Graphical Models, Distributed Fusion, and Sensor Networks

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One Group's Journey

- The launch: Collaboration with Albert Benveniste and Michele Basseville
  - Initial question: what are wavelets ***really*** good for (in terms that a card-carrying statistical signal processor would like)
    - What does optimal inference mean and look like for multiresolution models (whatever they are)
  - The answer (at least **our** answer): Stochastic models defined on multiresolution ***trees***

MR tree models as a cash cow

- MR models on trees admit really fast and scalable algorithms that involve propagation of statistics up and down (more generally throughout the tree)
  - Generalization of Levinson
  - Generalization of Kalman filters and RTS smoothers
  - Calculation of likelihoods
  - ...

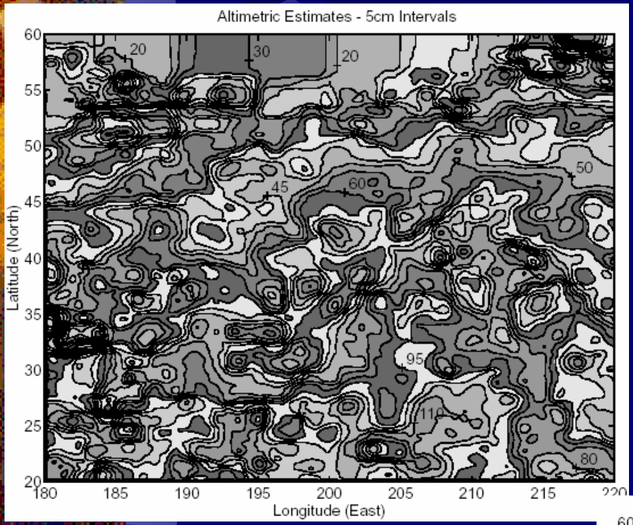
Milking that cow for all it's worth

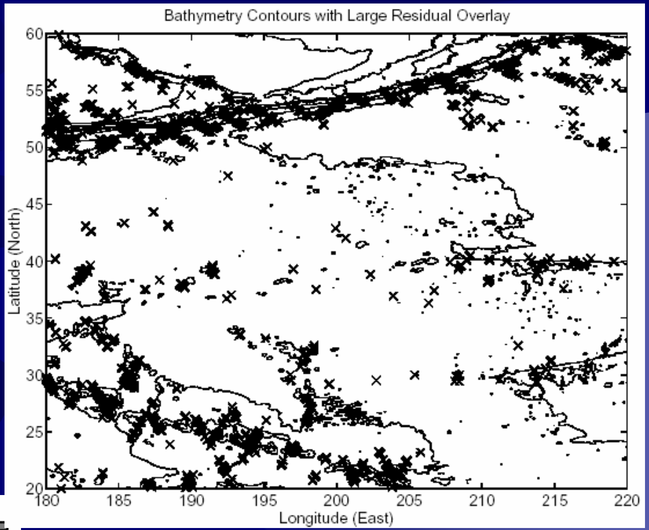
- Theory
  - Old control theorists never die: Riccati equations, MR system theory, etc.
  - MR models of Markov processes and fields
  - Stochastic realization theory and "internal models"
  - MR internal wavelet representations
  - New results on max-entropy covariance extension

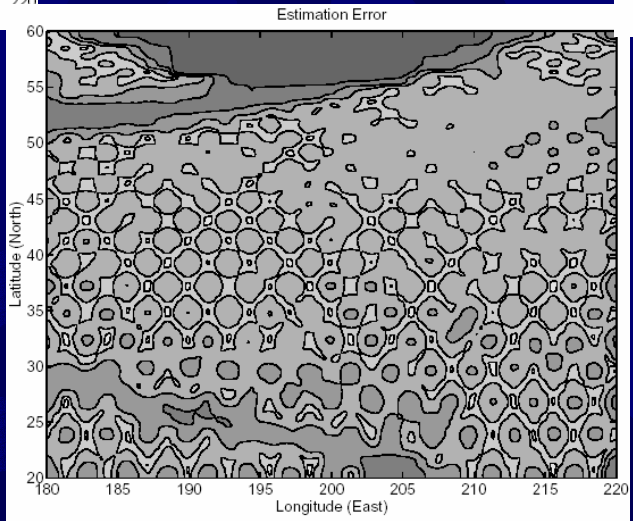
Keep on milking…

- Applications
  - Computer vision/image processing
    - Motion estimation in image sequences
    - Image restoration and reconstruction
  - Geophysics
    - Oceanography
    - Groundwater hydrology
    - Helioseismology (???)
    - Other fields I don’t understand and probably can’t spell

One F'rinstance







Sadly, cows can't fly (no matter how hard they flap their ears)

- The dark side of trees is the same as the bright side: No loops
- Try #1: Pretend the problem isn't there
  - If the real objectives are at coarse scales, then fine-scale artifacts may not matter
- Try #2: Beat the dealer
  - Cheating: Averaging multiple trees
  - Theoretically precise cheating: Overlapping trees
- Try #3: Partial (and later, total) surrender
  - Put the &#%!*@# loops in!!
  - Now we're playing on the same field (sort of) as AI graphical model-niks and statistical physicists

Graphical Models 101

- ***G*** = (V, E) = a graph
  - V = Set of vertices
  - $E \subset V \times V$ = Set of edges
  - **C** = Set of cliques
- Markovianity on ***G*** (Hammersley-Clifford)

$$P(\{x_S \mid s \in V\}) \propto \prod_{C \subset \mathrm{C}} \psi_C(x_C)$$

- Objectives

$$\text{Estimation:} \quad \text{Compute} \quad P_s(x_s)$$

$$\text{Optimization:} \quad \arg\max \quad P(\{x_S \mid s \in V\})$$

For trees: Optimal algorithms compute reparameterizations

For Estimation

$$P(\{x_s \mid s \in \mathcal{V}\}) = \prod_{s \in \mathcal{V}} P_s(x_s) \prod_{(s,t) \in \mathcal{E}} \frac{P_{st}(x_s, x_t)}{P_s(x_s) P_t(x_t)}$$

