PDE's on the Space of Patches for Image Denoising and Registration

David Tschumperlé* - Luc Brun*

Patch-based Image Representation, Manifolds and Sparsity, Rennes/France, April 2009.

* GREYC IMAGE (CNRS UMR 6072), Caen/France

Presentation Layout

- Definition of a Patch Space Γ.
- Patch-based Tikhonov Regularization.
- Patch-based Anisotropic Diffusion PDE's.
- Patch-based Lucas-Kanade registration.
- Conclusions & Perspectives.

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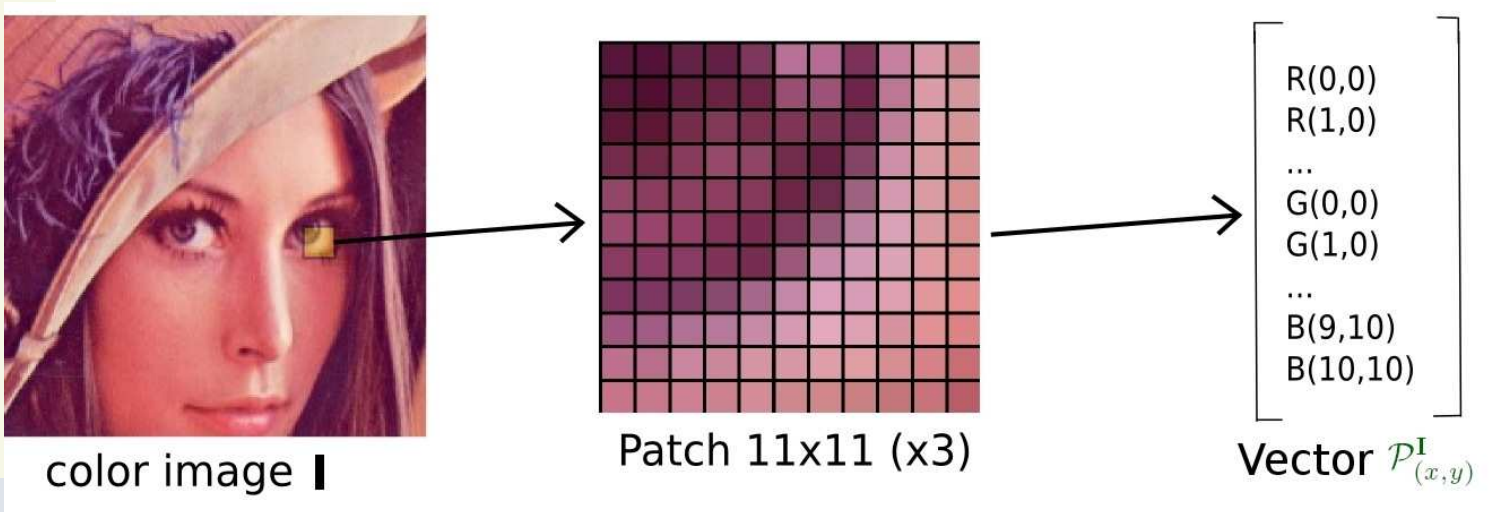
Located Patch of an Image

- Considering a 2D image $\mathbf{I} : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}^n$ ($n = 3$, for color images).

- An image patch $\mathcal{P}^{\mathbf{I}}_{(x,y)}$ is a discretized $p \times p$ neighborhood of $\mathbf{I}$, which can be ordered as a np^2-dimensional vector :

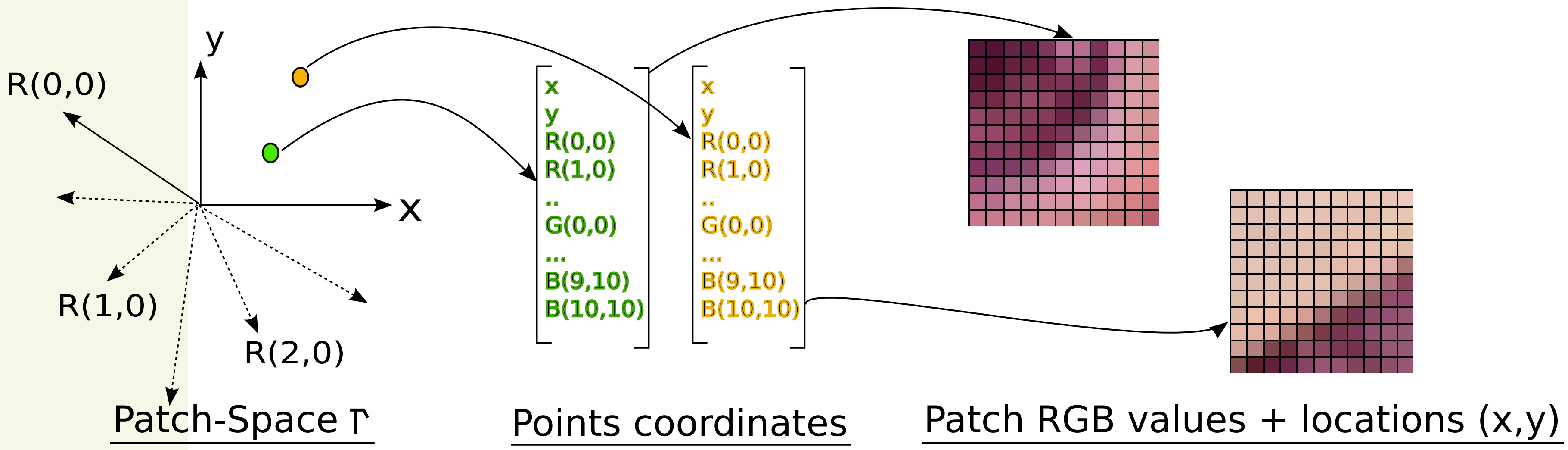
$$\mathcal{P}^{\mathbf{I}}_{(x,y)} = \left(I_{1(x-q,y-q)}, \ldots, I_{1(x+q,y+q)}, I_{2(x-q,y-q)}, \ldots, I_{n(x+q,y+q)}\right)$$



- We define a located patch as the $(np^2 + 2)$-D vector $(x, y, \lambda\mathcal{P}^{\mathbf{I}}_{(x,y)})$ ($\lambda > 0$ balances importance of spatial/intensity features).

Space Γ of Located Patches

- $\Gamma = \Omega \times \mathbb{R}^{np^2}$ defines a (np^2+2)-dimensional space of located patches.



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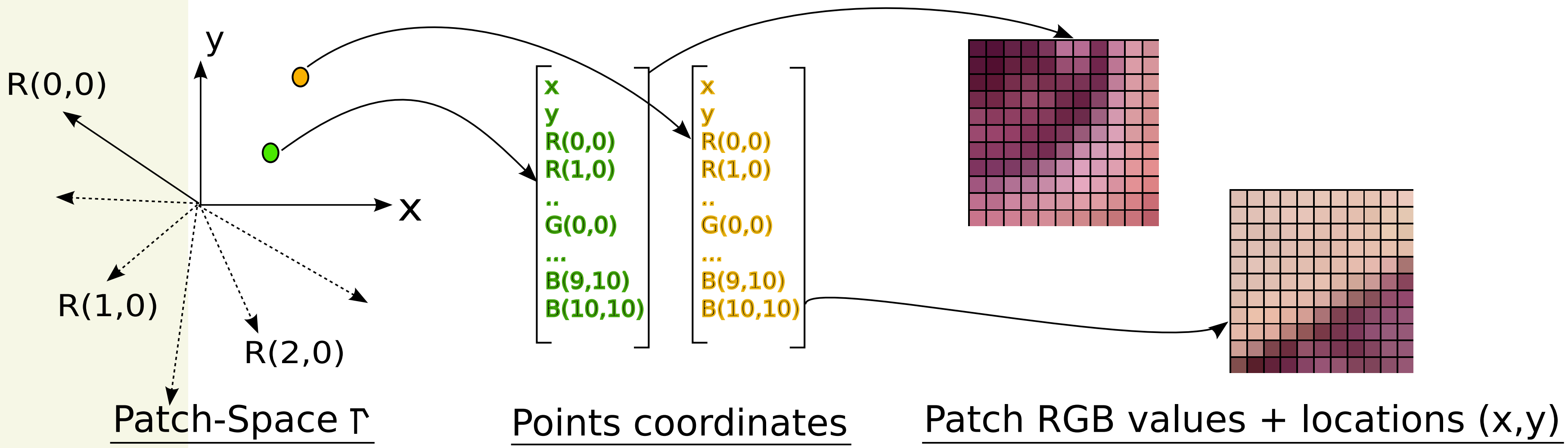


- The Euclidean distance between two points $p_1, p_2 \in \Gamma$ measures a spatial & intensity dissimilarity between corresponding located patches :

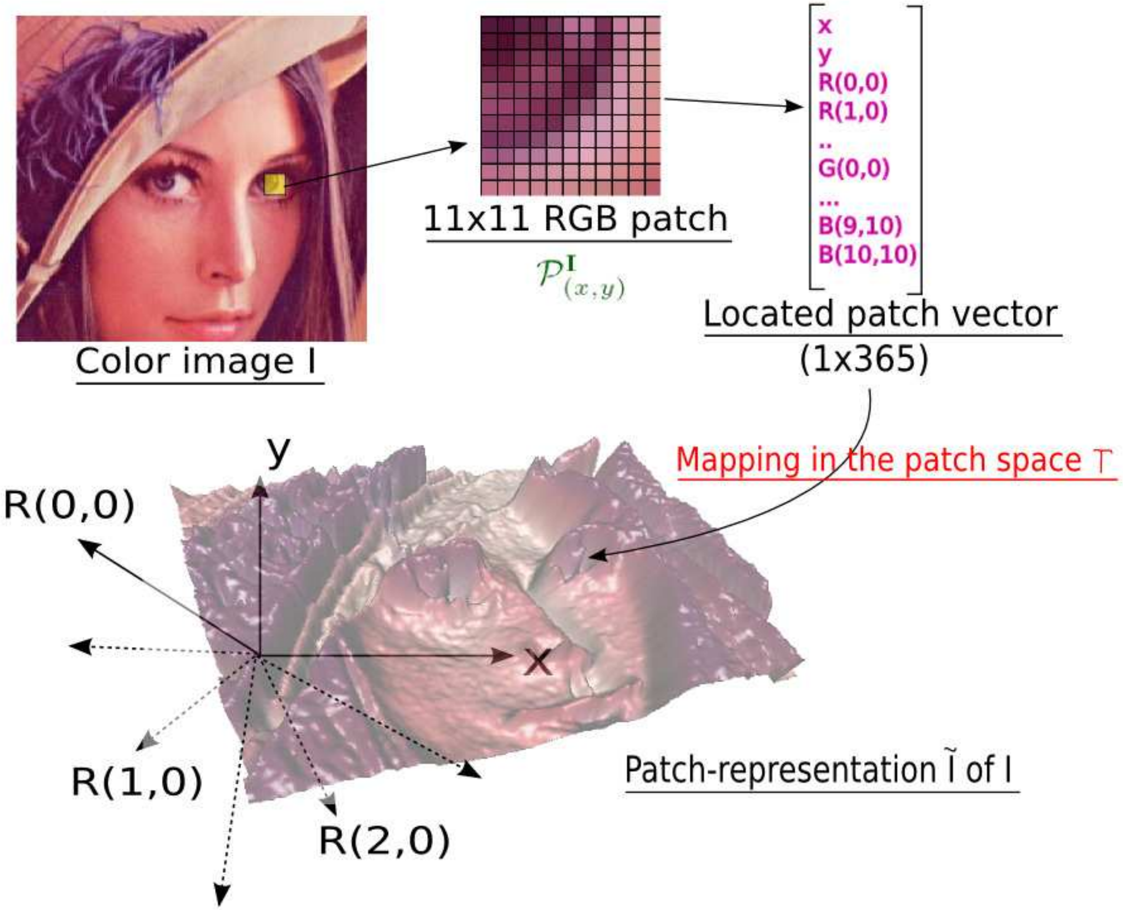
$$d(p_1, p_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + \lambda^2 \text{SSD}(\mathcal{P}_1, \mathcal{P}_2)}$$

