + 2m_{p+3,q+2} + m_{p+1,q+4}\right)] \\
&\quad + B[-4\,a\left(m_{p+4,q+1} + 2m_{p+2,q+3} + m_{p,q+5}\right)] \\
&\quad + C[-4\,a\left(m_{p+4,q} + 2m_{p+2,q+2} + m_{p,q+4}\right)] \\
L^{\omega_x}_{\mathtt{m}_{\mathtt{pq}}} &= L^{\omega_x}_{1\mathtt{m}_{\mathtt{pq}}} - 4\,a(m_{p+4,q+1} + 2\,m_{p+2,q+3} + m_{p,q+3} + m_{p+2,q+1} + m_{p,q+5}) \\
L^{\omega_y}_{\mathtt{m}_{\mathtt{pq}}} &= L^{\omega_y}_{1\mathtt{m}_{\mathtt{pq}}} + 4\,a(m_{p+3,q} + m_{p+1,q+2} + m_{p+5,q} + 2\,m_{p+3,q+2} + m_{p+1,q+4}) \\
L^{\omega_z}_{\mathtt{m}_{\mathtt{pq}}} &= L^{\omega_z}_{1\mathtt{m}_{\mathtt{pq}}} = p m_{p-1,q+1} - q m_{p+1,q-1}
\end{aligned}
$$

$$\mathrm{L}_{m_{pq}} = \mathrm{L}_1 + 4a\mathrm{L}_2$$

- L_1 has the same analytical form as for the uniformly weighted moments
- The second term with L_2 is a resultant of the weighting function and determines the maximum order of the moments required
- No new terms in last component, behaviour wrt optic axis rotations unmodified
- Visual features based on translation and scale invariance property are affected

Simulation Results | 4d

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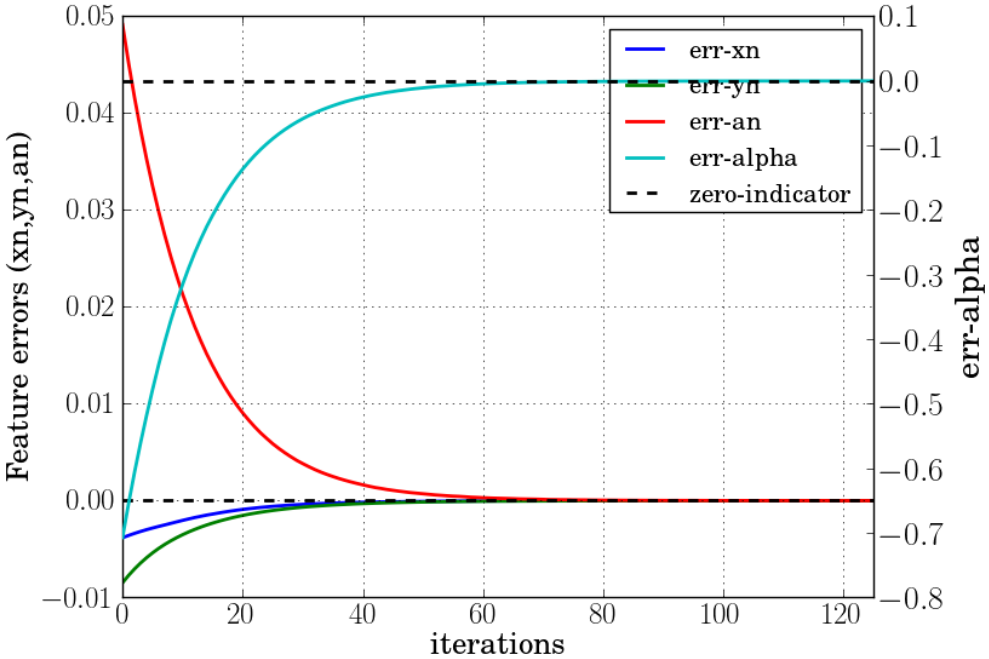