For Optimization

$$P(\{x_s \mid s \in \mathcal{V}\}) \propto \prod_{s \in \mathcal{V}} \overline{P}_s(x_s) \prod_{(s,t) \in \mathcal{E}} \frac{\overline{P}_{st}(x_s, x_t)}{\overline{P}_s(x_s) \overline{P}_t(x_t)}$$

$$\overline{P}_s(x_s) = \max_{\{x_t \mid t \neq s\}} P(\{x_s \mid s \in \mathcal{V}\})$$

Algorithms that do this on trees

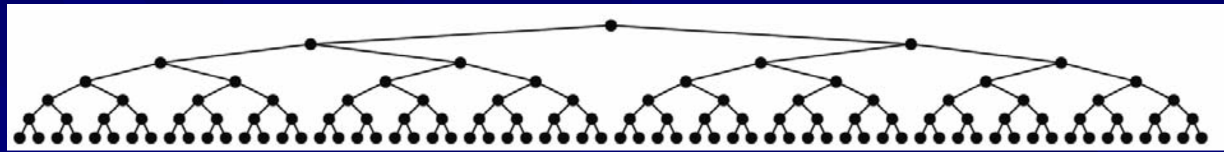
- *Message-passing* algorithms for "estimation" (marginal computation)
  - Two-sweep algorithms (leaves-root-leaves)
    - For linear/Gaussian models, these are the generalizations of Kalman filters and smoothers
  - Belief propagation, sum-product algorithm
    - Non-directional (no root; all nodes are equal)
    - Lots of freedom in message scheduling
- *Message-passing* algorithms for "optimization" (MAP estimation)
  - Two sweep: Generalization of Viterbi/dynamic programming
  - Max-product algorithm

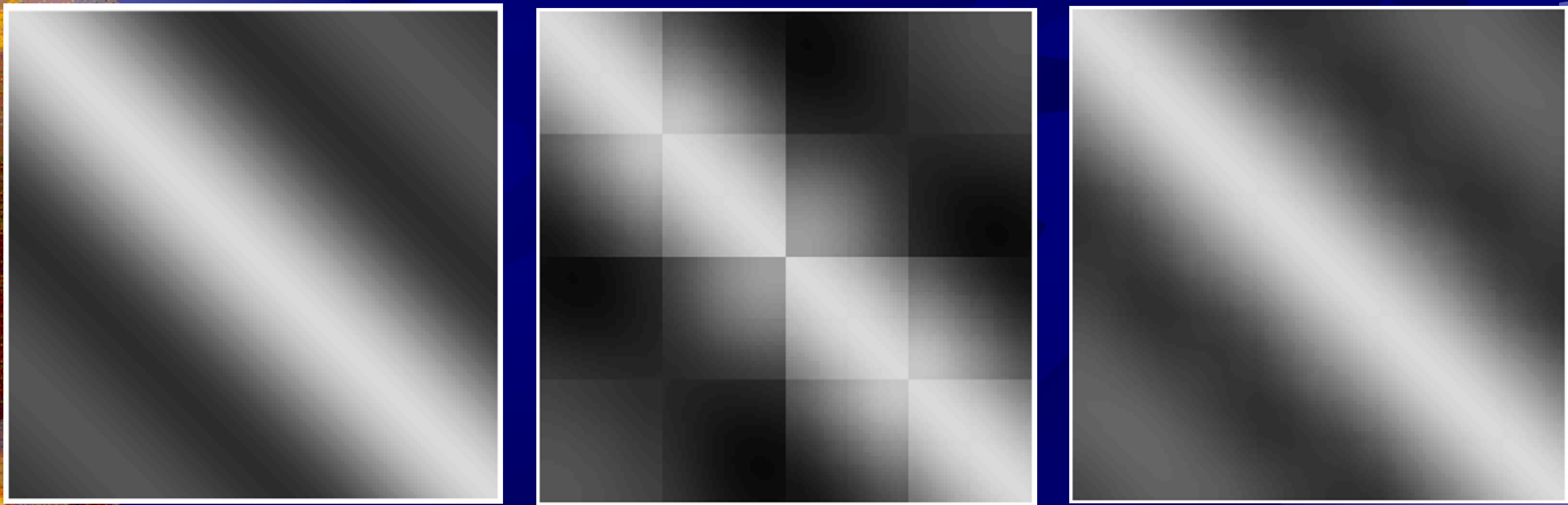
What do people do when there are loops?

- One well-oiled approach
  - Belief propagation (and max-product) are algorithms whose local form is well defined for any graph
  - So why not just use these algorithms?
- Well-recognized limitations
  - The algorithm fuses information based on invalid assumptions of conditional independence
  - Think Chicken Little, rumor propagation,…
  - Do these algorithms converge?
  - If so, what do they converge to?