(SSD = Sum of Squared Differences)

Mapping an Image $\mathbf{I}$ on the Patch Space Γ

- We define $\tilde{\mathbf{I}} : \Gamma \rightarrow \mathbb{R}^{np^2+1}$, a mapping of the image $\mathbf{I}$ on Γ :

$$\forall \mathbf{p} \in \Gamma, \quad \tilde{\mathbf{I}}_{(\mathbf{p})} = \begin{cases} (\mathcal{P}^{\mathbf{I}}_{(x,y)}, 1) & \text{if} \quad \mathbf{p} = (x, y, \mathcal{P}^{\mathbf{I}}_{(x,y)}) \\ \vec{0} & \text{elsewhere} \end{cases}$$



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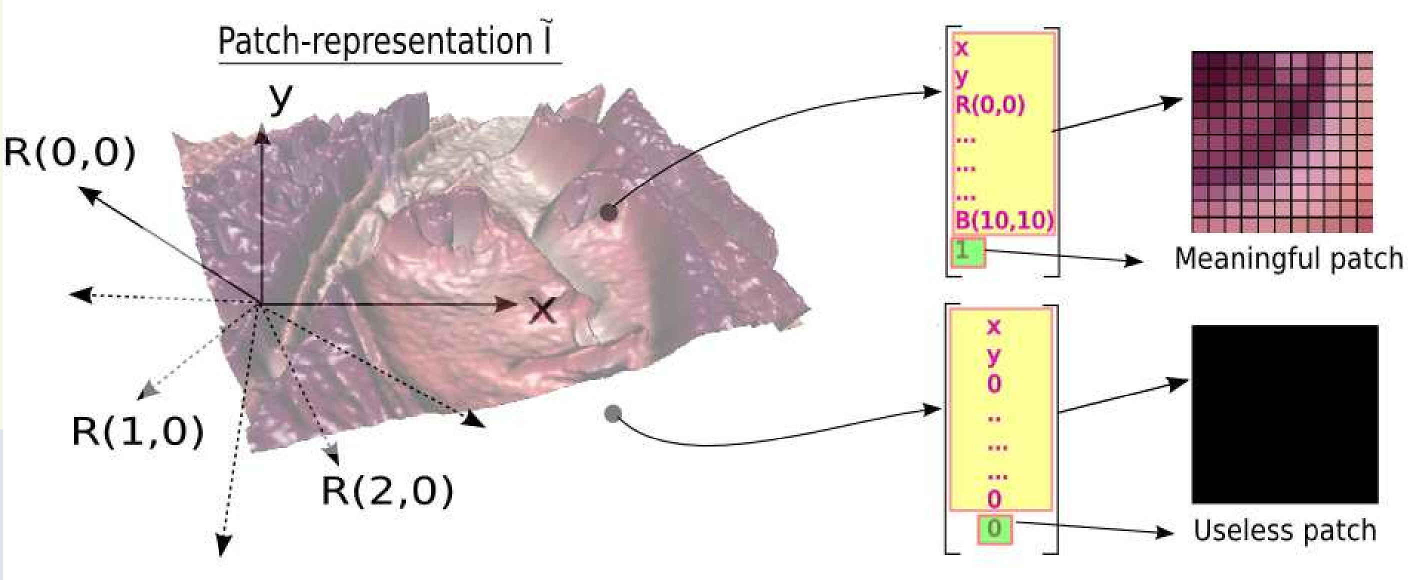
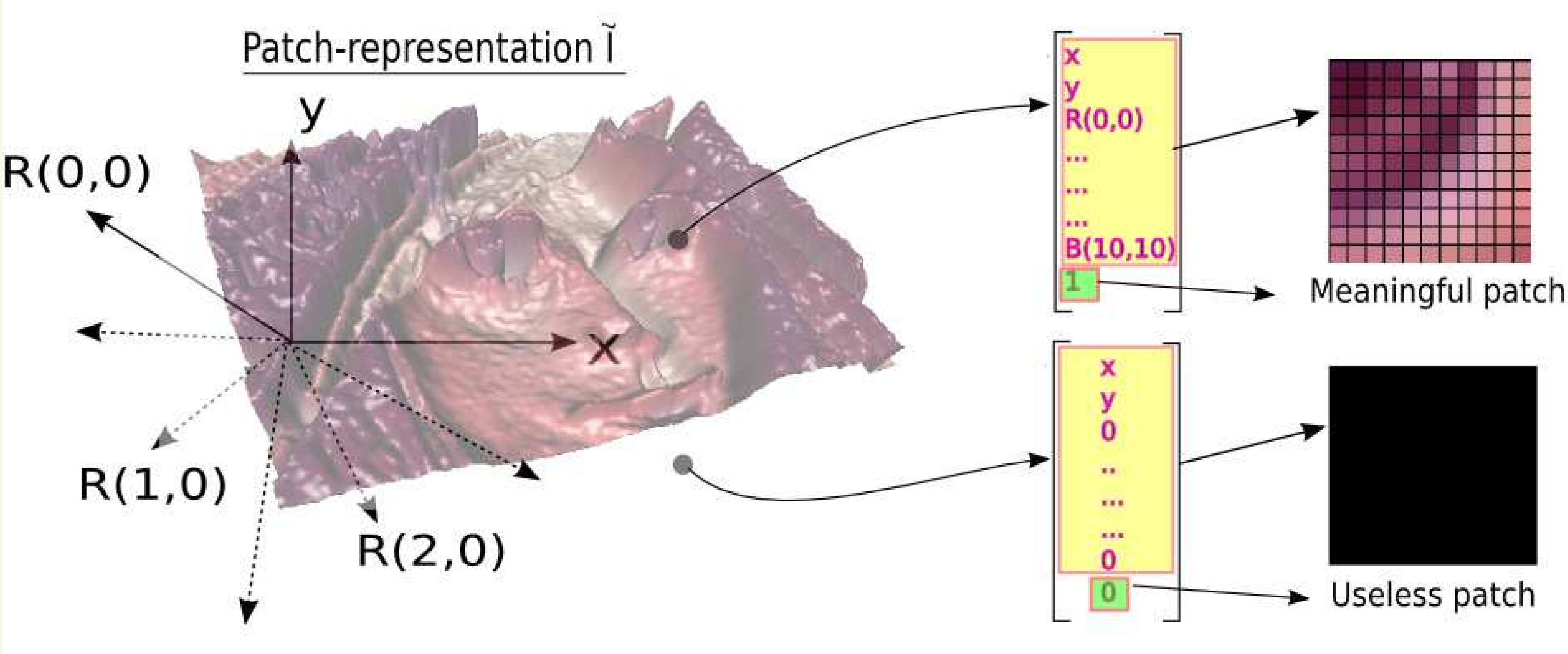
- The last value of $\tilde{\mathbf{I}}_{(\mathbf{p})}$ models the meaningfulness of a located patch p.
  All patches coming from the original image $\mathbf{I}$ have the same unit weight.



$\Rightarrow$ $\tilde{\mathbf{I}}$ is a patch-based representation of $\mathbf{I}$ in Γ, as an implicit surface.

Inverse Mapping to the Image Domain Ω

- Question : Is it possible to retrieve $\mathbf{I}$ from $\tilde{\mathbf{I}}$?

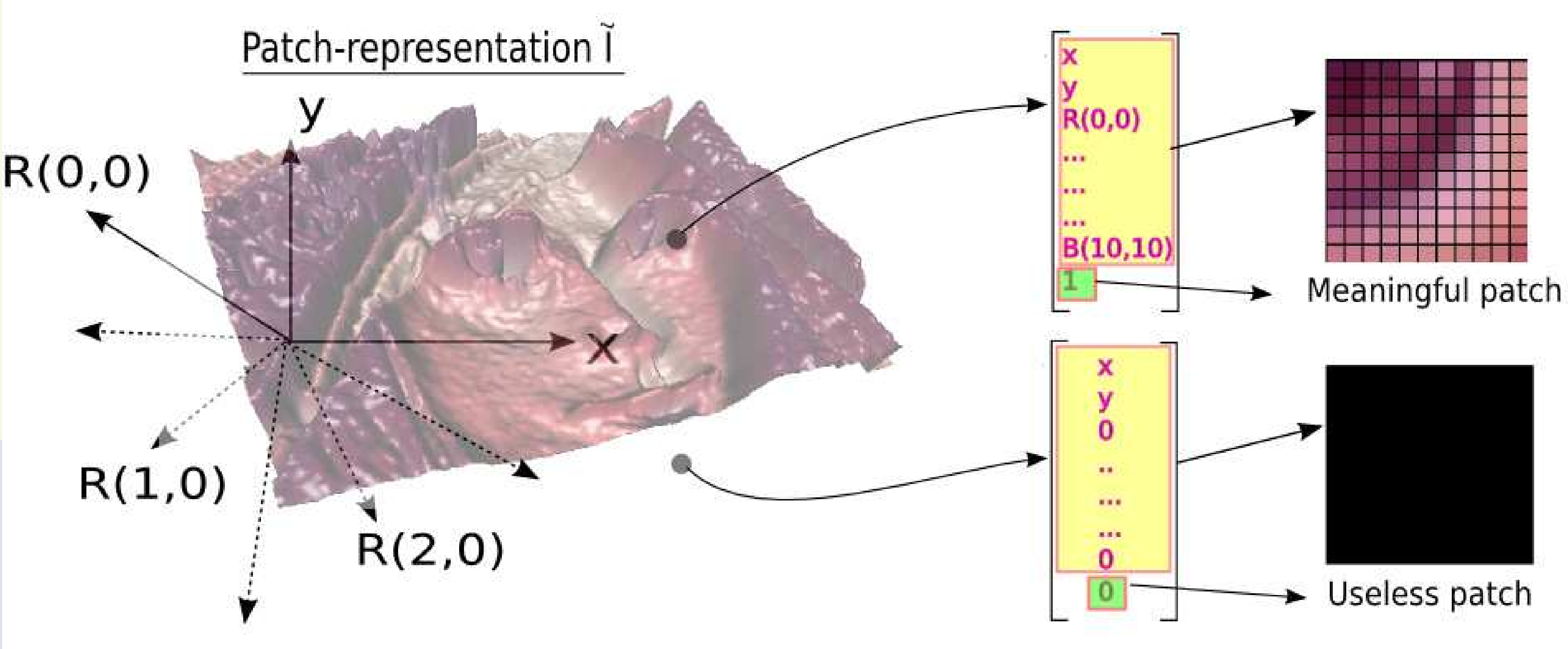


Inverse Mapping to the Image Domain Ω

- Question : Is it possible to retrieve $\mathbf{I}$ from $\tilde{\mathbf{I}}$? **YES !**