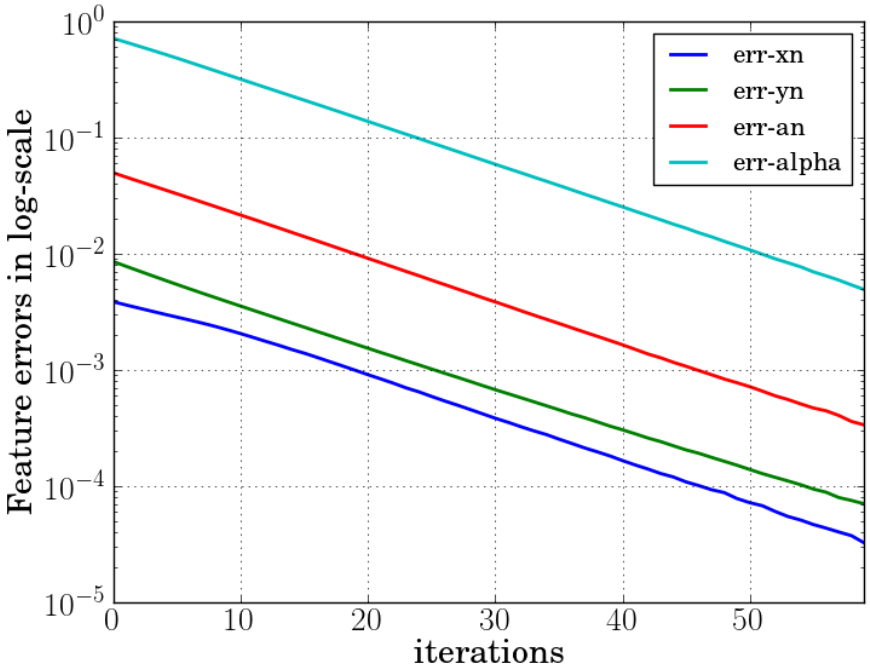

Comparison without weights

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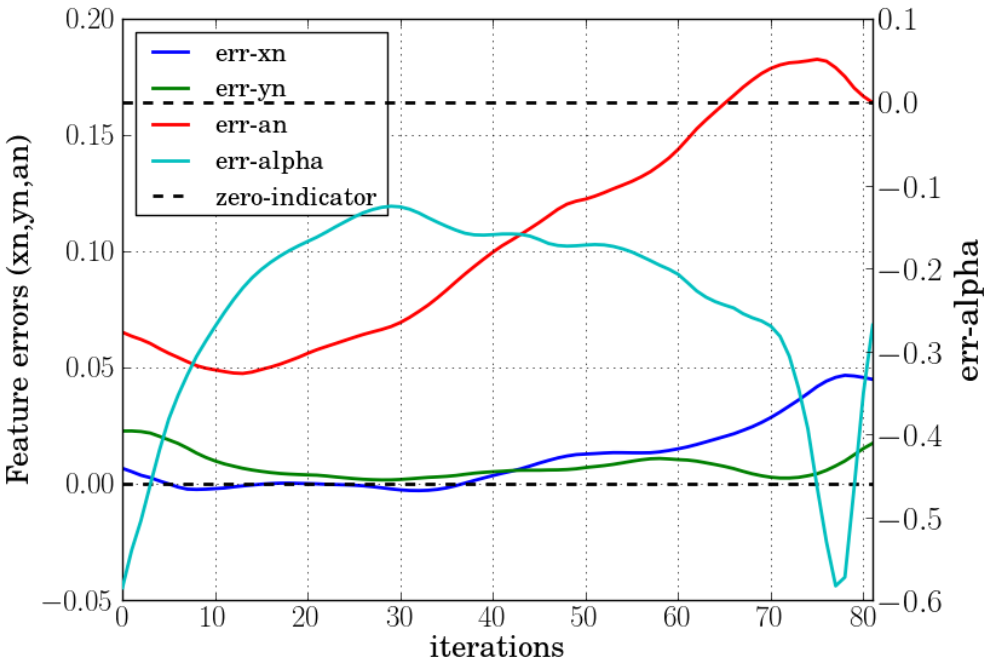
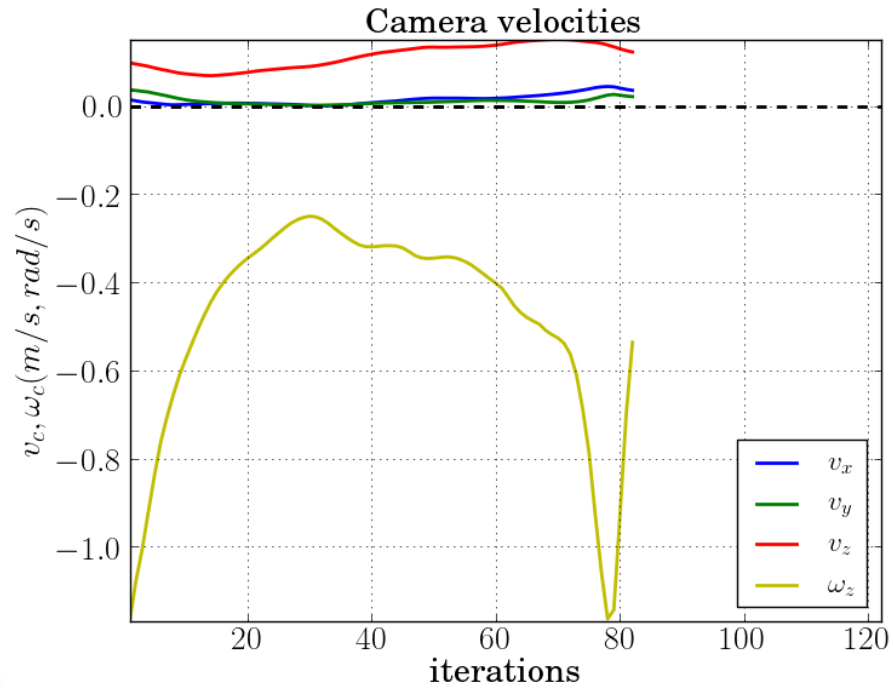
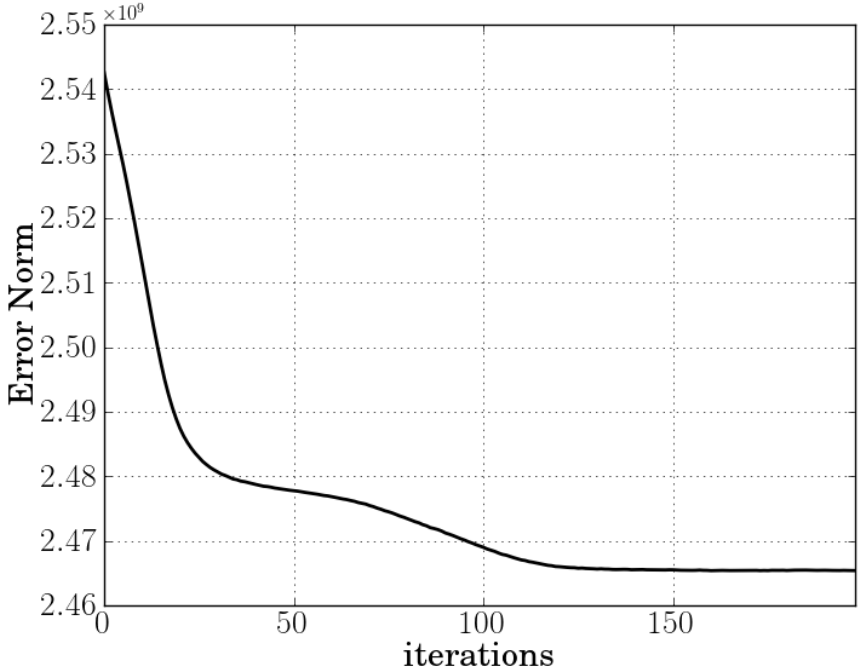
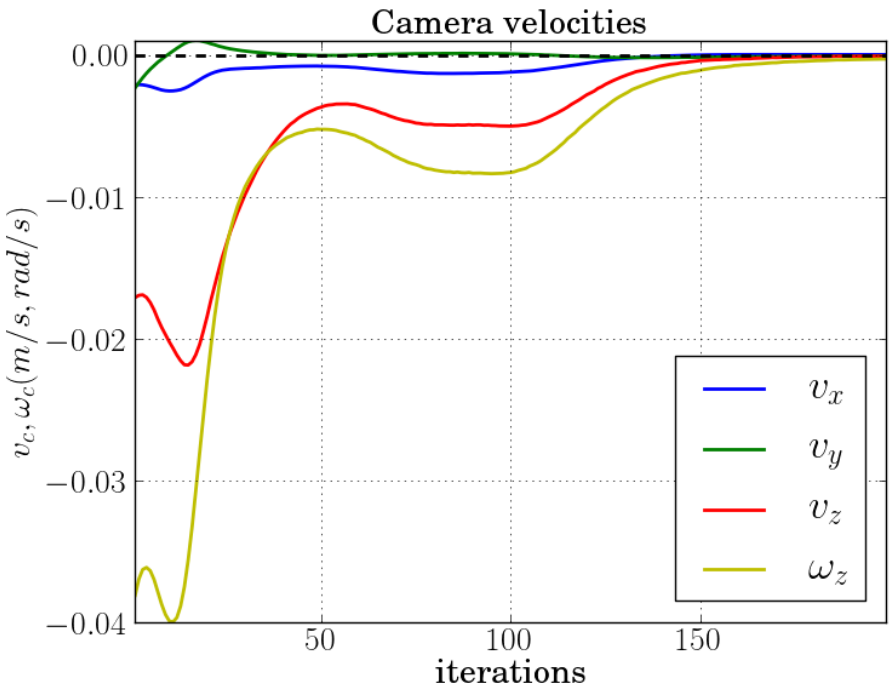