Near trees can help cows at least to hover…

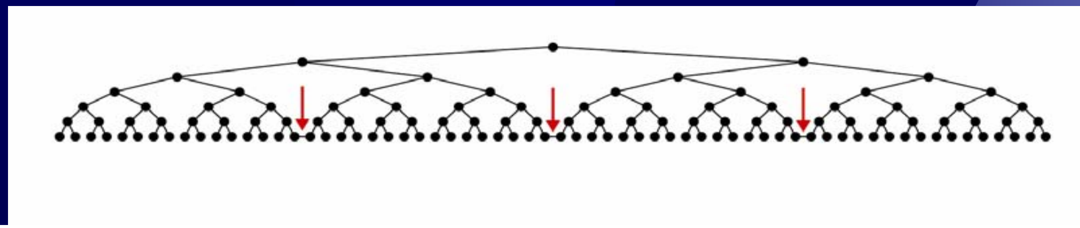
Tree





Exact Covariance　　Tree Covariance　　Near-Tree Covariance

Near-Tree



Something else we've been doing: Tree-reparameterization

- For *any* embedded acyclic structure:

For Estimation

$$P(\{x_s \mid s \in \mathcal{V}\}) = \prod_{s \in \mathcal{V}} T_s(x_s) \prod_{(s,t) \in \mathcal{E}_{tree}} \frac{T_{st}(x_s, x_t)}{T_s(x_s) T_t(x_t)} \times \text{Remainder}$$

For Optimization

$$P(\{x_s \mid s \in \mathcal{V}\}) \propto \prod_{s \in \mathcal{V}} \bar{T}_s(x_s) \prod_{(s,t) \in \mathcal{E}_{tree}} \frac{\bar{T}_{st}(x_s, x_t)}{\bar{T}_s(x_s) \bar{T}_t(x_t)} \times \text{Remainder}$$

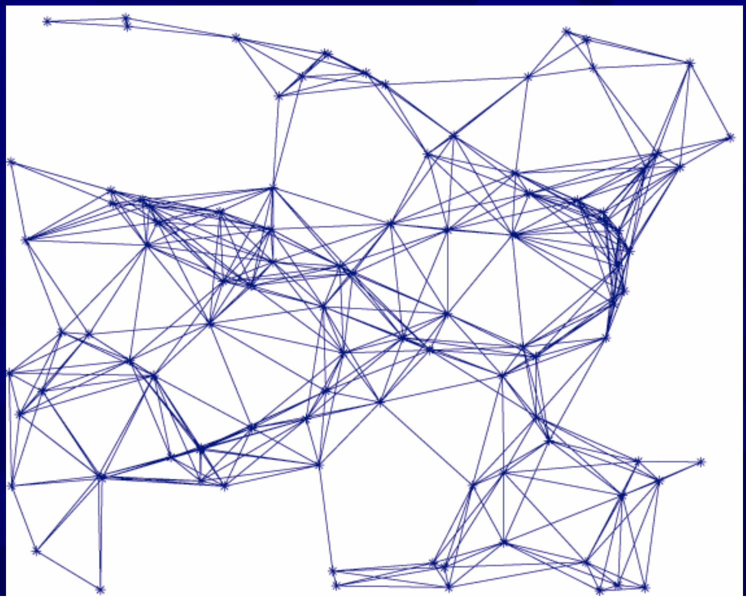
$$\bar{T}_s(x_s) = \max_{\{x_t \mid t \neq s\}} T(\{x_s \mid s \in \mathcal{V}\})$$

So what does any of this have to do with distributed fusion and sensor networks?

- Well, we ***are*** talking about passing messages and fusing information
- But there are special issues in sensor networks that add some twists and require some thought
  - And that also lead to new results for graphical models more generally

A first example: Sensor Localization and Calibration

- Variables at each node can include
  - Node location, orientation, time offset
- Sources of information
  - Priors on variables (single-node potentials)
  - Time of arrival (1-way or 2-way), bearing, and ***absence of signal***
    - These enter as edge potentials
    - Modeling absence of signals may be needed for well-posedness, but it also leads to denser graphs



Even this problem raises new challenges

- BP algorithms require sending messages that are likelihood functions or prob. distributions
  - That's fine if the variables are discrete or if we are dealing with linear-Gaussian problems
  - More generally very little was available in the literature (other than brute-force discretization)
- Our approach: Nonparametric Belief Propagation (NBP)

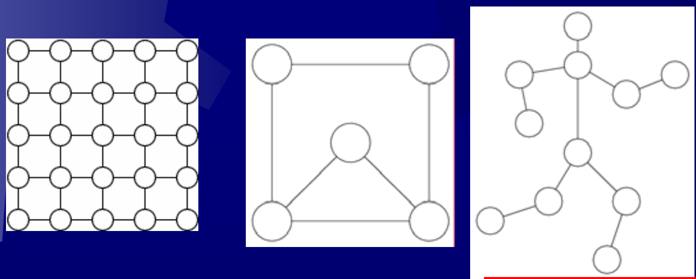
Nonparametric Inference for General Graphs

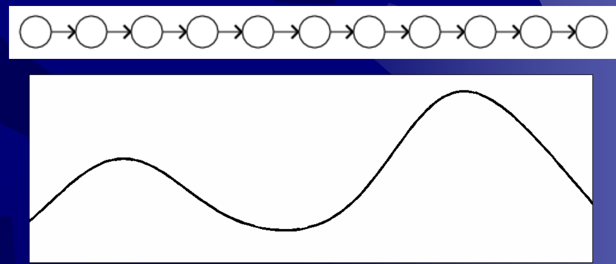
Belief Propagation

- General graphs
- Discrete or Gaussian

Particle Filters

- Markov chains
- General potentials



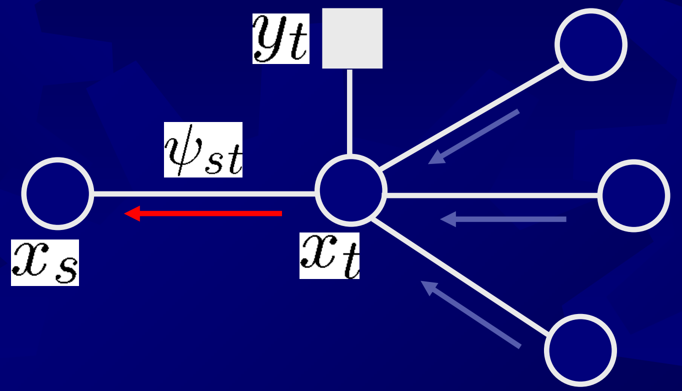


Nonparametric BP

- General graphs
- General potentials

Problem: **What is the product of two collections of particles?**

Nonparametric BP



$$m_{ts}(x_s) = \alpha \int_{x_t} \psi_{s,t}(x_s, x_t)\psi_t(x_t, y_t) \prod_{u \in \Gamma(t) \setminus s} m_{ut}(x_t)\, dx_t$$

