Computability Abstractions for Fault-tolerant Asynchronous Distributed Computing

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under the supervision of
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Outline

Distributed Computing

Motivations, Problems and Contributions

Synchrony weakened by message adversaries vs asynchrony restricted by failure detectors

A Hierarchy of Iterated Models from Messages to Memory

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Distributed Computing
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Agreement Problems
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In nearly all computing environments

- resources are (physically) **distributed**
  (multicores, clusters, grid, cloud, web, p2p...)

We need models to describe these environments.
We need to know what can be computed in such conditions.
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Asynchrony vs. Synchrony

- Heterogeneous systems and unpredictable networks;

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Not always possible (failures);

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We would like to have algorithms for asynchronous systems in which the relative speeds of processes are finite but unbounded.
Wait-free \((n - 1)\)-Resilient Models

- This thesis considers wait-free solvability in \((n - 1)\)-resilient systems:
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  - whatever the number of failures,
  - whatever the level of concurrency,
  - processes have to make progress.
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Communication via Messages and Distributed Objects
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- In order to achieve modularity, more complex shared objects can encapsulate solutions to building blocks problems. (consensus, shared data structures...)

The communication primitives available to the processes have an impact on what can be computed in asynchronous systems.
Agreement Problems

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Universal Construction

When consensus objects and registers are available, any shared object with a sequential specification can be implemented\(^a\).

\(^a\)Maurice Herlihy: Wait-Free Synchronization. ACM TOPLAS (1991)
Impossibilities and Failure Detectors

Consensus Impossibility

In the presence of failures, solving the consensus in an asynchronous system (message-passing\textsuperscript{a} or shared memory\textsuperscript{b} communication) is impossible.

\textsuperscript{a}Fischer, Lynch, Paterson: Impossibility of Distributed Consensus with One Faulty Process. J. ACM (1985)
\textsuperscript{b}Loui, Abu-Amara: Memory requirements for agreement among unreliable asynchronous processes. Advances in Computing Research (1987)

Implementing a Shared Memory

In an asynchronous message-passing system, if half of the processes can crash, it is impossible to implement a shared memory.
To work around this kind of impossibilities, the notion of failure detector\textsuperscript{1} has been introduced.

\textsuperscript{1}Tushar Deepak Chandra, Sam Toueg: Unreliable Failure Detectors for Reliable Distributed Systems. J. ACM (1996)
Impossibilities and Failure Detectors

- To work around this kind of impossibilities, the notion of failure detector\(^1\) has been introduced.
- A failure detector provides system-controlled read-only variables giving some information to the processes on the failures in the current execution.

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- A failure detector provides system-controlled read-only variables giving some information to the processes on the failures in the current execution.
- Failure detectors can be compared on the possibility to simulate one with another.
- Any problem solvable with a failure detector has an associated weakest failure detector\(^2\).

\(^1\)Tushar Deepak Chandra, Sam Toueg: Unreliable Failure Detectors for Reliable Distributed Systems. J. ACM (1996)
\(^2\)Prasad Jayanti, Sam Toueg: Every problem has a weakest failure detector. PODC 2008
Distributed Computing

Motivations, Problems and Contributions
The Weakest Failure Detector for \(k\)-Set Agreement in Wait-Free Message-Passing Systems
Iterated Models to Study Asynchronous Computability
Leading Questions During this Thesis and Contributions

Synchrony weakened by message adversaries vs asynchrony restricted by failure detectors

A Hierarchy of Iterated Models from Messages to Memory

Conclusion and Perspectives
The starting point of this thesis was the quest for the weakest failure detector for the $k$-set agreement in asynchronous message-passing systems.

It is known:

- in asynchronous shared memory$^3, 4$,
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- In the case $k = 1$ \(^5,\)

\(^3\)Piotr Zielinski: Anti-Omega: the weakest failure detector for set agreement. Distributed Computing (2010)
The Weakest Failure Detector for \( k \)-Set Agreement in Wait-Free Message-Passing Systems

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- in the case \( k = n - 1 \) \(^6\).

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Underlying Issues

- In the case $k = 1$, the failure detector $\Sigma^7$ is needed to prevent partitioning.