Comparison with pure luminance





Simulation Results | Large rotational motions

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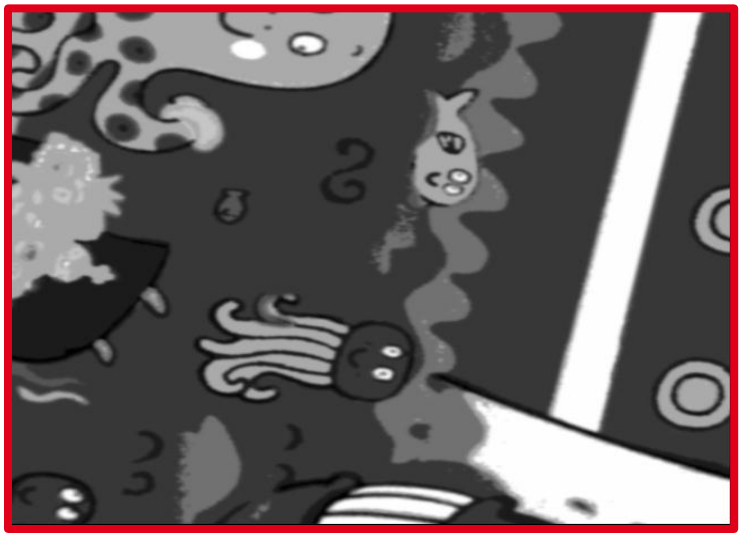