$\Rightarrow$ (1) Find the most significant patches $\mathbf{p} = (x, y, \mathcal{P}) \in \Gamma$ for each location $(x, y) \in \Omega$:

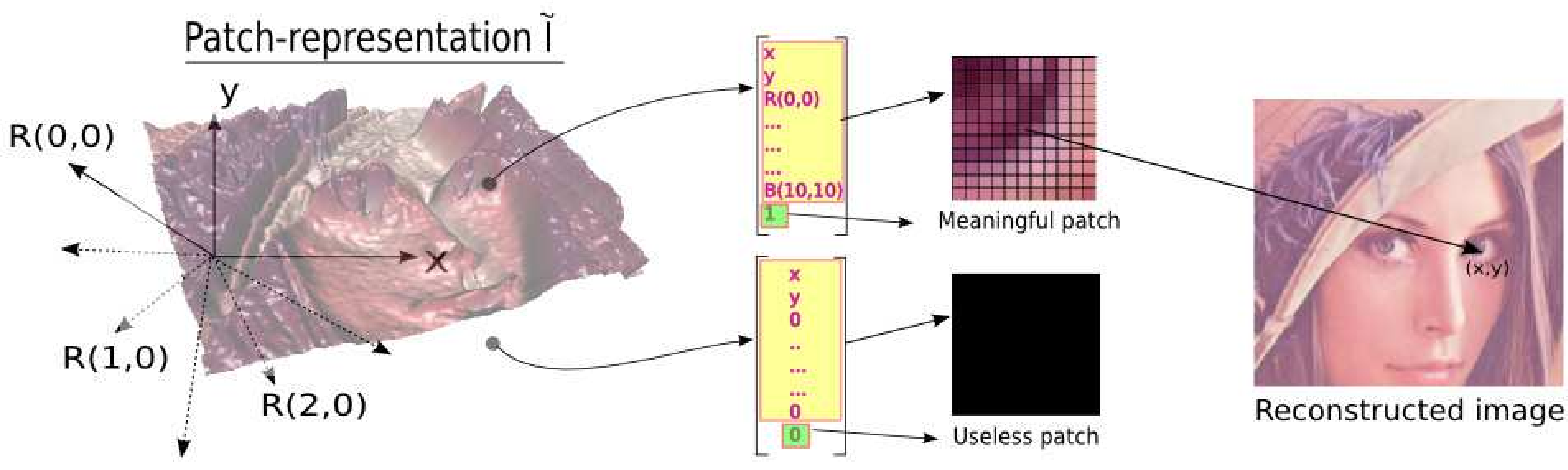
$$\mathcal{P}^{\tilde{\mathbf{I}}}_{sig(x,y)} = \text{argmax}_{\mathbf{q} \in \mathbb{R}^{np^2}} \ \tilde{I}_{np^2+1}(x, y, \mathbf{q})$$



Inverse Mapping to the Image Domain Ω

⇒ (2) Get the central pixel of these patches, and normalize it by its meaningfulness :

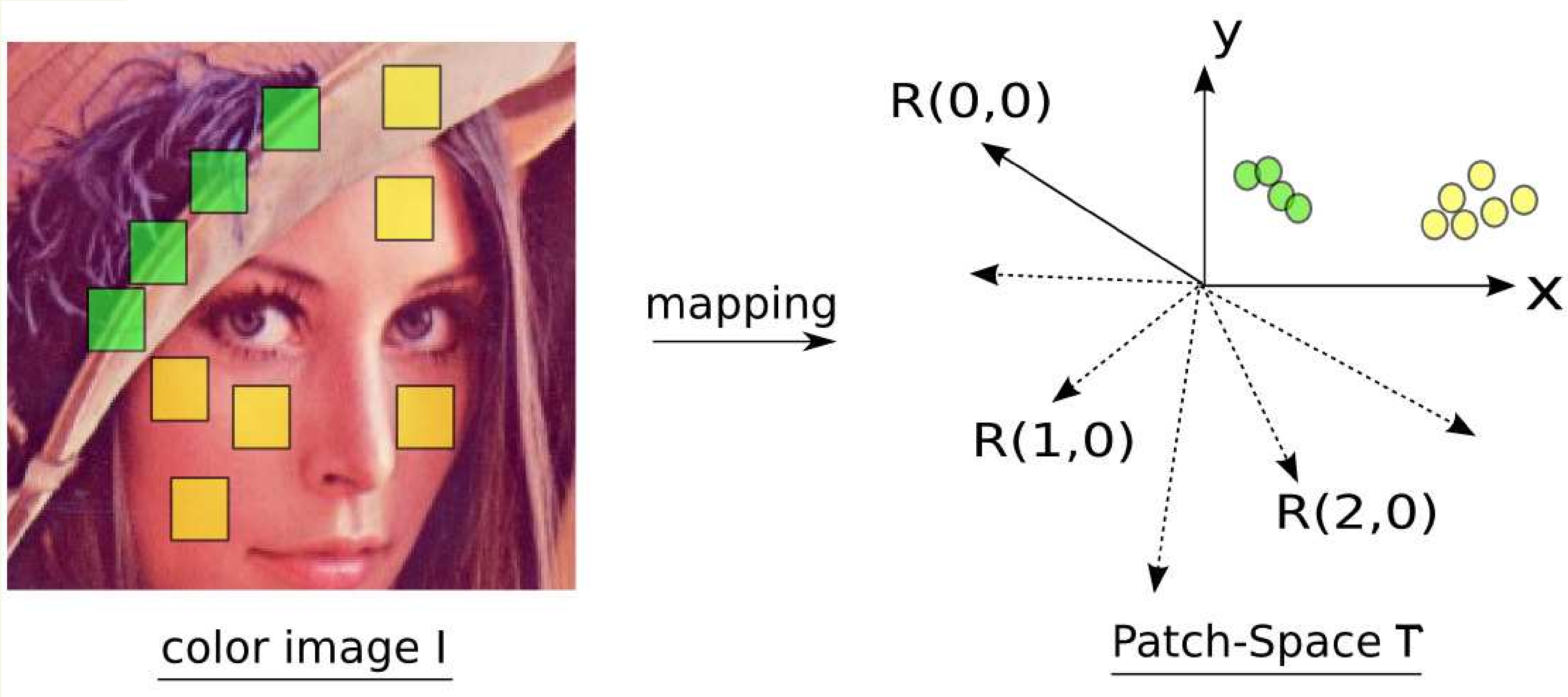
$$\forall (x,y) \in \Omega, \qquad \hat{I}_{i(x,y)} = \frac{\tilde{I}_{ip^2+\frac{p^2+1}{2}}(x,y,\mathcal{P}^{\tilde{\mathbf{I}}}_{sig(x,y)})}{\tilde{I}_{np^2+1}(x,y,\mathcal{P}^{\tilde{\mathbf{I}}}_{sig(x,y)})}$$



(Other solutions may be considered, for instance : averaging spatially-overlapping meaningful patches).

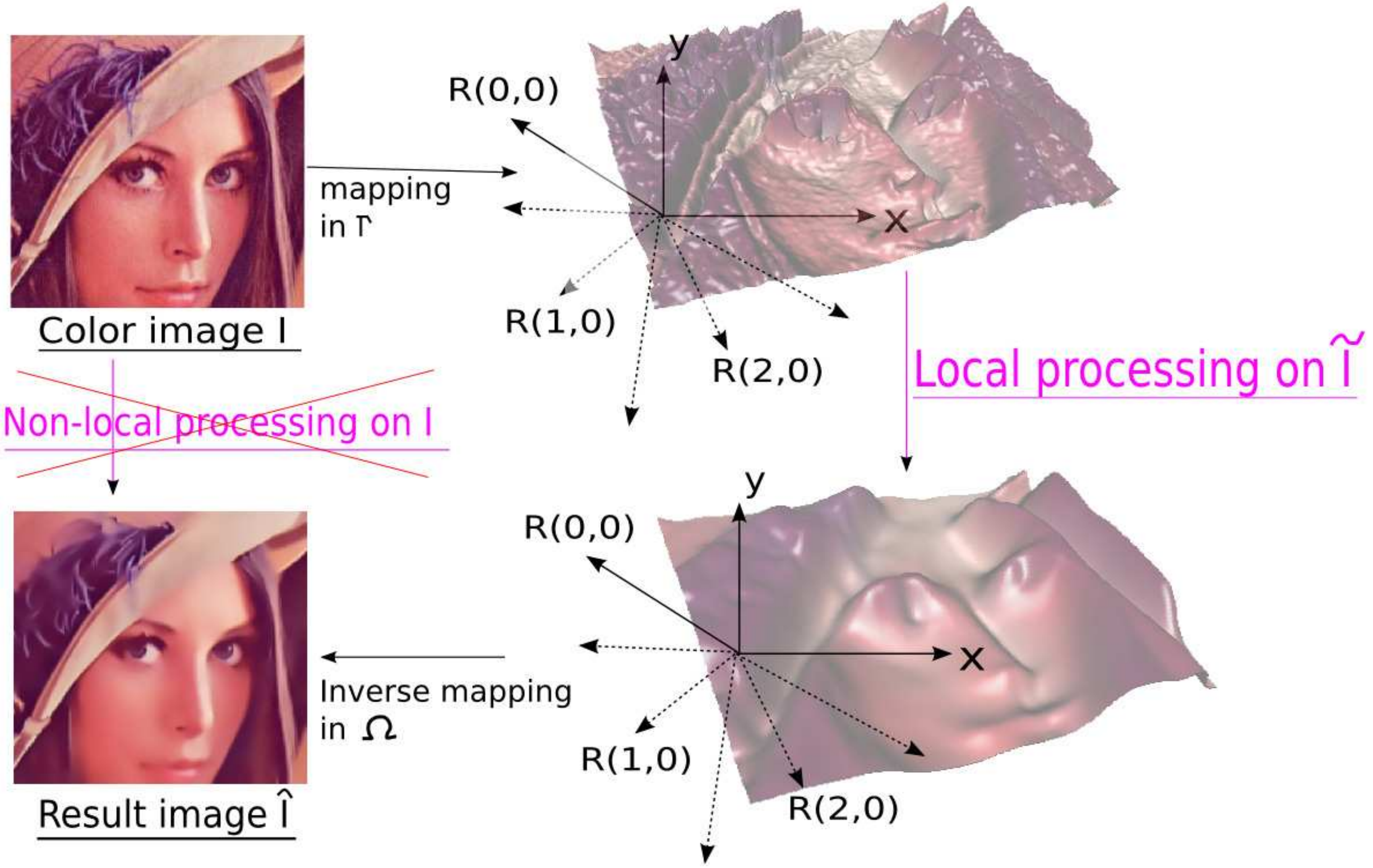
From Non-Local to Local processing

- Mapping $\mathbf{I}$ in Γ transforms a non-local processing problem into a local one.



- Local or semi-local measures of $\tilde{\mathbf{I}}$ in Γ (gradients,curvatures,...) will be related to non-local features of the original image $\mathbf{I}$ (patch dissimilarity, variance,...).

Main Idea of this Talk



⇒ Apply **local algorithms** on $\tilde{I}$ in order to build their **patch-based counterparts**.