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8. François Bonnet, Michel Raynal: On the road to the weakest failure detector for k-set agreement in message-passing systems. TCS (2011)
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Underlying Issues

- In the case $k = 1$, the failure detector $\Sigma^7$ is needed to prevent partitioning.
- It is known that when $1 < k < n - 1$, $\Sigma$ is not needed: it can be solved without shared memory and in presence of partitioning.
- $\Sigma_k^8$ that prevent the system from partitioning in more than $k$ sets across the execution has been proved necessary for any value of $k$.

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$^8$François Bonnet, Michel Raynal: On the road to the weakest failure detector for k-set agreement in message-passing systems. TCS (2011)
Iterated models allow us to consider more structured set of executions while preserving the asynchronous shared memory computability.
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- The failure-free synchronous message-passing model weakened by the message-adversary TOUR \(^9\);

\(^9\)Yehuda Afek, Eli Gafni: Asynchrony from Synchrony. ICDCN 2013
Iterated models allow us to consider more structured set of executions while preserving the asynchronous shared memory computability.

- The failure-free synchronous message-passing model weakened by the message-adversary TOUR \(^9\);
- The iterated immediate snapshot model\(^{10}\).

\(^9\)Yehuda Afek, Eli Gafni: Asynchrony from Synchrony. ICDCN 2013
\(^{10}\)Elizabeth Borowsky, Eli Gafni: A Simple Algorithmically Reasoned Characterization of Wait-Free Computations (Extended Abstract). PODC 1997
Leading Questions During this Thesis

- How do computability degrades when allowing a limited but dynamic partitioning?
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- How to express, in the iterated models, the computability brought by failure detectors in asynchronous systems?
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- How do computability degrades when allowing a limited but dynamic partitioning?
- What is needed to solve agreement problems in presence of partitioning?
- How to express, in the iterated models, the computability brought by failure detectors in asynchronous systems?
- What can we build between message-passing and shared memory communication?

How can we compare and link some of the numerous models?
Articles Published During this Thesis

SSS’11
Relations Linking Failure Detectors Associated with k-Set Agreement in Message-Passing Systems

OPODIS’12
When and How Process Groups Can Be Used to Reduce the Renaming Space

Euro-Par’12
From a Store-Collect Object and $\Omega$ to Efficient Asynchronous Consensus

NCA’12
Chasing the Weakest Failure Detector for k-Set Agreement in Message-Passing Systems

SIROCCO’12
Increasing the Power of the Iterated Immediate Snapshot Model with Failure Detectors

CISIS’12
A Simple Asynchronous Shared Memory Consensus Algorithm Based on Omega and Closing Sets

SIROCCO’13
Simultaneous Consensus vs Set Agreement: A Message-Passing-Sensitive Hierarchy of Agreement Problems

PODC’13
Synchrony Weakened by Message Adversaries vs Asynchrony Restricted by Failure Detectors

TCS 512 (2013)
Trust-aware peer sampling: Performance and privacy tradeoffs

AINA’14
A Simple Broadcast Algorithm for Recurrent Dynamic Systems

LATIN’14
Computing in the Presence of Concurrent Solo Executions

SIROCCO’14
Reliable Shared Memory Abstraction on Top of Asynchronous Byzantine Message-Passing Systems

OPODIS’14
Distributed Universality

Computability Abstractions for Fault-tolerant Asynchronous Distributed Computing

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Distributed Computing

Motivations, Problems and Contributions

Synchrony weakened by message adversaries vs asynchrony restricted by failure detectors

- Two fundamental Failure Detectors: $\Sigma$ and $\Omega$
- Message Adversaries: Weakening the Synchronous Crash-free Model
- Equivalence Results and Questions

A Hierarchy of Iterated Models from Messages to Memory

Conclusion and Perspectives
Michel Raynal, Julien Stainer:

_Synchrony Weakened by Message Adversaries

_vs.

_Asynchrony Restricted by Failure Detectors._

PODC 2013: 166-175
Preventing Partitioning: $\Sigma$

- $\Sigma$ provides each process with a set of process identities called quorum;
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Preventing Partitioning: $\Sigma$

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$\Sigma$ is the weakest failure detector to simulate a memory in the asynchronous message-passing system $\mathcal{AMP}[\emptyset]$.