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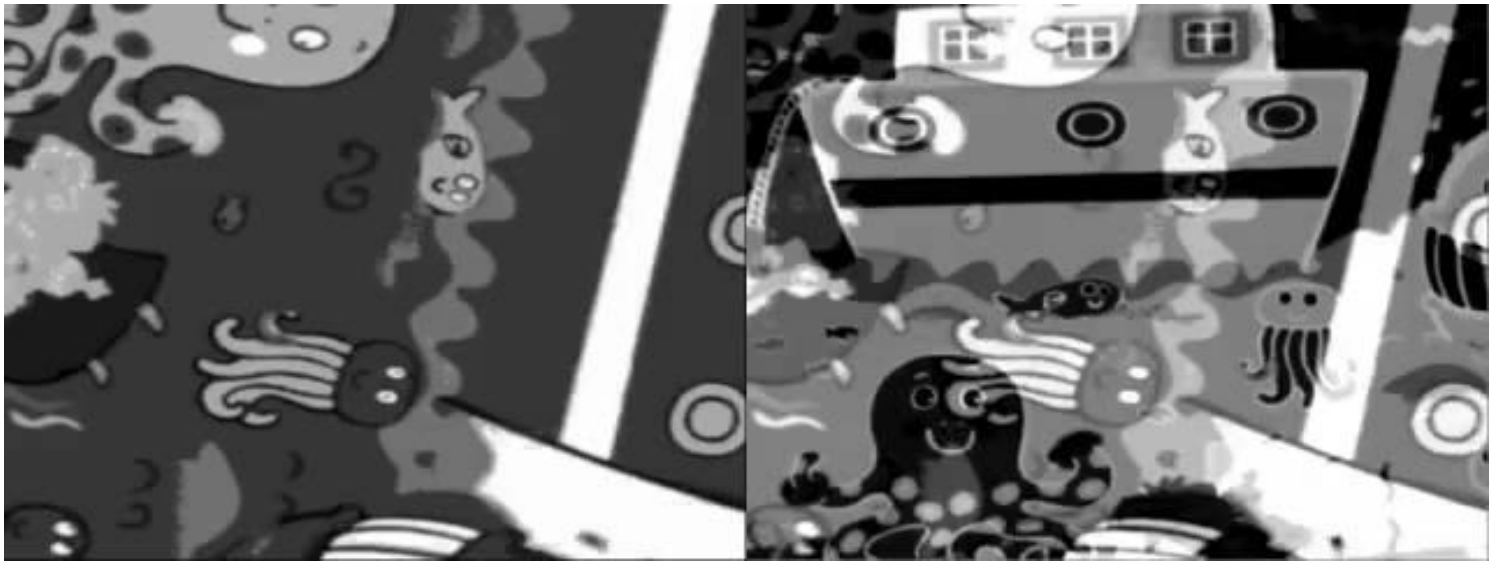
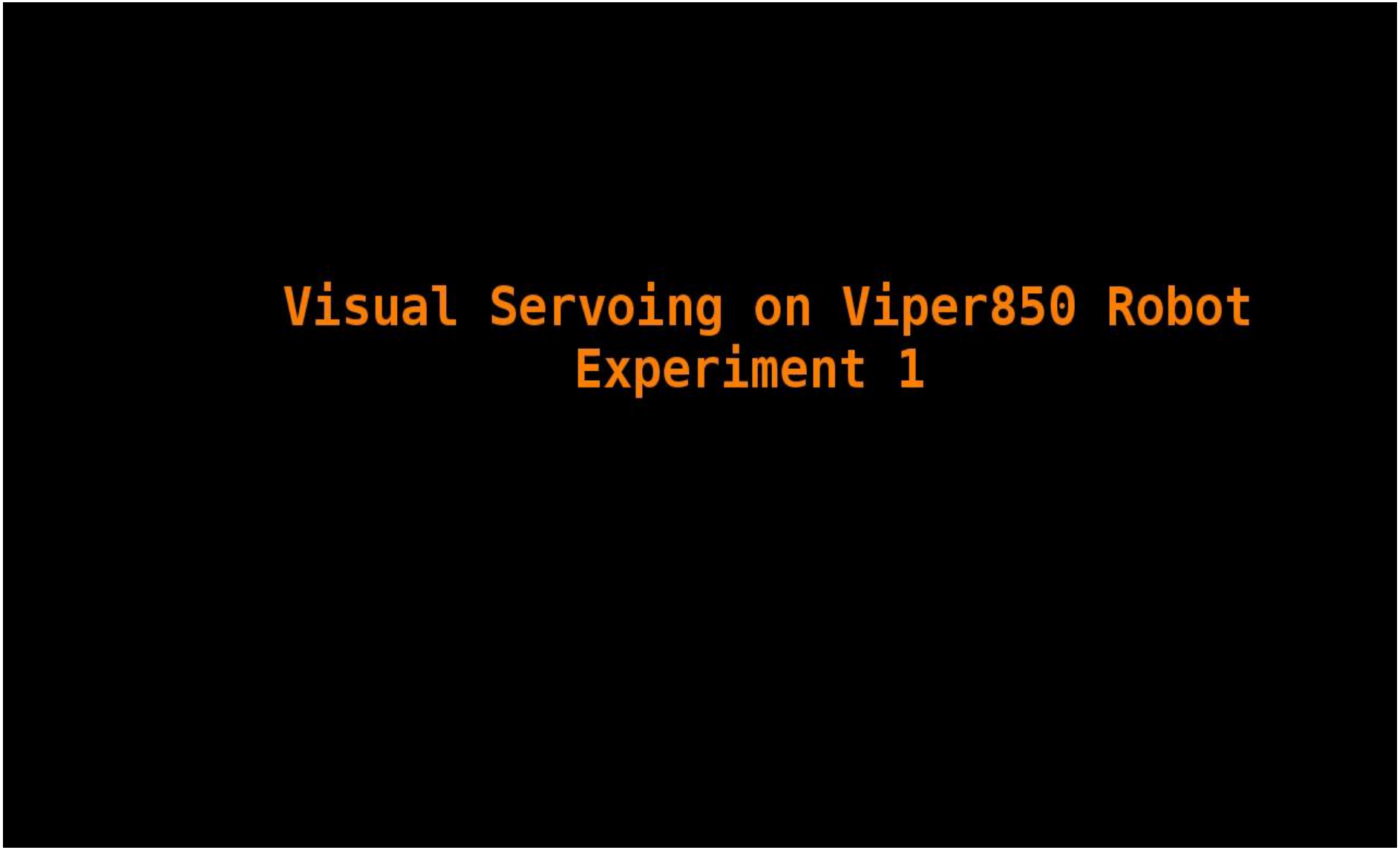