⇒ Find **correspondences** between non-local and local algorithms.

What Local Algorithms to Apply in Γ ?

$\Rightarrow$ **PDE's and variational methods** are good candidates.

- They are purely local or semi-local.
- They are adaptive to local image informations (non-linear).
- They are often expressed independently on the data dimension.
- They give interesting solutions for a wide range of different (local) problems.

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⇒ In this talk :

- **Diffusion PDE's** for image denoising.
- **PDE's for image registration**, coming from a variational formulation.

Presentation Layout

- **Definition of a Patch Space Γ.**

⇒ **Patch-based Tikhonov Regularization.**

- **Patch-based Anisotropic Diffusion PDE's.**

- **Patch-based Lucas-Kanade registration.**

- **Conclusions & Perspectives.**

Tikhonov Regularization in Γ

- We minimize the classical Tikhonov regularization functional for $\tilde{\mathbf{I}}$ in Γ :

$$E(\tilde{\mathbf{I}}) = \int_{\Gamma} \|\nabla \tilde{\mathbf{I}}_{(\mathbf{p})}\|^2 \, d\mathbf{p}$$

where $\|\nabla \tilde{\mathbf{I}}_{(\mathbf{p})}\| = \sqrt{\sum_{i=1}^{np^2+1} \|\nabla \tilde{I}_{i(\mathbf{p})}\|^2}$

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where $\|\nabla \tilde{\mathbf{I}}_{(\mathbf{p})}\| = \sqrt{\sum_{i=1}^{np^2+1} \|\nabla \tilde{I}_{i(\mathbf{p})}\|^2}$

- The Euler-Lagrange equations of E give the desired minimizing flow for $\tilde{\mathbf{I}}$:

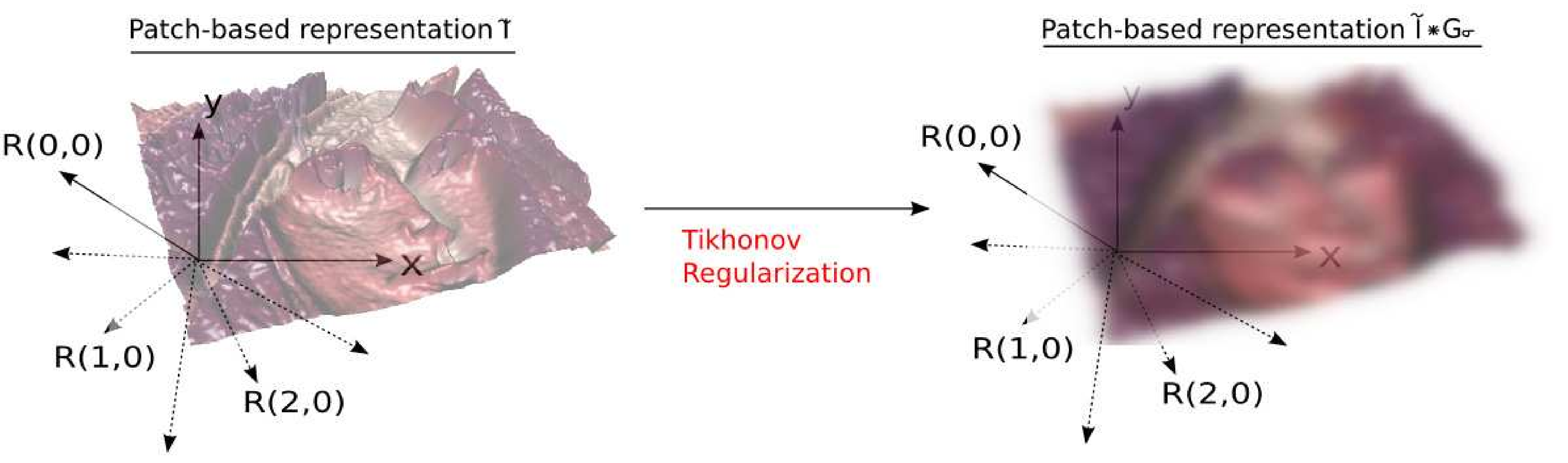
$$\begin{cases} \tilde{\mathbf{I}}_{[t=0]} = \tilde{\mathbf{I}}^{noisy} \\ \frac{\partial \tilde{I}_i}{\partial t} = \Delta \tilde{I}_i \end{cases}$$

$\Rightarrow$ Heat flow in the high-dimensional space of patches Γ.

Solution to the Tikhonov Regularization in Γ

- This high-dimensional heat flow has an explicit solution (at time t) :

$$\tilde{\mathbf{I}}^{[t]} = \tilde{\mathbf{I}}^{noisy} * G_\sigma \quad \text{with} \quad \forall \mathbf{p} \in \Gamma,\ \ G_{\sigma(\mathbf{p})} = \frac{1}{(2\pi\sigma^2)^{\frac{np^2+2}{2}}}\, e^{-\frac{\|\mathbf{p}\|^2}{2\sigma^2}} \quad \text{and} \quad \sigma = \sqrt{2\,t}.$$



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- Simplification : As $\tilde{\mathbf{I}}^{noisy}$ vanishes almost everywhere (except on the original located patches of $\mathbf{I}$), the convolution simplifies to :

$$\tilde{\mathbf{I}}^{[t]}_{(x,y,\mathcal{P})} = \int_\Omega \tilde{\mathbf{I}}^{noisy}_{(p,q,\mathcal{P}^{\mathbf{I}^{noisy}}_{(p,q)})}\, G_{\sigma(p-x,q-y,\mathcal{P}^{\mathbf{I}^{noisy}}_{(p,q)}-\mathcal{P})}\, dp\, dq$$

$\Rightarrow$ Computing the solution does not require to build an explicit representation of the patch-based representation $\tilde{\mathbf{I}}$.

Inverse mapping of the Tikhonov Regularization in Γ

- Finding the most significant patches in Γ : the flow preserves the locations of the local maxima. The inverse mapping of $\tilde{\mathbf{I}}^{[t]}$ on Ω is then :

$$\forall (x,y) \in \Omega, \quad \mathbf{I}^{[t]}_{(x,y)} = \frac{\int_\Omega \mathbf{I}^{noisy}_{(p,q)} \; w_{(x,y,p,q)} dp\, dq}{\int_\Omega w_{(x,y,p,q)} \; dp\, dq}$$

with $w_{(x,y,p,q)} = \frac{1}{2\pi\sigma^2} \, e^{-\frac{(x-p)^2+(y-q)^2}{2\sigma^2}} \times \frac{1}{(2\pi\sigma^2)^{\frac{np^2}{2}}} \, e^{-\frac{\|\mathcal{P}_{\mathbf{I}^{noisy}_{(x,y)}} - \mathcal{P}_{\mathbf{I}^{noisy}_{(p,q)}}\|^2}{2\sigma^2}}$

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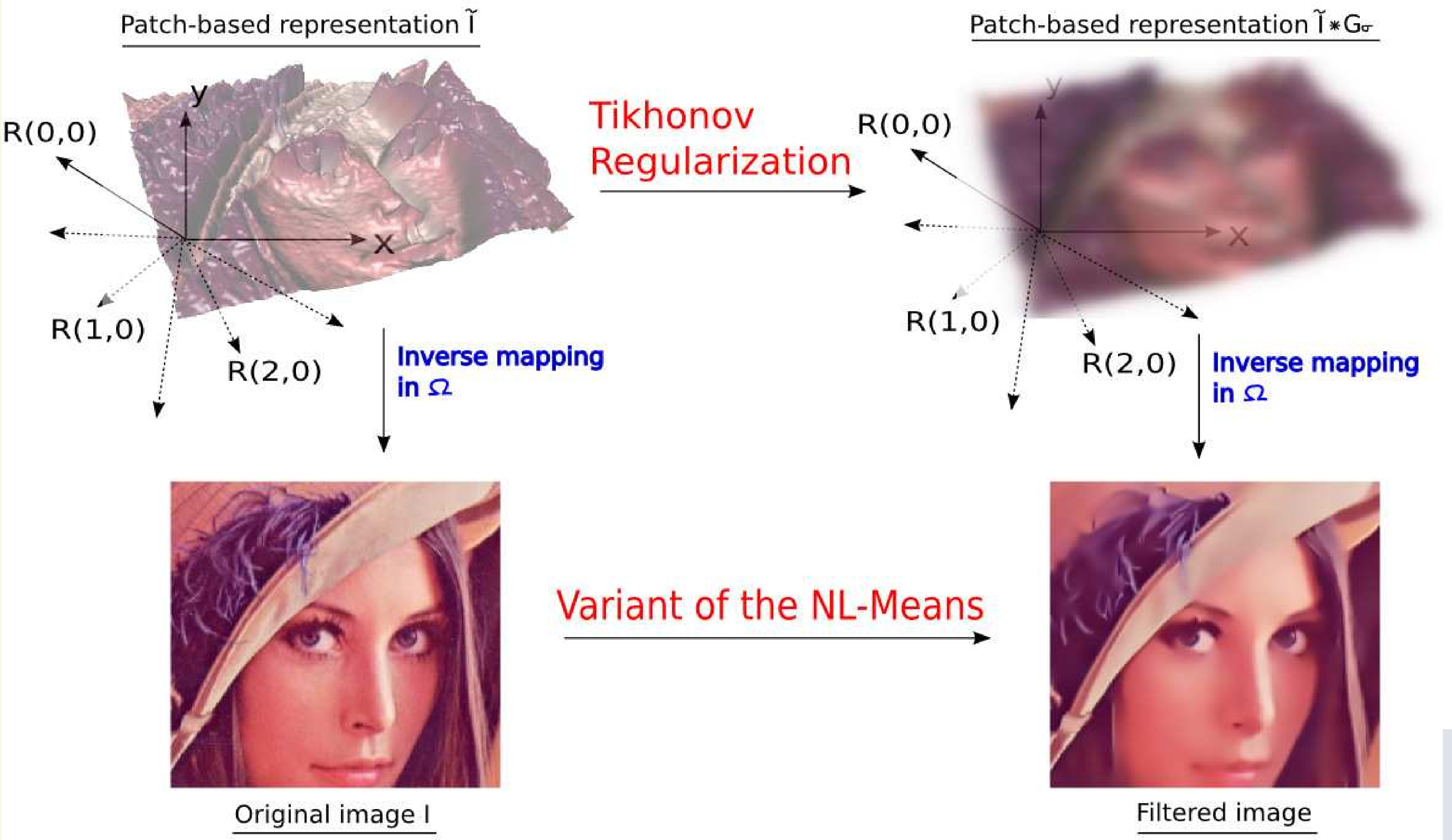
⇒ Variant of the NL-means algorithm (Buades-Morel:05)
with an additional weight depending on the spatial distance between patches in Ω.

⇒ NL-means is an **isotropic diffusion process** in the space of patches Γ.