Stochastic update of kernel based messages:

I. Message Product: Draw samples of x_t from the product of all incoming messages and the local observation potential

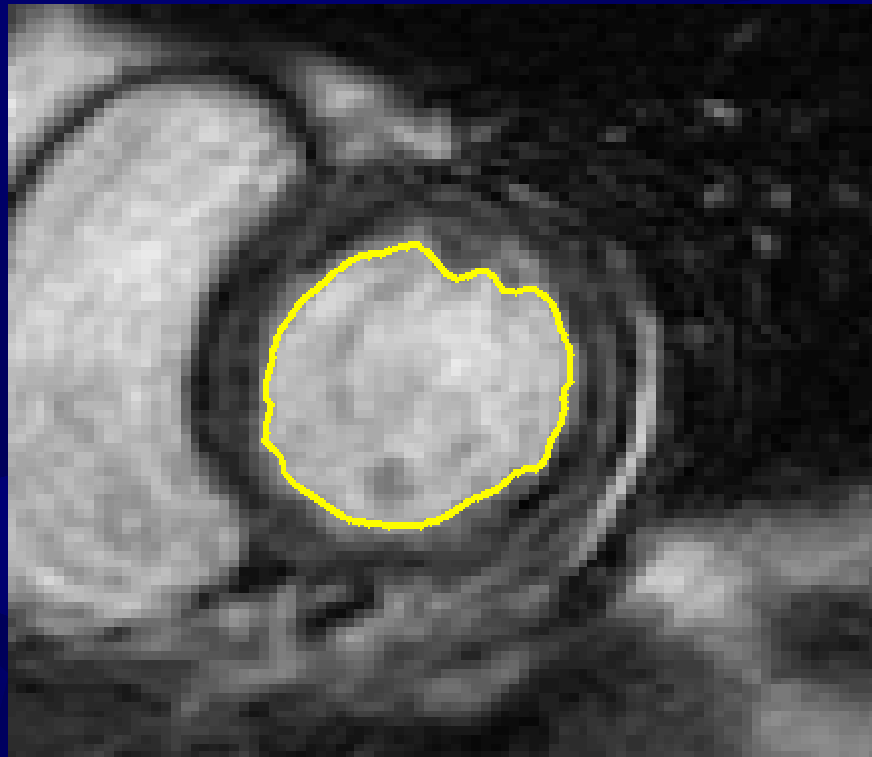
II. Message Propagation: Draw samples of x_s from the compatibility function, $\psi_{st}(x_s, x_t)$, fixing x_t to the values sampled in step I

→ Samples form new kernel density estimate of outgoing message (determine new kernel bandwidths)

NBP particle generation

- Dealing with the explosion of terms in products
  - How do we sample from the product without explicitly constructing it?
- The key issue is solving the label sampling problem (which kernel)
  - Solutions that have been developed involve
    - Multiresolution Gibbs sampling using KD-trees
    - Importance sampling