Camera View

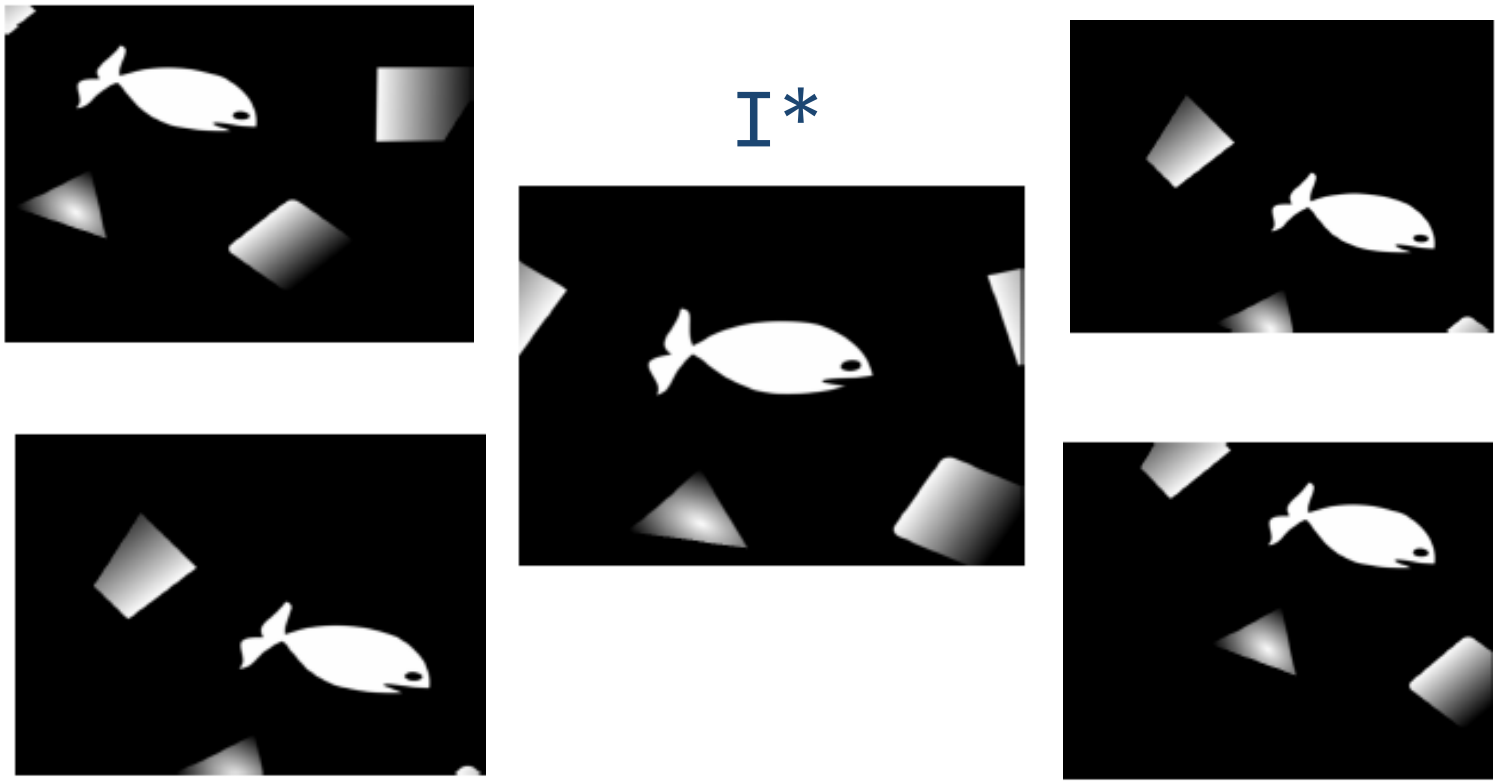
Difference Image

Experimental Results | 4d

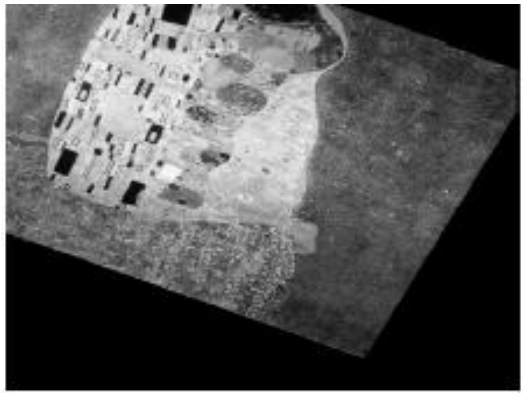


Empirical Convergence Analysis | Synthetic Case

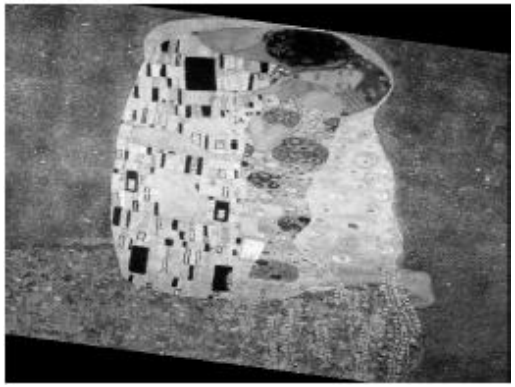
- To empirically compare the convergence with non-weighted moments and pure photometric method
  - 243 different initial poses at 3 different depths
  - Synthetic imagery to simulate varying levels appearance and diappearance phenomenon

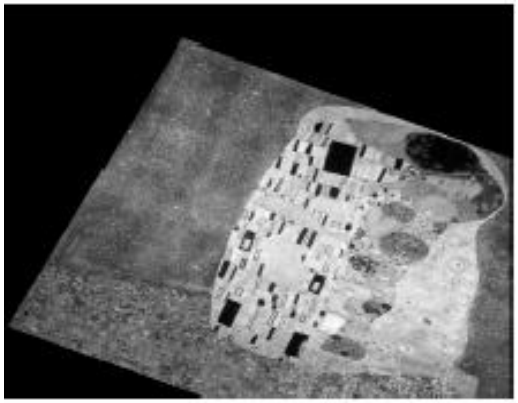


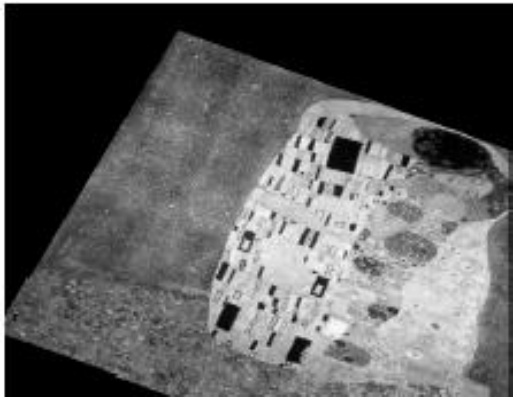
Empirical Convergence Analysis | Textured case