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Breaking Symmetry: $\Omega$

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  - the same for each process;
  - correct.
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$\Omega$ is the weakest failure detector to solve the consensus in the asynchronous shared memory system $\text{ASM}[\emptyset]^a$

$\langle \Sigma, \Omega \rangle$ is the weakest failure detector to solve the consensus in the asynchronous message-passing system $\text{AMP}[\emptyset]^b$

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^bCarole Delporte-Gallet, Hugues Fauconnier, Rachid Guerraoui, Vassos Hadzilacos, Petr Kouznetsov, Sam Toueg: The weakest failure detectors to solve certain fundamental problems in distributed computing. PODC 2004
The Synchronous Message-passing Model: $SMP[\emptyset]$

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  - they **compute locally** their new states.
The Synchronous Message-passing Model: $\text{SMP}[\emptyset]$

- The execution is stripped in a sequence of rounds;
- each round is made of three phases:
  - processes send messages to each other,
  - they receive the round messages addressed to them,
  - they compute locally their new states.
- There are no process failures.
Message Adversaries

- The adversary removes messages in $\mathcal{SM}[\emptyset]$. 
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- The adversary removes messages in $SMP[\emptyset]$.
- Properties define the patterns of messages that can be removed
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Adversaries weaken the synchronous crash-free model $SMP[\emptyset]$. 
Shared Memory from Synchrony: the Adversary TOUR

- TOUR \textsuperscript{11} can remove any message but it preserves a tournament in any round.
- In any round and between any pair of processes, it has to let one of the two messages exchanged untouched.

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SOURCE can remove any message but it eventually preserves all messages sent by a given source.
Ω from Synchrony: the Adversary SOURCE

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Ω from Synchrony: the Adversary SOURCE

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QUORUM can remove any message but in each round each process receives messages from an entire quorum:

- in any two rounds $r_1$ and $r_2$, for any two processes $p_1$ and $p_2$, there is a process $p_3$ such that:
  - $p_1$ receives the message of $p_3$ during $r_1$ and
  - $p_2$ receives the message of $p_3$ during $r_2$;

- There is at least one process that is infinitely often able to send messages (directly or not) to any other process.
Equivalence Results

\[ \text{SMP}[\text{adv} : \emptyset] \simeq \text{AMP}[\text{nocrash}] \simeq \text{ASM}[\text{nocrash}] \]

\[ \text{SMP}[\text{adv} : \text{SOURCE} + \text{QUORUM}] \simeq \text{AMP}[\text{fd} : \Sigma + \Omega] \]

\[ \text{SMP}[\text{adv} : \text{SOURCE} + \text{TOUR}] \simeq \text{ASM}[\text{fd} : \Omega] \]

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(computability w.r.t. colorless tasks)
Remarks and Perspectives

- Expressing the calculability of the two asynchronous models associated with failure detectors through message adversaries gives us a new way to compare them in a common framework.
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- **Strongly correct processes** play an essential role in dynamic systems.

- What are the message adversaries that allow agreement tasks to be solved?
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- Expressing the calculability of the two asynchronous models associated with failure detectors through message adversaries gives us a new way to compare them in a common framework.
- **Strongly correct processes** play an essential role in dynamic systems.

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- Is there a matching message adversary for any failure detector?
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- **Strongly correct processes** play an essential role in dynamic systems.

- What are the message adversaries that allow agreement tasks to be solved?
- Is there a matching message adversary for any failure detector?
- What happens when considering colored tasks?
Distributed Computing

Motivations, Problems and Contributions

Synchrony weakened by message adversaries vs asynchrony restricted by failure detectors

A Hierarchy of Iterated Models from Messages to Memory
  Wait-free Models and Solo Executions
  $d$-Solo Models
  The Colorless Algorithm in the $d$-solo model
  The $(d, \epsilon)$-Approximate Agreement Problem
  A Strict Hierarchy from Shared Memory to Message-Passing
  Status and Further Investigation

Conclusion and Perspectives
Maurice Herlihy, Sergio Rajsbaum, Michel Raynal, Julien Stainer:

Computing in the Presence of Concurrent Solo Executions.