Examples: Hand-tracking and contour tracking using level sets

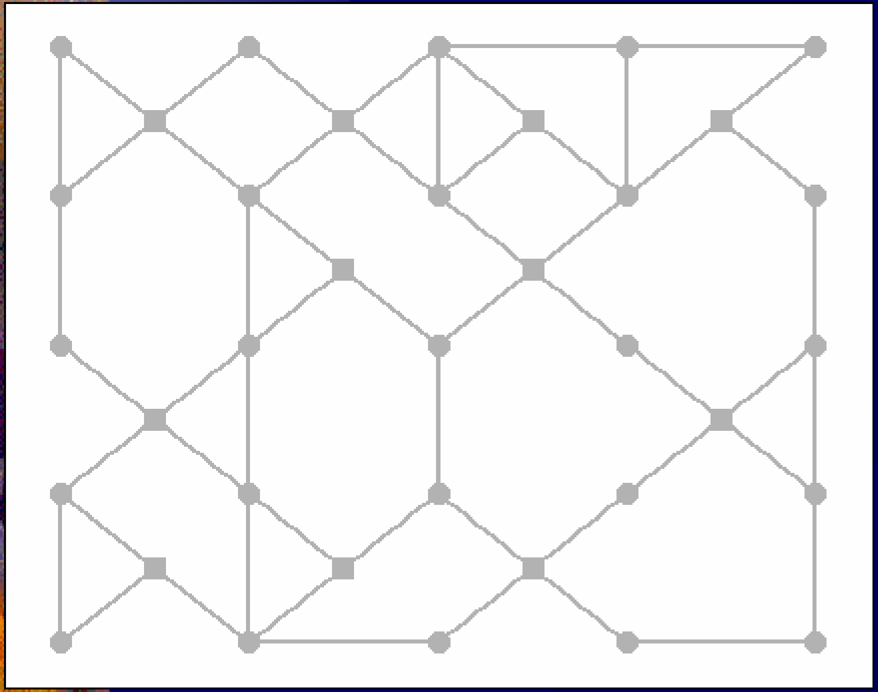


Communications-sensitive message-passing

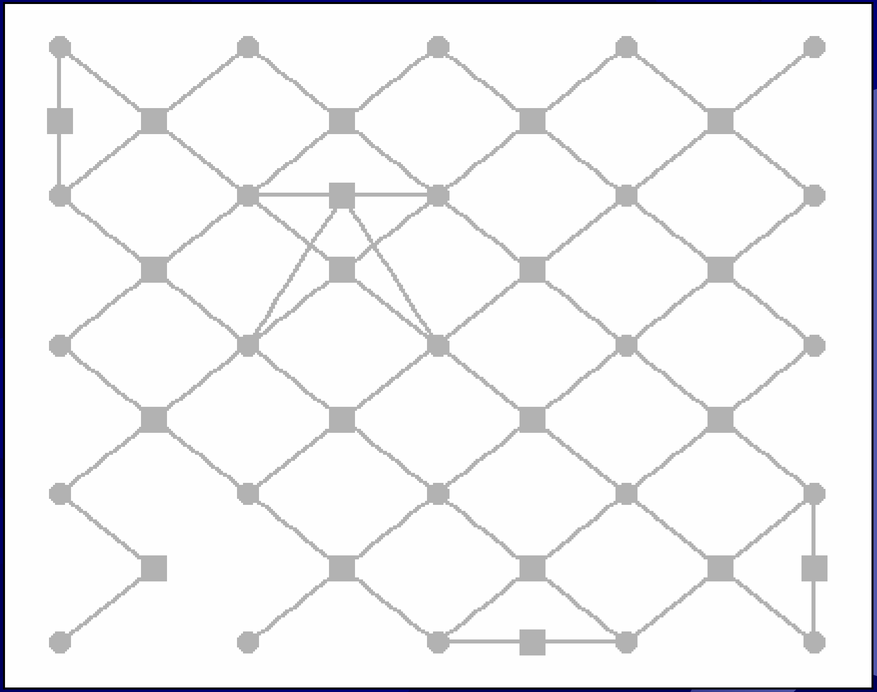
- Objective:
  - Provide each node with computationally simple (and completely local) mechanism to decide if sending a message is worth it
  - Need to adapt the algorithm in a simple way so that each node has a mechanism for updating its beliefs when it doesn’t receive a full set of messages
- Simple rule:
  - Don’t send a message if the K-L divergence from the previous message falls below a threshold
  - If a node doesn’t receive a message, use the last one sent (which requires a bit of memory: to save the last one sent)

Illustrating comms-sensitive message-passing dynamics

Organized network data association



Self-organization with region-based representation



Empirical observations

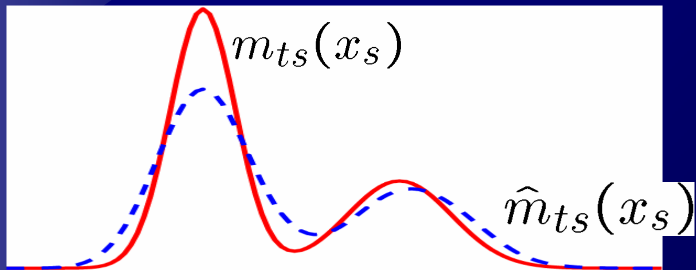
- Sharp transitions in performance as a function of message tolerance threshold
- Dynamics of messaging provides scenario-dependent adaptivity automatically
- However:
  - Where is the ***theory*** to explain this behavior and provide design guidelines?
  - This approach bases censoring ***solely*** on the information as measured by the ***transmitting node***, with no attention paid to the objectives of the ***receiving node***

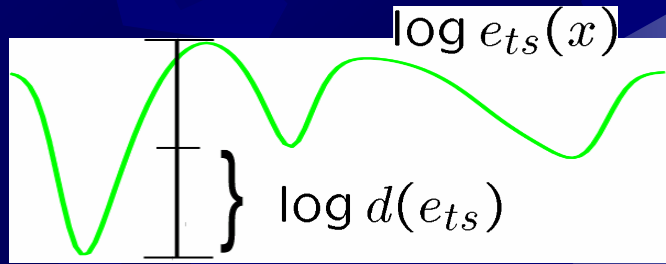
How different are BP messages?

- Message “error” as ratio (or, difference of log-messages)

$$e_{ts}(x_s) = m_{ts}(x_s) / \widehat{m}_{ts}(x_s)$$

$$\log e_{ts}(x_s) = \log m_{ts}(x_s) - \log \widehat{m}_{ts}(x_s)$$





- One (scalar) measure
  - Dynamic range
  - Equivalent log-form

$$d(e_{ts}) = \sup_{a,b} \sqrt{e_{ts}(a)/e_{ts}(b)}$$

$$\log d(e_{ts}) = \inf_{\alpha} \sup_{x} |\log \alpha m_{ts}(x) - \log \widehat{m}_{ts}(x)|$$
$$= \inf_{\alpha} \sup_{x} |\log \alpha - \log e_{ts}(x)|$$

Why dynamic range?

- Satisfies sub-additivity condition

$M_t(x) \propto m_{ut}(x) \cdot m_{st}(x)$ $\widehat{M}_t(x) \propto \widehat{m}_{ut}(x) \cdot \widehat{m}_{st}(x)$

$$\Rightarrow \quad \log d(E_t) = \log d(M_t / \widehat{M}_t) \\ \leq \log d(e_{ut}) + \log d(e_{st})$$