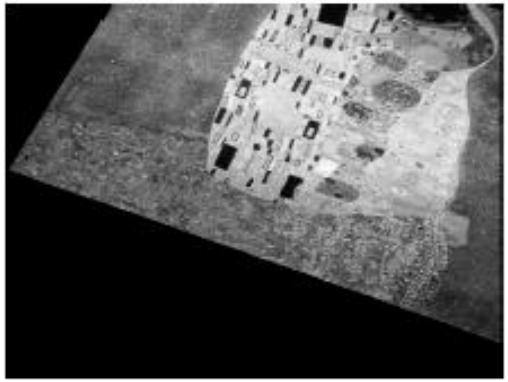


I^*









Empirical Convergence Analysis | Discussion

Synthetic Case			
Exp Set #	Non-weighted	Pure luminance	Weighted moments
1	0%	85.1%	96.29%
2	0%	85.1%	93.82%
3	0%	85.1%	92.59%
Avg.	**0%**	**85.1%**	**96.29%**

Textured Case			
Exp Set #	Non-weighted	Pure luminance	Weighted moments
1	51.85%	0%	100%
2	55.55%	0%	100%
3	60.49%	0%	96.29%
Average	**55.96%**	**0%**	**98.76%**

- The uniformly weighted moments are unable to handle the energy inflow from the periphery of the image
- The proposed weighting strategy equips moments to succeed in such cases
- A small number of failure cases due to the existence of an equilibrium point other than the desired one
- For textured case, failure in pure luminance case more due to unexpected noise in planar model rather than the appearance and disappearance. But it is interesting to note that the moments are unaffected by the same

Simulation Results | 6dof

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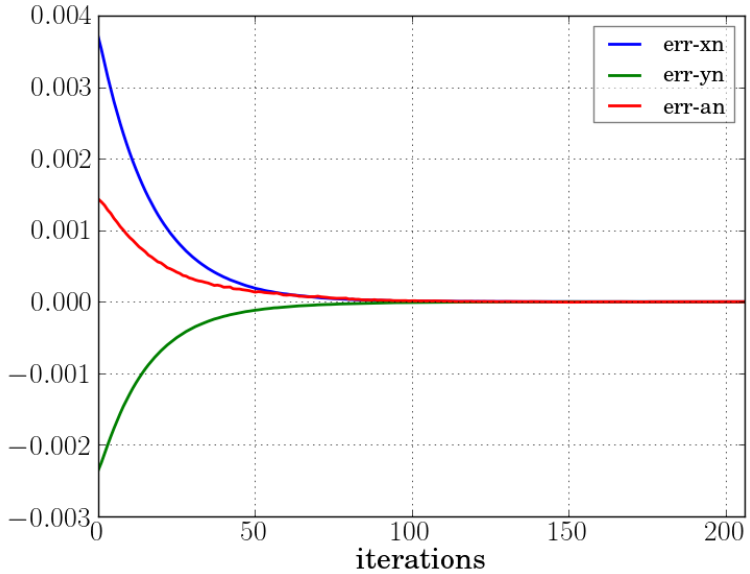
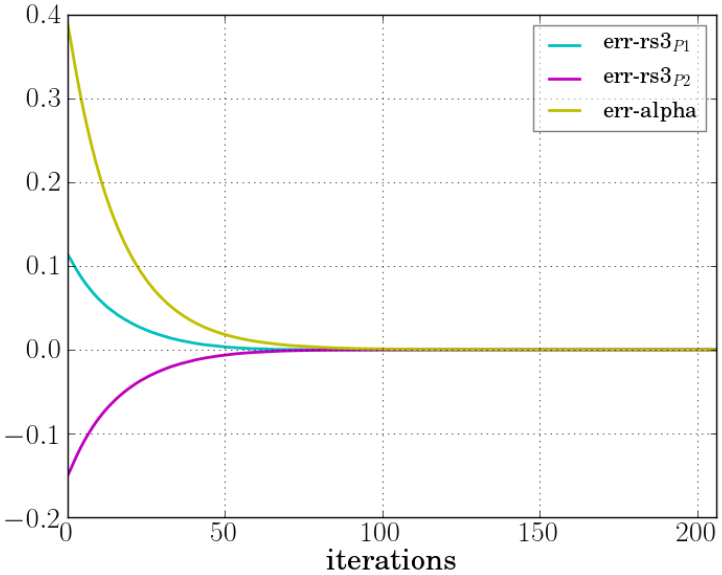


Errors - translation



Errors - rotation

Comparison with non-weighted | 6dof

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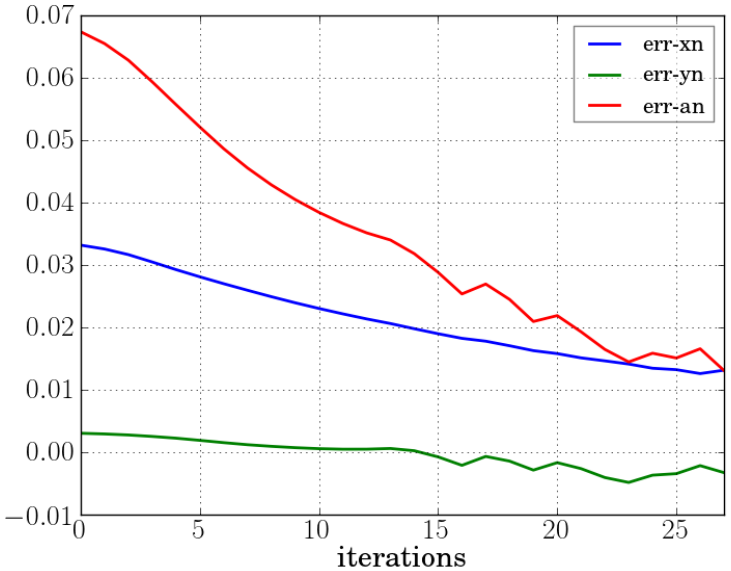
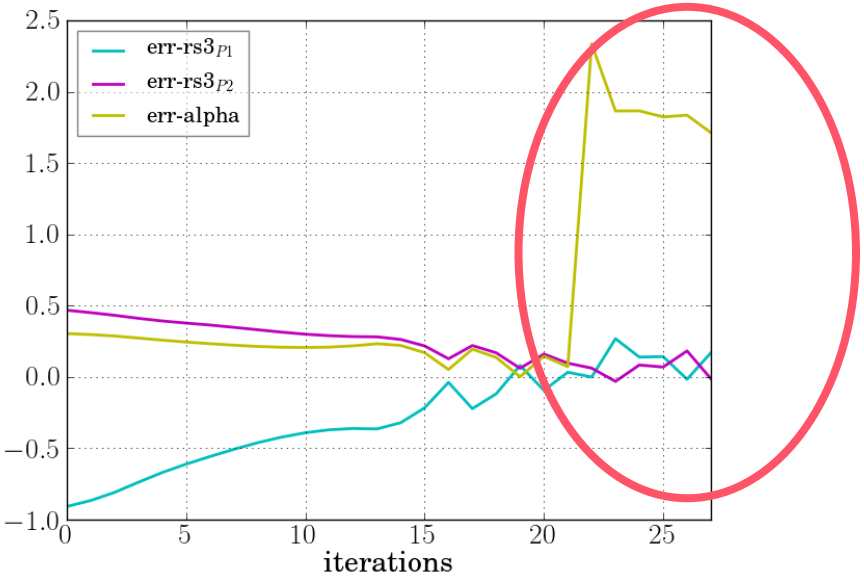