LATIN 2014: 214-225
Wait-free Algorithms and Solo Executions

- **Wait-free** models in both meanings:
  - as a **progress condition**: each process makes progress in a finite number of steps, whatever the level of concurrence;
  - slow and crashed processes are indistinguishable: some processes may have to behave as if they were alone; do not have access to other processes inputs.
Wait-free Algorithms and Solo Executions

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What could be computed in intermediate models in which up to $d$ processes may run solo?


**d-Solo Models**

- An iterated model generalizing the *iterated immediate snapshot* model.
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- The execution is stripped in a sequence of **rounds**;
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- each process writes a value and retrieves the previously or simultaneously written values.
$d$-Solo Models

- An iterated model generalizing the **iterated immediate snapshot** model.
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- a **one-shot communication object** for each round;
- each process writes a value and retrieves the previously or simultaneously written values.
- in the $d$-solo model, the first set of simultaneous accesses can **miss each other**.
  - If they do, then this set contains at most $d$ processes.

A spectrum of models that spans from message-passing ($d = n$) to shared memory ($d = 1$).
From the Immediate Snapshot Object... 

- Each process $p$ provides a value $v_p$ to the object and retrieves a set of values (a view).
From the Immediate Snapshot Object...

- Each process $p$ provides a value $v_p$ to the object and retrieves a set of values (a view).
- As with the immediate snapshot object, any ordered partition $(\pi_1, \ldots, \pi_x)$ of the set of the processes accessing the object describe a valid behavior for the object:
  - the view of any process belonging to $\pi_i$ is $\bigcup_{j \leq i} \{(p, v_p), p \in \pi_j\}$. 

From the Immediate Snapshot Object...
From the Immediate Snapshot Object...
Additionally, any ordered partition \((\rho_1, \ldots, \rho_x)\) of the set of processes accessing the object describe another authorized behavior for the object if \(|\rho_1| \leq d\):
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Additionally, any ordered partition \((\rho_1, \ldots, \rho_x)\) of the set of processes accessing the object describe another authorized behavior for the object if \(|\rho_1| \leq d\):

- if \(i > 1\), then the view of any process belonging to \(\rho_i\) is \(\bigcup_{j \leq i} \{(p, v_p), p \in \rho_j\}\);
- the view of a process \(p\) of \(\rho_1\) is \(\{v_p\}\).
... to the $CO^d$ Communication Object
... to the $CQ^d$ Communication Object
The Subdivided Complex of the Possible States after one Access to an Immediate Snapshot Object
The Subdivided Complex of the Possible States after one Access to an Immediate Snapshot Object
The Additional Possible States after one Access to a $CO^2$ Object
The Colorless Algorithm in the $d$-solo model

- We consider the case of a colorless algorithm:
  - processes do not use their identities during the computation;
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- We consider the case of a colorless algorithm:
  - processes do not use their identities during the computation;
  - they use the object as a set: during each round a process writes the last view it retrieved (initially its input value) ignoring writers identities and multiple occurrences of the same view;
  - It allows us to describe all the possible states of the system after the execution of $R$ rounds by a subdivided complex without coloring vertices with process identities.
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- processes do not use their identities during the computation;
- they use the object as a set: during each round a process writes the last view it retrieved (initially its input value) ignoring writers identities and multiple occurrences of the same view;
- they compute their output from their view after $R$ rounds.
The Colorless Algorithm in the \( d \)-solo model

- We consider the case of a colorless algorithm:
  - processes do not use their identities during the computation;
  - they use the object as a set: during each round a process writes the last view it retrieved (initially its input value) ignoring writers identities and multiple occurrences of the same view;
  - they compute their output from their view after \( R \) rounds.