Tikhonov Regularization in the Patch Space Γ



(Useless) Results (Tikhonov Regularization in Γ)

Noisy color image

(Useless) Results (Tikhonov Regularization in Γ)

Tikhonov regularization in the image domain Ω

(= isotropic smoothing)

(Useless) Results (Tikhonov Regularization in Γ)

Tikhonov regularization in the 5×5 patch space Γ

($\approx$ Non Local-means algorithm)

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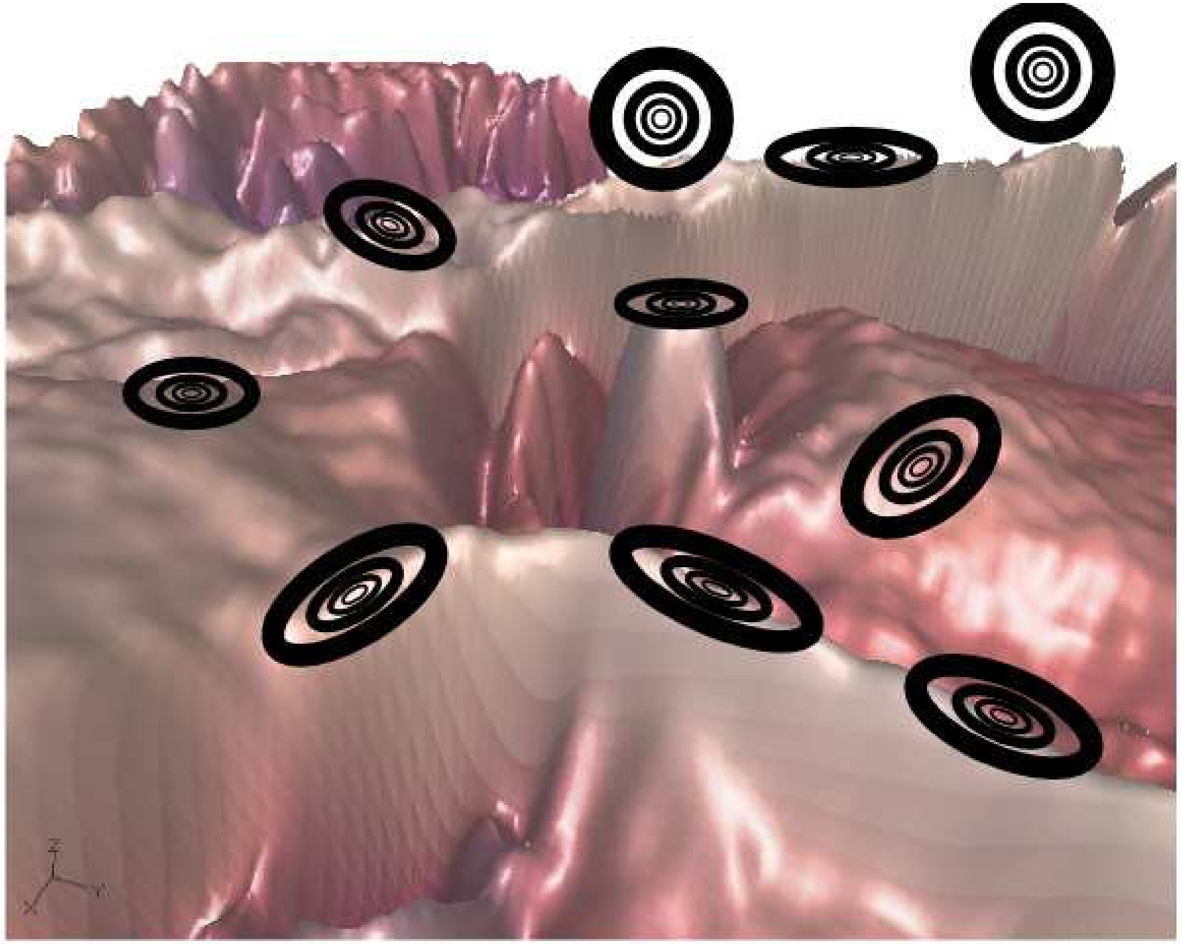
Behavior of Isotropic Diffusion in Γ

- Isotropic diffusion in Γ (NL-means) does not take care of the geometry of the patch mapping $\tilde{\mathbf{I}}$: The smoothing is done homogeneously in all directions.

What We Want to Do : Anisotropic Diffusion

- Anisotropic diffusion would adapt the smoothing kernel to the local geometry of the patch mapping $\tilde{\mathbf{I}}$.



⇒ This **anisotropic behavior** can be described with **diffusion tensors**.

Introducing Diffusion Tensors

- A second-order tensor is a symmetric and semi-positive definite $p \times p$ matrix. (p is the dimension of the considered space).
- It has p positive eigenvalues λ_i and p orthogonal eigenvectors $\mathbf{u}^{[i]}$:

$$\mathbf{T} = \sum_i \lambda_i \, \mathbf{u}^{[i]} {\mathbf{u}^{[i]}}^T$$

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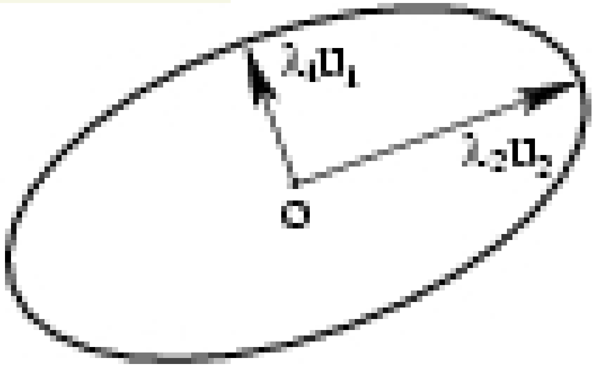

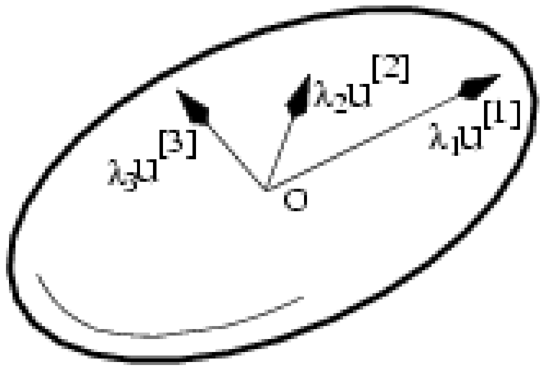

2×2 Tensor (e.g. in Ω) | 3×3 Tensor | $(np^2 + 2) \times (np^2 + 2)$ Tensor

- Diffusion tensors describe how much pixel values locally diffuse along given orthogonal orientations, i.e. the "geometry" of the performed smoothing.

- A tensor field $\mathbf{T}$ can describe locally the amplitudes and the orientations of the desired smoothing.

- The smoothing itself can be performed with the application of this diffusion PDE :

$$\frac{\partial I_{(\mathbf{p})}}{\partial t} = \text{trace}\left(\mathbf{T}_{(\mathbf{p})}\mathbf{H}_{(\mathbf{p})}\right) \qquad (\mathbf{H}_{(\mathbf{p})} \text{ is the Hessian matrix } : \mathbf{H}_{i,j(\mathbf{p})} = \frac{\partial^2 I_{(\mathbf{p})}}{\partial x_i \partial x_j})$$

Diffusion Tensors in Anisotropic Diffusion PDE's

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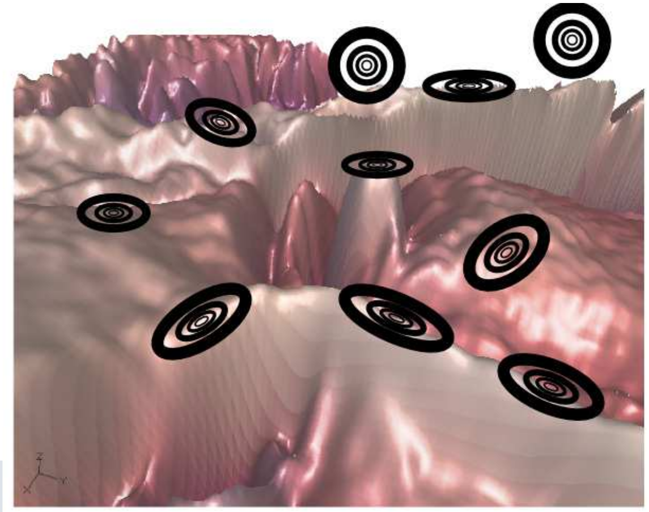
Isotropic tensor field in $\Gamma \Rightarrow$ Isotropic smoothing



Anisotropic tensor field in $\Gamma \Rightarrow$ Anisotropic smoothing

$\Rightarrow$ How to design the tensor field $\mathbf{T}$? $\Rightarrow$ **from the structure tensor field $\mathbf{J}_\sigma$.**

Structure Tensors in the Patch Space Γ

- The structure tensor field $\mathbf{J}_\sigma : \Omega \rightarrow \mathrm{P}(np^2+2)$ tells about local geometric features (local contrast, structure orientation) of $\tilde{\mathbf{I}}$:

$$\tilde{\mathbf{J}}_\sigma = \sum_{i=1}^{np^2+1} \nabla \tilde{I}_{i\sigma} \nabla \tilde{I}_{i\sigma}^T \qquad \text{where} \qquad \nabla \tilde{I}_{i\sigma} = \nabla \tilde{I}_i * G_\sigma$$

$\Rightarrow$ Very useful extension of the notion of “gradient” for multi-dimensional datasets.

(Silvano Di-Zenzo:86, Joachim Weickert:98) used it for 2D images.

$\Rightarrow$ Here, we consider a $np^2 \times np^2$ structure tensor !

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- The diffusion tensor field $\mathbf{T}$ is then designed from $\mathbf{J}_\sigma$:

$$\forall \mathbf{p} \in \Gamma, \qquad \tilde{\mathbf{D}}_{(\mathbf{p})} = \frac{1}{\sqrt{\beta^2 + \text{trace}(\tilde{\mathbf{J}}_{\sigma(\mathbf{p})})}} \left(\mathbf{I}_d - \tilde{\mathbf{u}}_{(\mathbf{p})} \tilde{\mathbf{u}}_{(\mathbf{p})}^T \right)$$

where $\tilde{\mathbf{u}}_{(\mathbf{p})}$ is the main eigenvector of $\tilde{\mathbf{J}}_{\sigma(\mathbf{p})}$.

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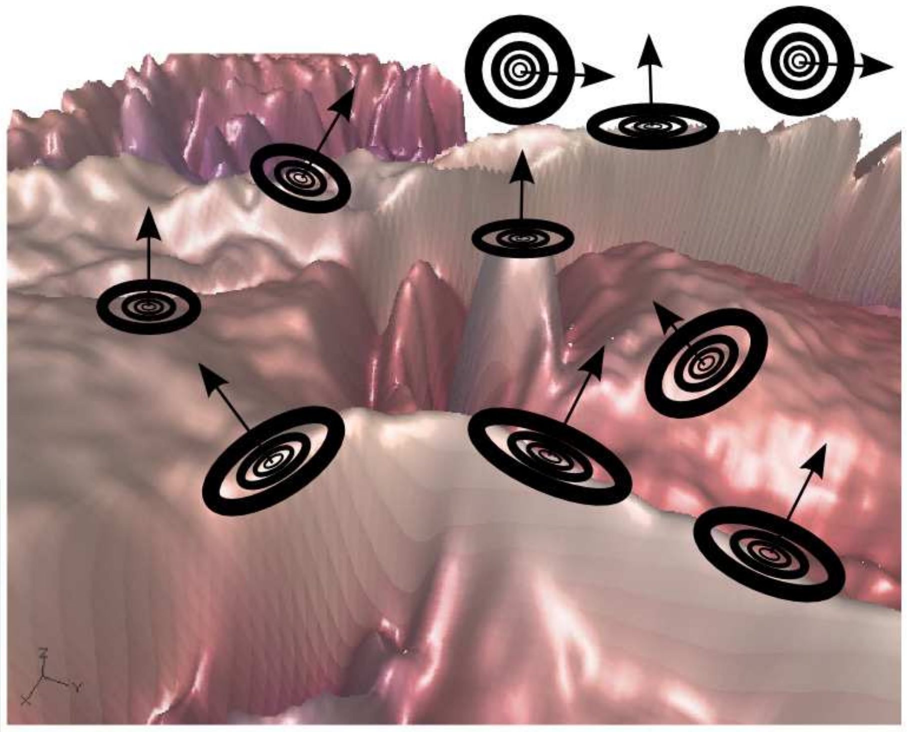

Approximation of the PDE solution

- **Problem :** Obtaining the PDE solution requires several iterations.
- But, we cannot afford to store the entire patch space Γ in computer memory (dim(Γ)=365 for 11x11 color patches).