Errors - translation



Errors - rotation

Comparison with raw luminance | 6dof

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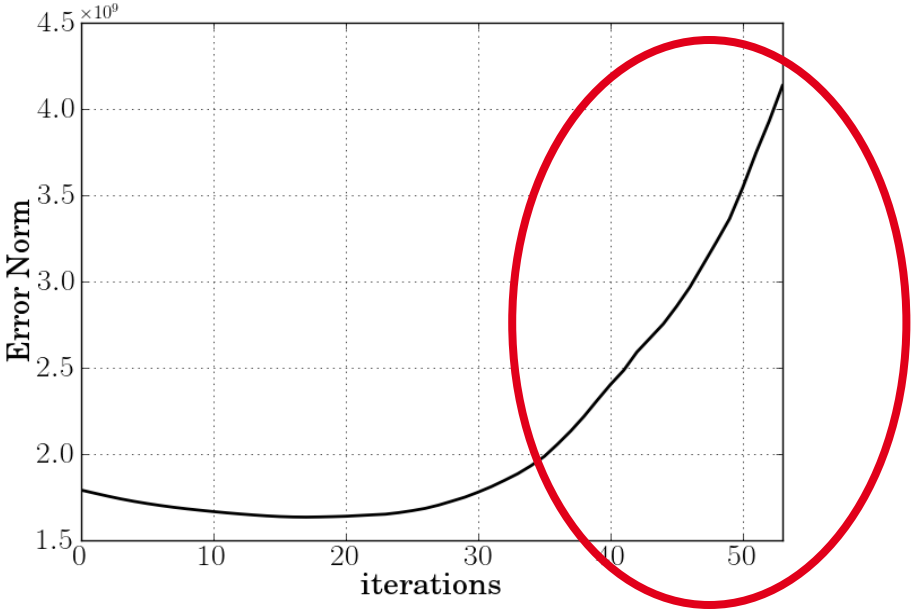
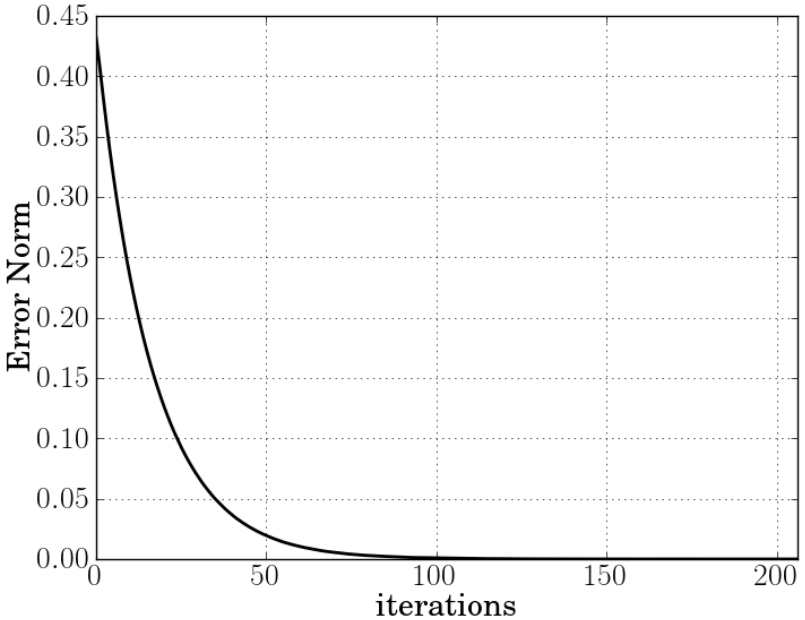
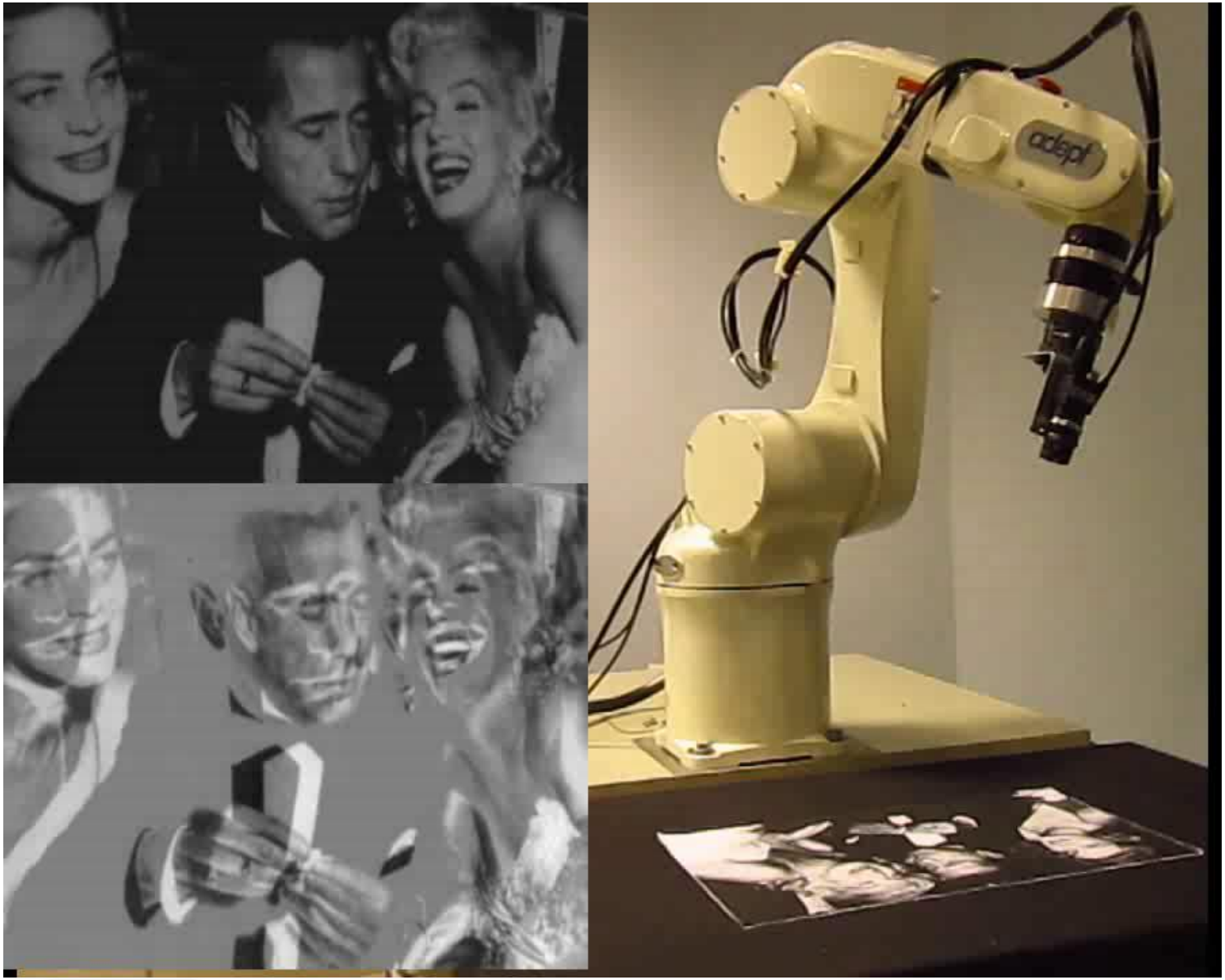