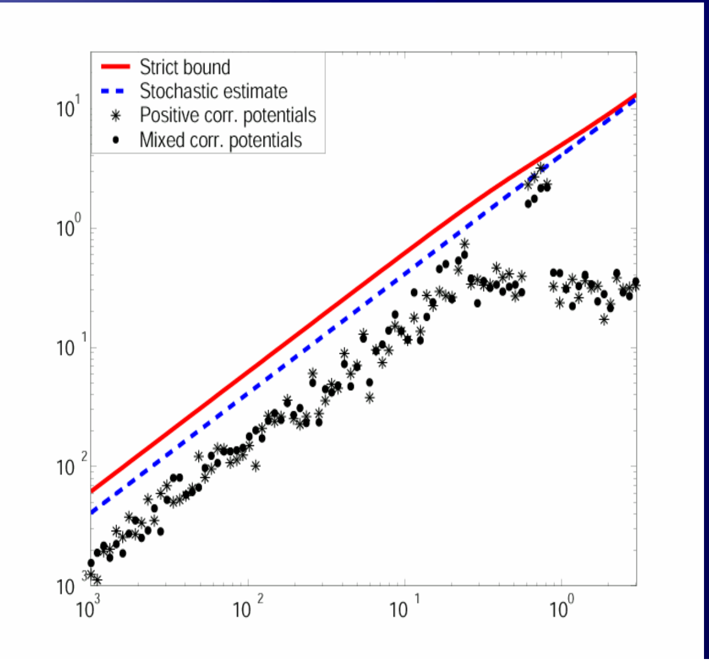
- Message errors contract under edge potential strength/mixing condition

Results using this measure

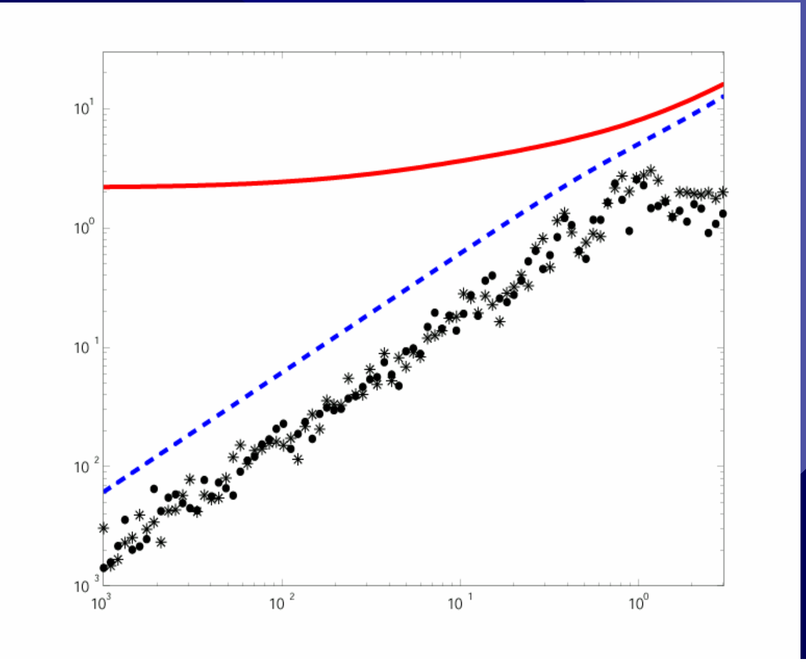
- Best known convergence results for loopy BP
  - Result also provides result on relative locations of multiple fixed points
- Bounds and stochastic approximations for effects of (possibly intentional) message errors

Experiments

- Relatively weak potential functions
  - Loopy BP guaranteed to converge
  - Bound and estimate behave similarly



- Stronger potentials
  - Loopy BP ***not*** guaranteed to converge
  - Estimate may still be useful



Communicating particle sets

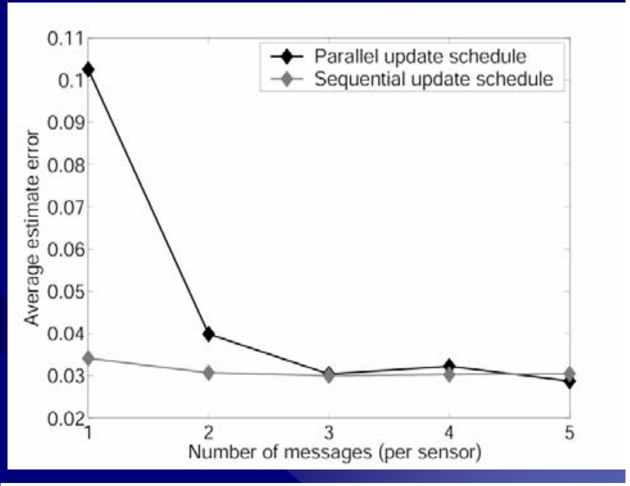
- Problem: transmit *N iid* samples $x_i \sim p(x)$
- Sequence of samples:
  - Expected cost is ¼ N*R*H(p)
    - H(p) = differential entropy
    - R = resolution of samples
- Set of samples
  - Invariant to reordering
    - We can reorder to reduce the transmission cost
  - Expected cost is ¼ N*R*H(p) – log(N!)
  - Entropy reduced for any deterministic order
    - In 1-D, “sorted” order
    - In > 1-D, can be harder, but…

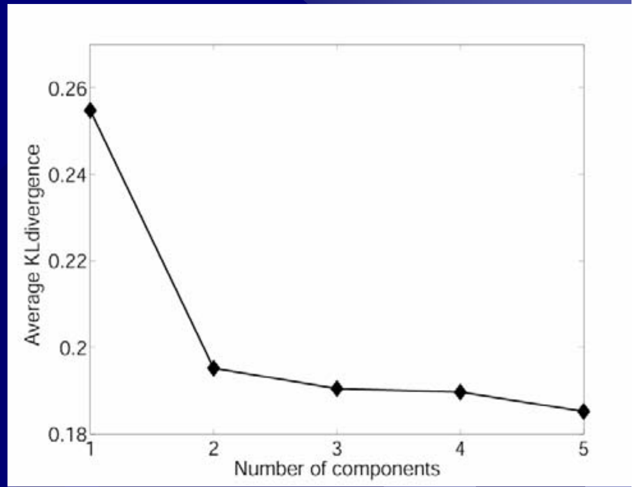
Trading off error vs communications

- KD-trees
  - Tree-structure successively divides point sets
    - Typically along some cardinal dimension
  - Cache statistics of subsets for fast computation
  - Example: cache means and covariances
- Can also be used for approximation…
  - Any cut through the tree is a density estimate
  - Easy to optimize over possible cuts
    - Communications cost
    - Upper bound on error (KL, max-log, etc)

Examples – Sensor localization

- Many inter-related aspects
  - Message schedule
    - Outward “tree-like” pass
    - Typical “parallel” schedule
  - # of iterations (messages)
    - Typically require very few (1-3)
    - Could replace by msg stopping criterion
  - Message approximation / bit budget
    - Most messages (eventually) “simple”
      - unimodal, near-Gaussian
    - Early messages & poorly localized sensors
      - May require more bits / components…





How can we take objectives of other nodes into account?