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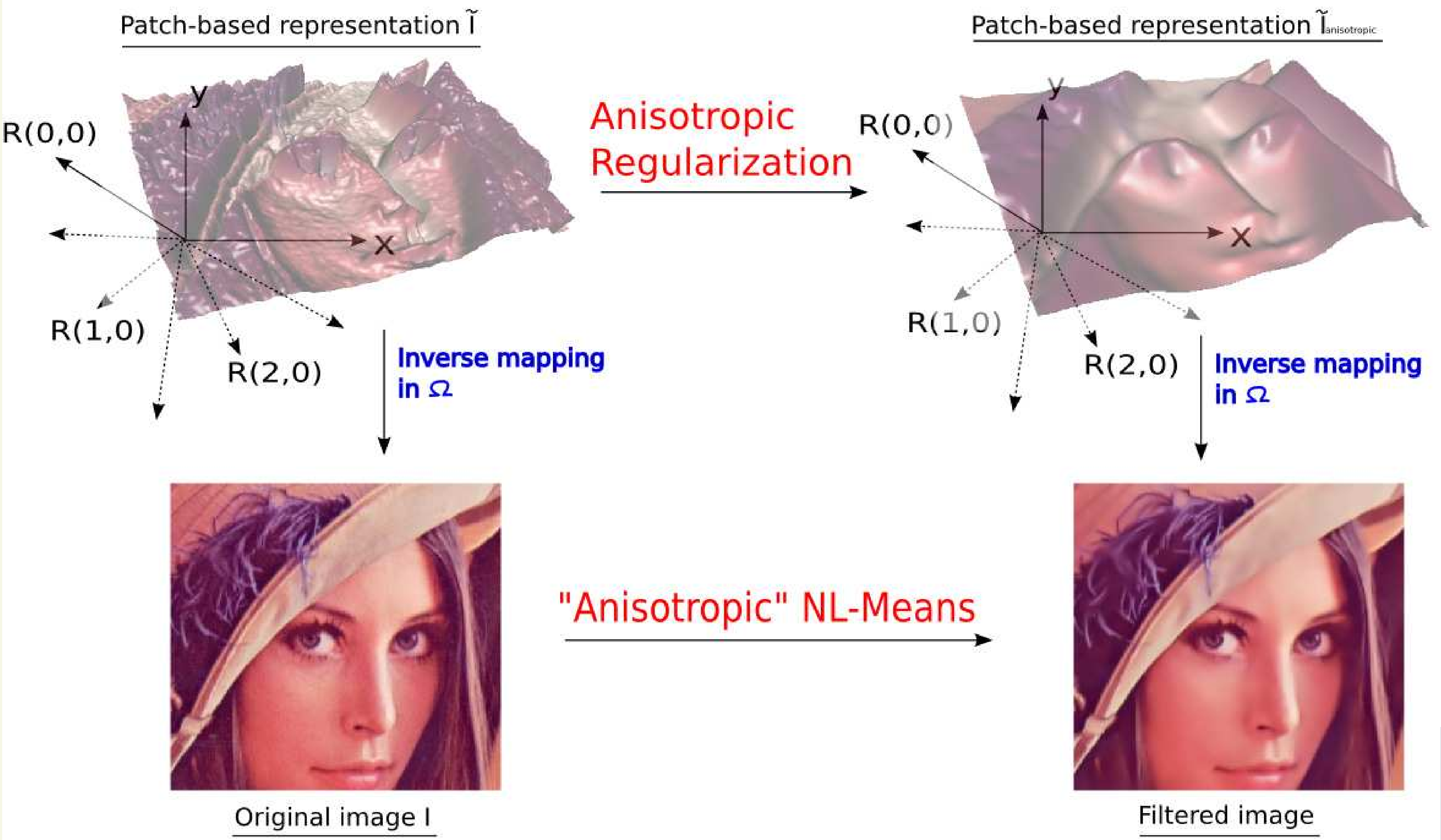
- But, we cannot afford to store the entire patch space Γ in computer memory (dim(Γ)=365 for 11x11 color patches).

⇒ Solution of the PDE can be approximated by one iteration [Tschumperle-Deriche:03] :

$$\tilde{\mathbf{I}}^{[t]}_{(\mathbf{p}_{(x,y)})} \approx \int_{(k,l)\in\Omega} \mathbf{I}^{[t=0]}_{(k,l)} G^{\tilde{\mathbf{D}}_{(\mathbf{p}_{(x,y)})}}_{dt(\mathbf{p}_{(x,y)}-\mathbf{q}_{(k,l)})} \, d_k d_l$$

⇒ Solution approximation + inverse mapping on Ω can be expressed in the image domain.

Anisotropic Diffusion in the Patch Space Γ



Anisotropic Diffusion in the Patch Space (Results)

Original image

Anisotropic Diffusion in the Patch Space (Results)

Anisotropic diffusion in the 7×7 patch space Γ

Anisotropic Diffusion in the Patch Space (Results)

Anisotropic diffusion in the image domain Ω

Anisotropic Diffusion in the Patch Space (Results)

Anisotropic diffusion in Ω

Anisotropic diffusion in the patch space Γ

Anisotropic Diffusion in the Patch Space (Results)

Noisy color image

Anisotropic Diffusion in the Patch Space (Results)

Bilateral filtering

(≈ NL-Means with 1×1 patches)

Anisotropic Diffusion in the Patch Space (Results)

Anisotropic diffusion PDE in the image domain Ω

Anisotropic Diffusion in the Patch Space (Results)

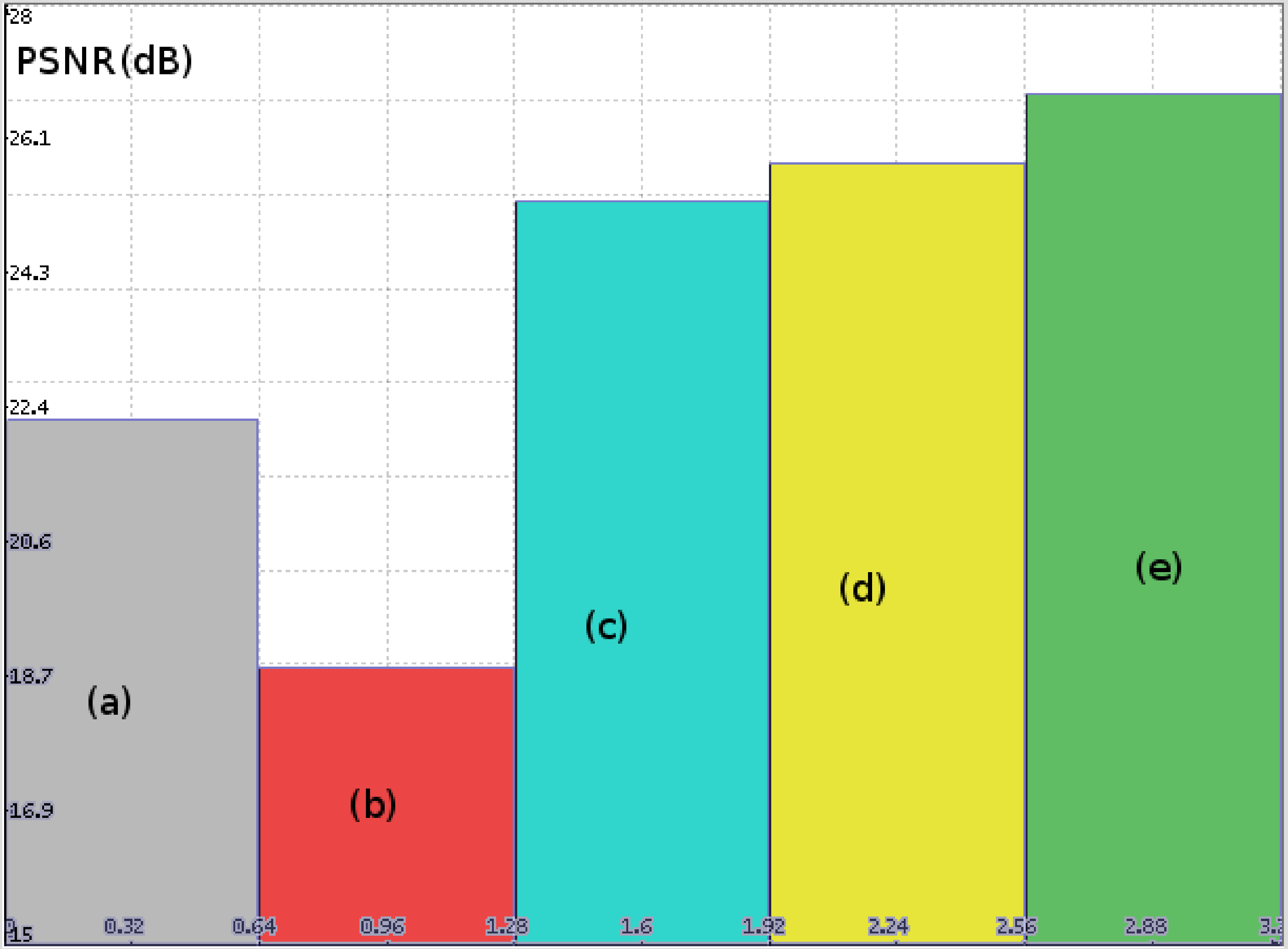
Isotropic diffusion PDE in the 5×5 patch-space Γ

($\approx$ NL-Means with 5×5 patches)

Anisotropic Diffusion in the Patch Space (Results)

Anisotropic diffusion PDE in the 5×5 patch-space Γ

Anisotropic Diffusion in the Patch Space (Results)



Corresponding PSNR compared to the noise-free version

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$\Rightarrow$ Patch-based Lucas-Kanade registration.