- It allows us to describe all the possible states of the system after the execution of \( R \) rounds by a subdivided complex without coloring vertices with process identities.
The Colorless Complex of the Possible States in the Colorless Algorithm (Immediate Snapshot)
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The Colorless Complex of the Additional Possible States in the Colorless Algorithm (\(CO^2\))
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Task Solvability in the $d$-Solo Model

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  - the (colorless) complex of all possible input configurations;
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- a monotonic carrier map associating each input configuration to a set of allowed output configurations.
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**Theorem**

A colorless task is solvable by a colorless algorithm in the $d$-solo model with $n$ processes if and only if there is a number of rounds $R \geq 0$ and a simplicial map from the $R$-iterated $d$-subdivision of the $n-1$ skeleton of the (colorless) input complex to the (colorless) output complex that is carried by the colorless task carrier map.
The \((d, \epsilon)\)-Approximate Agreement Problem

- Generalizing the \(\epsilon\)-Approximate Agreement that is universal for the shared memory model
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- Each process proposes a value from an Euclidian space.
- **Termination**: all correct processes decide in a finite number of steps.
- **Validity**: all the decided values belong to the convex hull of the set of proposed values.
- **Agreement**: there is a set \(S\) of up to \(d\) processes that can decide any valid value while other processes have to decide within a distance of \(\epsilon\) from the convex hull of the values decided by processes of \(S\).
The \((d, \epsilon)\)-Approximate Agreement Problem
$(d, \epsilon)$-Approximate Agreement in the $d$-Solo Models

- For any $\epsilon$, any $d$ and any $n$, if the volume of the $d$-faces of the input complex is bounded, there is a number of round $R$ such that the colorless algorithm solves the $(d, \epsilon)$-approximate agreement problem in the $d$-solo model.
(\(d, \epsilon\))-Approximate Agreement in the \(d\)-Solo Models

- For any \(\epsilon\), any \(d\) and any \(n\), if the volume of the \(d\)-faces of the input complex is bounded, there is a number of round \(R\) such that the colorless algorithm solves the \((d, \epsilon)\)-approximate agreement problem in the \(d\)-solo model.

- For any \(\epsilon\), any \(d\) and any \(n, n > d\), if there is a simplex of the input complex containing a large enough regular \(d\)-simplex, then the \((d, \epsilon)\)-approximate agreement problem is impossible to solve in the \((d + 1)\)-solo model.
$(d, \epsilon)$-Approximate Agreement in the $d$-Solo Models

- For any $\epsilon$, any $d$ and any $n$, if the volume of the $d$-faces of the input complex is bounded, there is a number of round $R$ such that the colorless algorithm solves the $(d, \epsilon)$-approximate agreement problem in the $d$-solo model.

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Since these conditions are compatible, the hierarchy of the $d$-solo models is strict.
A Strict Hierarchy Spanning from Shared Memory to Message-Passing

\[(1, \epsilon) - SAA \quad \overset{\perp}{\sim} \quad ACS^1 \sim ARW\]

\[(2, \epsilon) - SAA \quad \overset{\perp}{\sim} \quad ACS^2\]

\[(d, \epsilon) - SAA \quad \overset{\perp}{\sim} \quad ACS^d\]

\[(d + 1, \epsilon) - SAA \quad \overset{\perp}{\sim} \quad ACS^{d+1}\]

\[(n - 1, \epsilon) - SAA \quad \overset{\perp}{\sim} \quad ACS^{n-1}\]

\[(n, \epsilon) - SAA \quad \overset{\perp}{\sim} \quad ACS^n \sim AMP\]
We built a hierarchy of iterated models, spanning from shared memory to message-passing.
Status and Further Investigation

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- Can we solve a stronger generalization of $\varepsilon$-agreement if we do not restrict to colorless algorithms?
- How does computability evolve if we allow more behavior for the communication object?
Distributed Computing

Motivations, Problems and Contributions

Synchrony weakened by message adversaries vs asynchrony restricted by failure detectors

A Hierarchy of Iterated Models from Messages to Memory

Conclusion and Perspectives
Summary

During this thesis, we explored

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The processes that infinitely often able to communicate, directly or not, with all the others have a special role across the different models.

Partitioning, if contained, is not an end. There are important tasks that are solvable without the ability to implement a memory.
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- the possible high level abstractions we can offer to ease programming against byzantine failures;
- the mathematical structure of the set of possible executions in presence of partitioning.
Thank you for your attention!