- Rapprochement of two lines of inquiry
  - Decentralized detection
  - Message passing algorithms for graphical models
- We're just starting, but what we now know:
  - When there are communications constraints and both local and global objectives, optimal design requires the sensing nodes to ***organize***
  - This organization in essence specifies a ***protocol*** for generating and interpreting messages
  - Avoiding the traps of optimality for decentralized detection for complex networks requires careful thought

A tractable and instructive case

- Directed set of sensing/decision nodes
  - Each node has its local measurements
  - Each node receives one or more bits of information from its “parents” and sends one or more bits to its “children”
  - Overall cost is a sum of costs incurred by each node based on the bits it generates and the value of the state of the phenomenon being measured
  - Each node has a local model of the part of the underlying phenomenon that it observes and for which it is responsible
    - Simplest case: the phenomenon being measured has graph structure compatible with that of the sensing nodes

Person-by-person optimal solution

- Iterative optimization of local decision rules: A message-passing algorithm!
- Each local optimization step requires
  - A pdf for the bits received from parents (based on the current decision rules at ancestor nodes)
  - A cost-to-go summarizing the impact of different decisions on offspring nodes based on their current decision rules

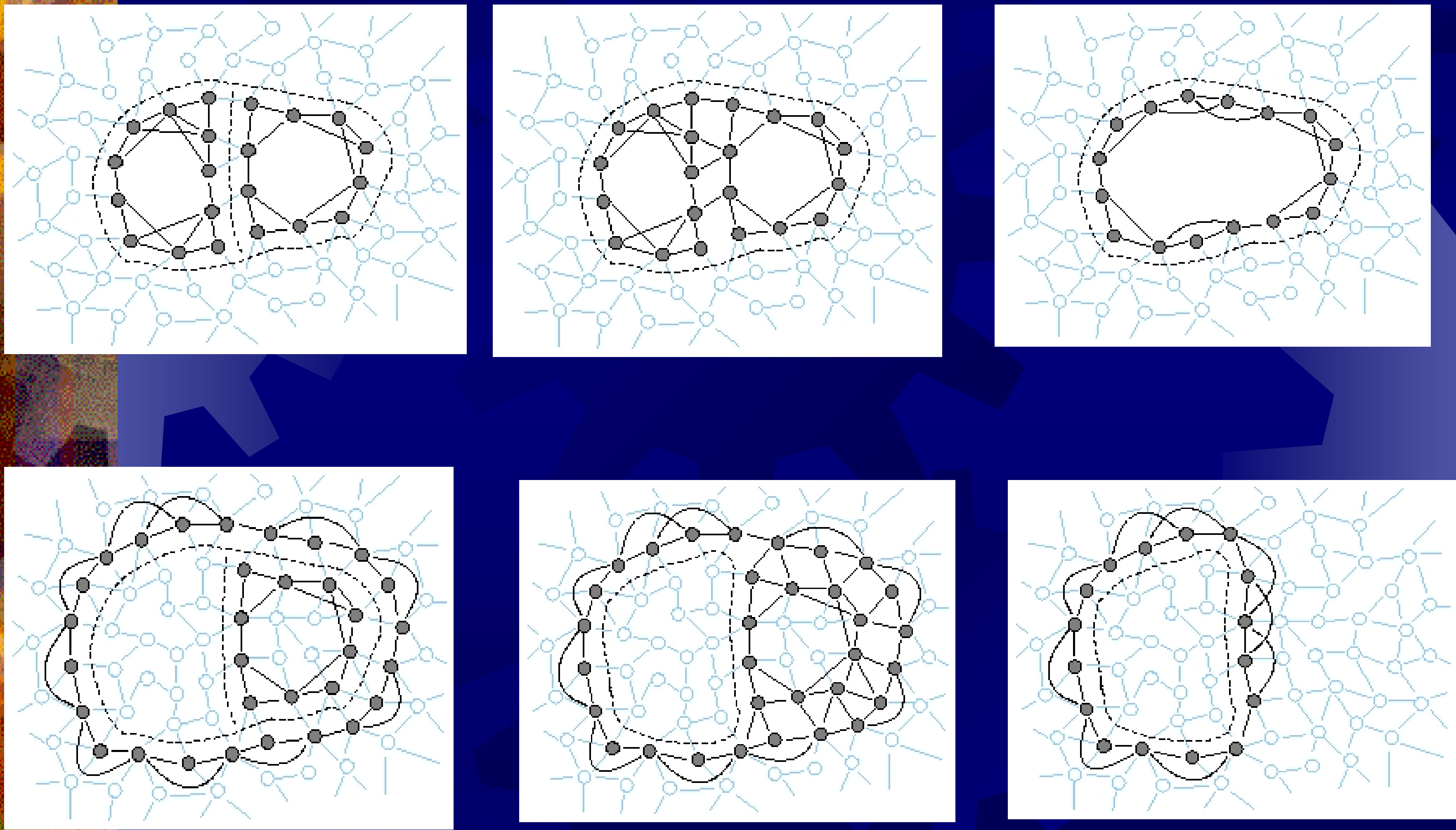
Two algorithmic structures

- Gauss-Seidel, e.g. sweeping from one end to the other and then back
  - Convergence guaranteed, as cost reduced at each stage
  - Very particular message scheduling
- Jacobi—Everyone updates at the same time
  - No convergence guarantees, but has same equilibria
  - Corresponds to the simplest message passing structure in BP: Everyone sends and receives messages at each iteration

What happens with more general networks?