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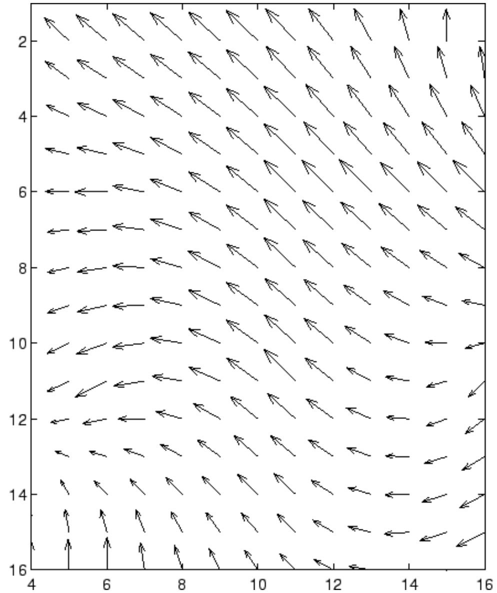
The image registration problem

- Given two images $\mathbf{I}^{t_1}$ and $\mathbf{I}^{t_2}$, find the displacement field $\mathbf{u} : \Omega \rightarrow \mathbb{R}^2$ from $\mathbf{I}^{t_1}$ to $\mathbf{I}^{t_2}$

Source image $\mathbf{I}^{t_1}$

Target image $\mathbf{I}^{t_2}$



Estimated displacement $\mathbf{u}$

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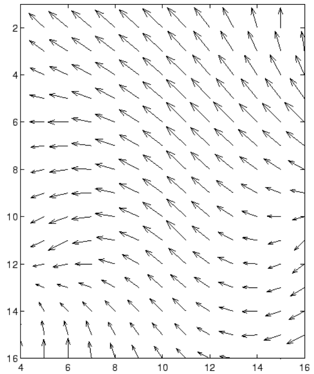

Estimated displacement $\mathbf{u}$

- The Lukas-Kanade registration method is based on the minimization of :

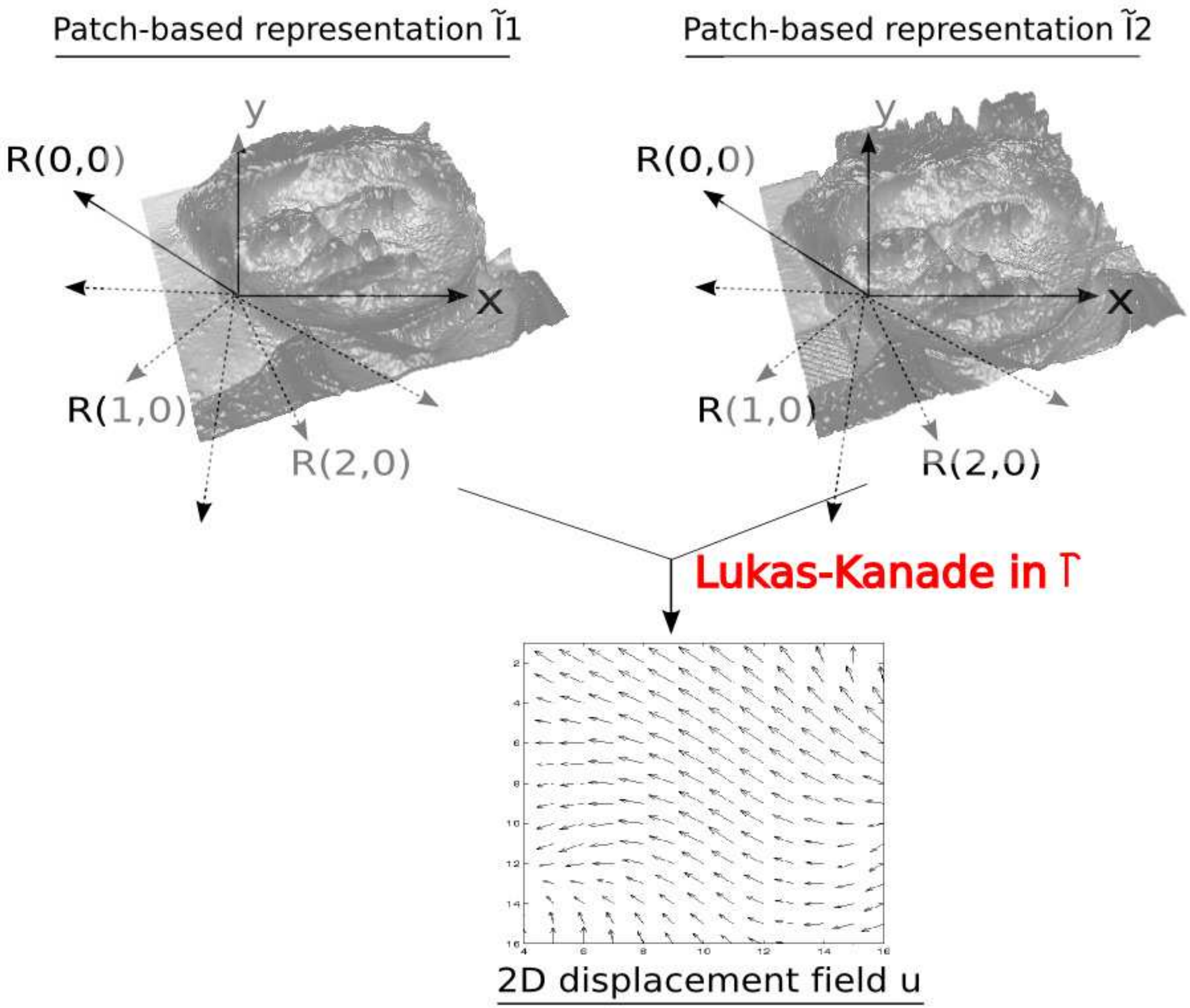
$$E(\mathbf{u}) = \int_{\Omega} \alpha \, \|\nabla \mathbf{u}_{(\mathbf{p})}\|^2 + \|\mathcal{D}_{(\mathbf{p},\mathbf{p}+\mathbf{u})}\|^2 \, d\mathbf{p}$$

- **Intensity preservation :**
  The intensity dissimilarity between warped $\mathbf{I}^{t_1}$ and $\mathbf{I}^{t_2}$ must be minimal.

$$\mathcal{D}_{(\mathbf{p},\mathbf{q})} = (\mathbf{I}^{t_1}_{\sigma(\mathbf{p})} - \mathbf{I}^{t_2}_{\sigma(\mathbf{q})}) \quad \text{where} \quad \mathbf{I}^{t_k}_{\sigma} = \mathbf{I}^{t_k} * G_{\sigma}$$

Transposition to the patch-space Γ

- We propose to solve the Lukas-Kanade problem with a dissimilarity measure defined in the patch space Γ, instead of on the image domain Ω



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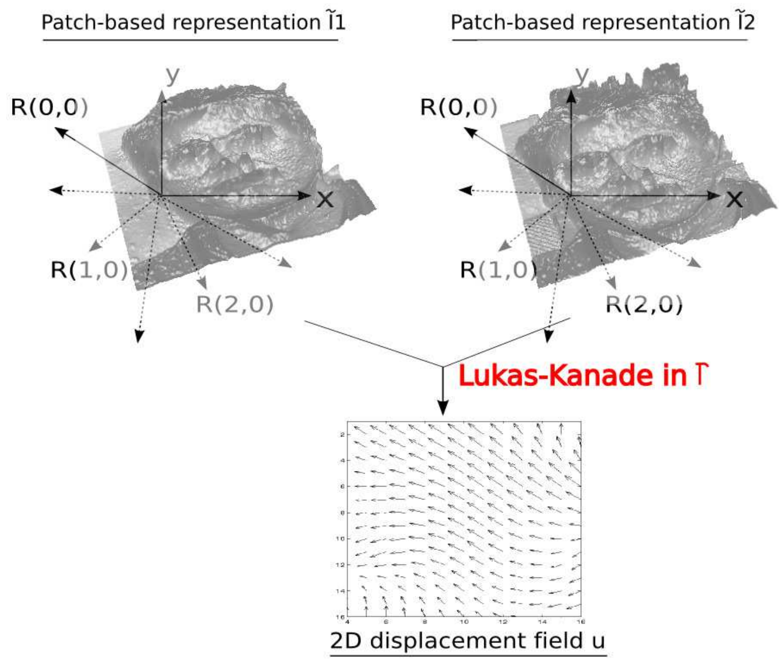
$$\mathcal{D}_{patch(\mathbf{p},\mathbf{q})} = (\tilde{\mathbf{I}}^{t_1}_{\sigma(\mathbf{p},\mathcal{P}^{\tilde{\mathbf{I}}^{t}1}_{max(\mathbf{p})})} - \tilde{\mathbf{I}}^{t_2}_{\sigma(\mathbf{q},\mathcal{P}^{\tilde{\mathbf{I}}^{t}2}_{max(\mathbf{q})})})$$

- i.e. Find the best 2D warp between patch representations $\tilde{\mathbf{I}}^{t_1}$ and $\tilde{\mathbf{I}}^{t_2}$.



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$\Rightarrow$ **Patch-preservation :**
The patch dissimilarity between warped $\mathbf{I}^{t_1}$ and $\mathbf{I}^{t_2}$ must be minimal.

$\Rightarrow$ **Bloc-matching-like** dissimilarity measure + **Smoothness constraints**.
(Classical bloc-matching gives the global minimum when smoothness $\alpha = 0$).

Minimizing PDE flow

- The Euler-Lagrange equations give the minimizing flow for the patch-based Lukas-Kanade functional :

$$\begin{cases} \mathbf{u}_{[t=0]} = \vec{0} \\ \dfrac{\partial u_{j(\mathbf{x})}}{\partial t} = \alpha \, \Delta u_j + \\ \displaystyle\sum_{i=1}^{np^2+1} \left(\tilde{I}^{t_1}_{\sigma i(\mathbf{x}, \mathcal{P}^{\mathbf{I}^{t_1}}_{(\mathbf{x})})} - \tilde{I}^{t_2}_{\sigma i(\mathbf{x}+\mathbf{u}, \mathcal{P}^{\mathbf{I}^{t_2}}_{(\mathbf{x}+\mathbf{u})})} \right) \left[\nabla \mathcal{G}_i \right]_{j(\mathbf{x}+\mathbf{u})} \end{cases}$$

where $\mathcal{G}_{i(\mathbf{x})} = \tilde{I}^{t_2}_{\sigma i(\mathbf{x}, \mathcal{P}^{\mathbf{I}^{t_2}}_{(\mathbf{x})})}$.