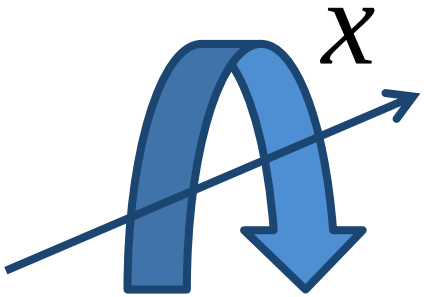
Raw luminance



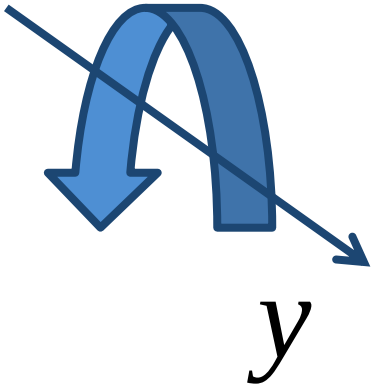
Weighted moments

Experimental Results | 6dof





Experimental Results | 6dof



Conclusions

TUNABLE VISUAL FEATURES

- Visual features can be selected so that they are optimal with respect to specific criteria through a useful generalization to shifted moments
- Interaction matrix based criteria were proposed for selection
- Studied their influence on system conditioning and improvements with different visual features and object shapes

PHOTOMETRIC MOMENTS

- A general analytical model for the interaction matrix for photometric (intensity-based) moments is developed
- Dense VS without image processing and visual tracking with mild ZBA violation

EXTRANEOUS IMAGE REGIONS

- Improved modelling scheme with weighting strategy that naturally eliminates ZBA and equips moments to handle extraneous regions
- Strategy is effective even for large rotations and convergence domain is better than non-weighted moments and raw luminance
- Spatial weighting results in alteration of translation invariance, which in turn affects the visual features which are based on this invariance property

Perspectives

- Search for a set of visual features such that $\mathbf{L_s} = \mathbf{I}_6$
- Singularity characterization and stability analysis
- Regain invariance properties (better weighting/alternate representation)

- Criteria selected using both current and reference images
- Find a strategy to combine moments (global) with geometric (local) features
  - Robustness in less overlap (more extraneous scene portions)
- Sequence moment-based features with raw luminance
  - Better convergence and assured accuracies at convergence

- Photometric moments for computer vision
  - Visual Tracking/Pose estimation : weighted similarity measures
  - Image descriptors based on image moments
- Extension to omnidirectional sensors
- For environment representation in appearance-based navigation

Publications

- Photometric moments: New promising candidates for visual servoing
  - IEEE/RSJ Int. Conference on Robotics and Automation (ICRA 2013), Germany

- Improving moments-based visual servoing with tunable visual features
  - IEEE/RSJ Int. Conference on Robotics and Automation (ICRA 2014), HongKong

- An Improved Modelling Scheme for Photometric Moments with Inclusion of Spatial Weights for Visual Servoing with Partial Appearance/Disappearance
  - IEEE/RSJ Int. Conference on Robotics and Automation (ICRA 2015), USA

Thank You