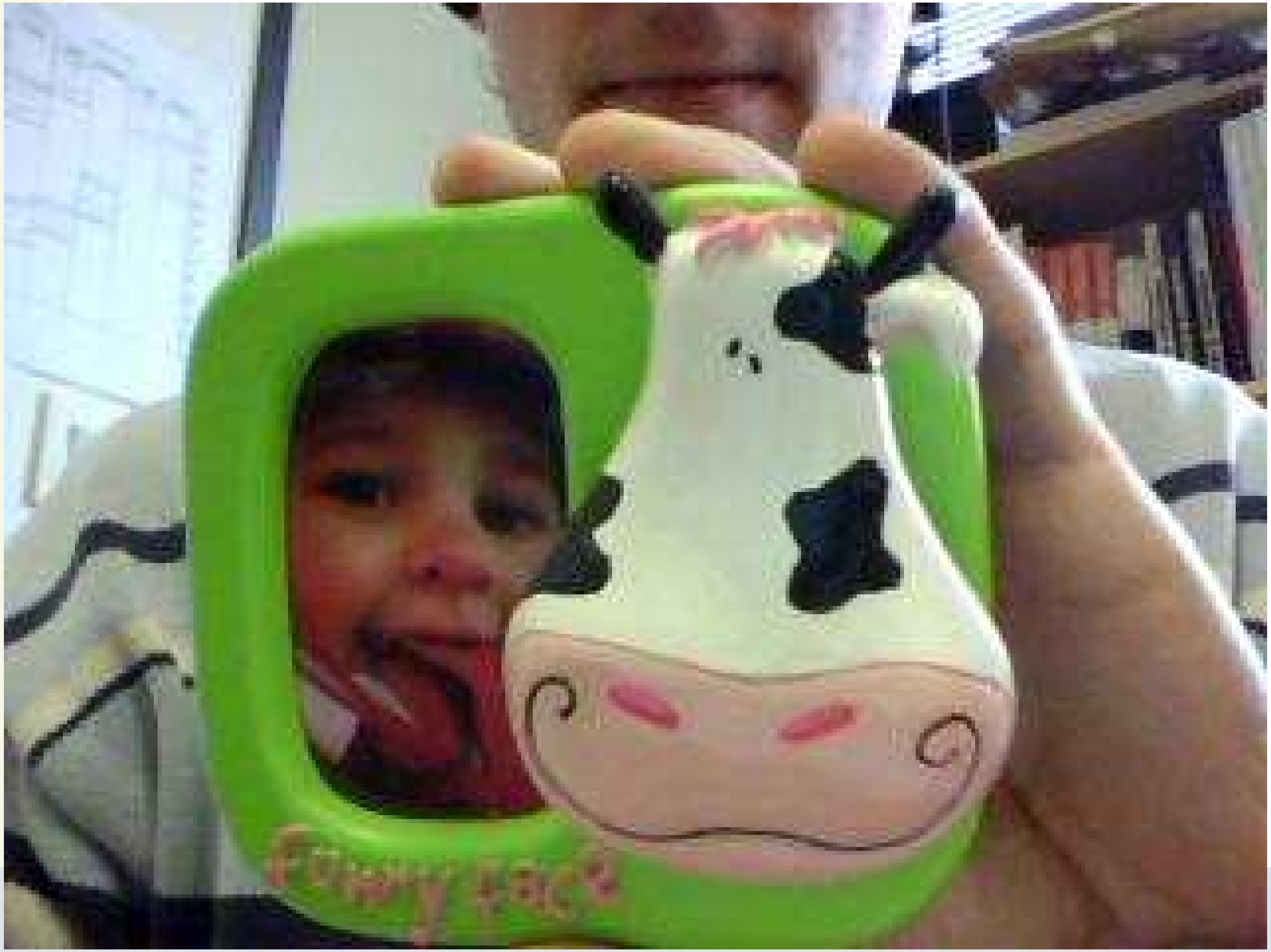
⇒ Local minimum of the functional.

- Resolution is done with a classical multi-scale approach (coarse to fine).

Patch-based Lukas-Kanade (Results)

Source color image

Patch-based Lukas-Kanade (Results)



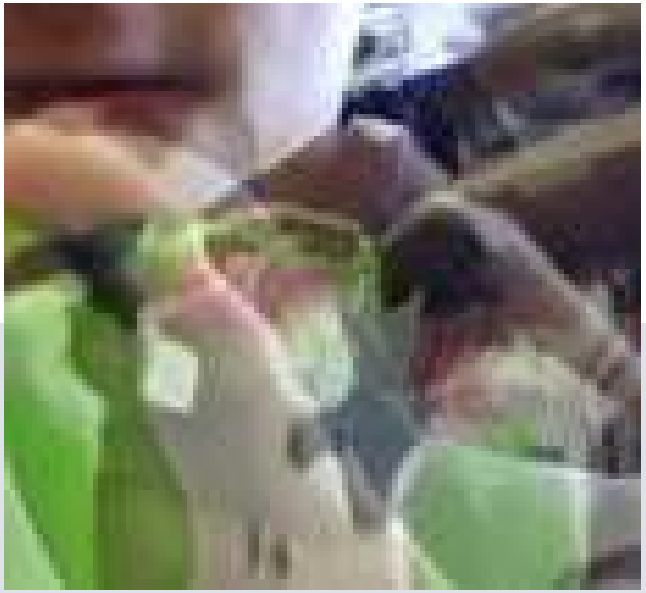
Target color image

Patch-based Lukas-Kanade (Results)

Estimated displacement

Warped source

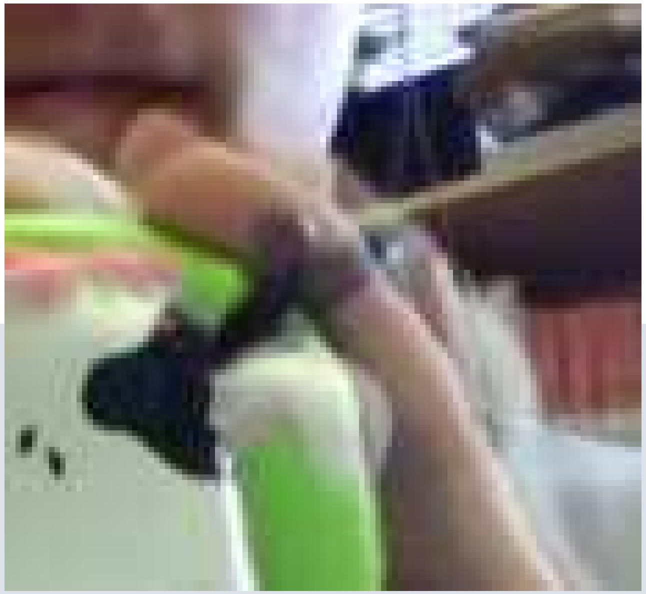
Result of the original Lukas-Kanade algorithm (smoothness $\alpha = 0.01$)



Patch-based Lukas-Kanade (Results)

Estimated displacement

Warped source



Result of the original Lukas-Kanade algorithm (smoothness $\alpha = 0.1$)

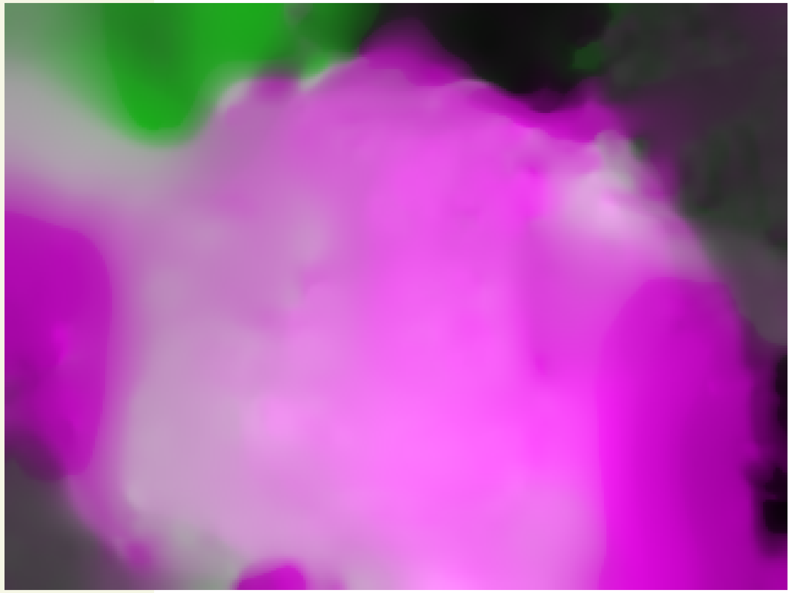
Patch-based Lukas-Kanade (Results)

Estimated displacement

Warped source

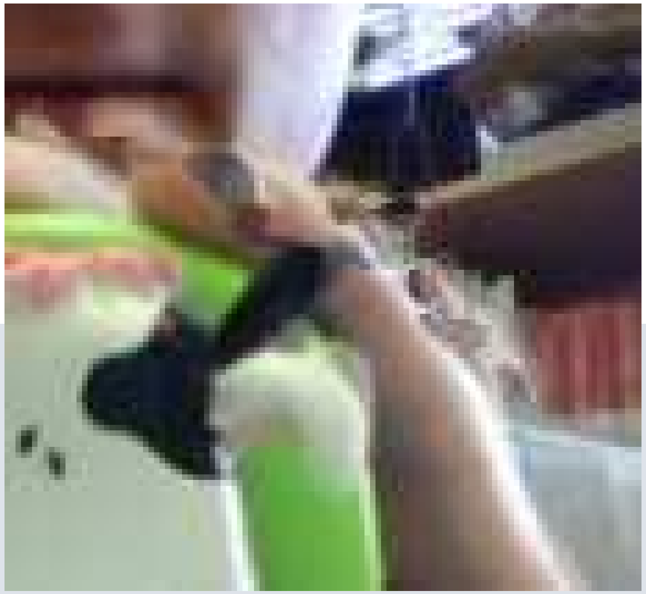
Result of the bloc-matching algorithm (7×7 patches)

Patch-based Lukas-Kanade (Results)



Estimated displacement

Warped source

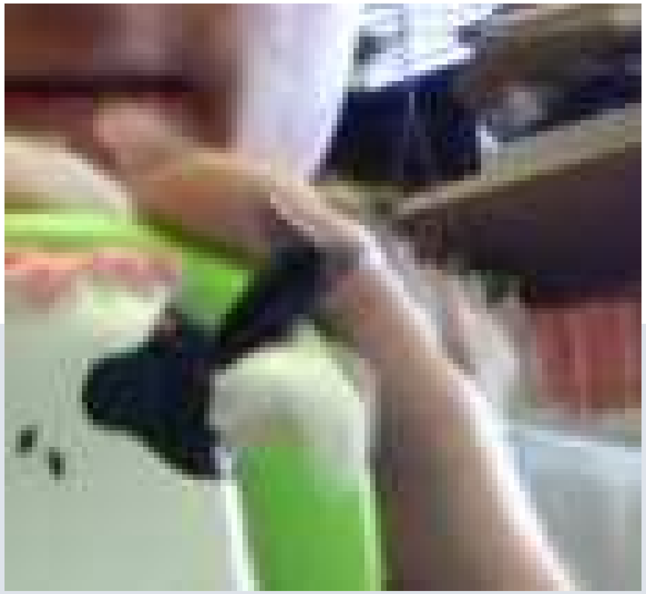


Result of the 7×7 Patch-Based Lukas-Kanade algorithm (smoothness $\alpha = 0$)

Patch-based Lukas-Kanade (Results)

Estimated displacement

Warped source



Result of the 7×7 Patch-Based Lukas-Kanade algorithm (smoothness $\alpha = 0.01$)

Presentation Layout

- Definition of a Patch Space Γ.
- Patch-based Tikhonov Regularization.
- Patch-based Anisotropic Diffusion PDE's.
- Patch-based Lucas-Kanade registration.

$\Rightarrow$ Conclusions & Perspectives.

Conclusions

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NL-means and Bilateral Filtering	$\Leftrightarrow$	**Isotropic diffusion in Γ.**
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NL-means and Bilateral Filtering	$\Leftrightarrow$	**Isotropic diffusion in Γ.**
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- (3) We applied more complex local methods on Γ to get more efficient non-local methods in Ω.

Anisotropic NL-means and Bilateral Filtering
Lukas-Kanade in Γ with smoothness constraint.

Perspectives

$\Rightarrow$ More local methods to transpose to the patch-space Γ !

- Texture-preserving inpainting (Perez-Criminisi) and Texture synthesis (Wei-Levoy) $\Longleftrightarrow$ Transport equations in Γ ?
- You are welcome to suggest other perspectives...

Questions ?

- Thanks for your patience !

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