- Basic answer: We'll let you know
- What we **do** know:
  - Choosing decision "rules" corresponds to specifying a graphical model consisting of
    - The underlying phenomenon
    - ***The sensor network (the part of the model we get to play with)***
    - The cost
  - For this reason
    - There are nontrivial issues in specifying globally compatible decision "rules"
    - Optimization (and for that matter cost evaluation) is intractable, for exactly the same reasons as inference for graphical models

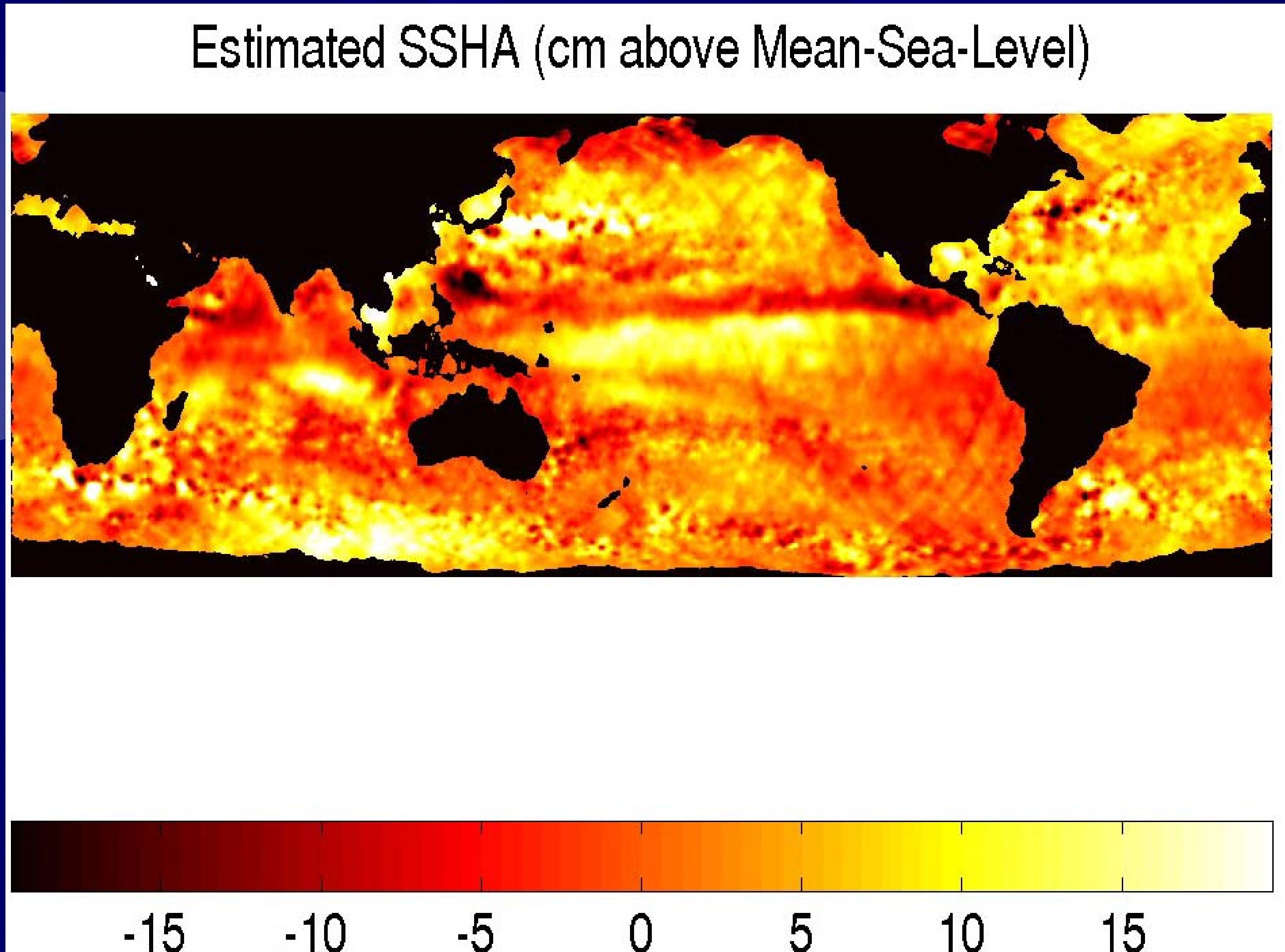
Alternate approach to approximate inference: Recursive Cavity Models



Comments

- Linear-Gaussian models
  - Graphical model specified by P^{-1}
  - Algorithm corresponds to ***information form*** of MR tree algorithms, with one additional step
    - Thinning approximations to maintain tractability
    - Leads to bounded errors under appropriate cross-boundary "mixing" conditions
- For sensor networks
  - Offers possibility of propagating information out from "seed" nodes
  - Computations at each stage involve information propagation around cavities

Recursive Cavity Modeling: Remote Sensing Application



Walk-sums, BP, and new algorithmic structures

- Focus (for now) on linear-Gaussian models
  - For simplicity normalize variables so that

    $$P^{-1} = I - R$$

  - R has zero diagonal
  - Non-zero off-diagonal elements correspond to edges in the graph
    - Values equal to partial correlation coefficients

Walk-sums, Part II

- For "walk-summable" models

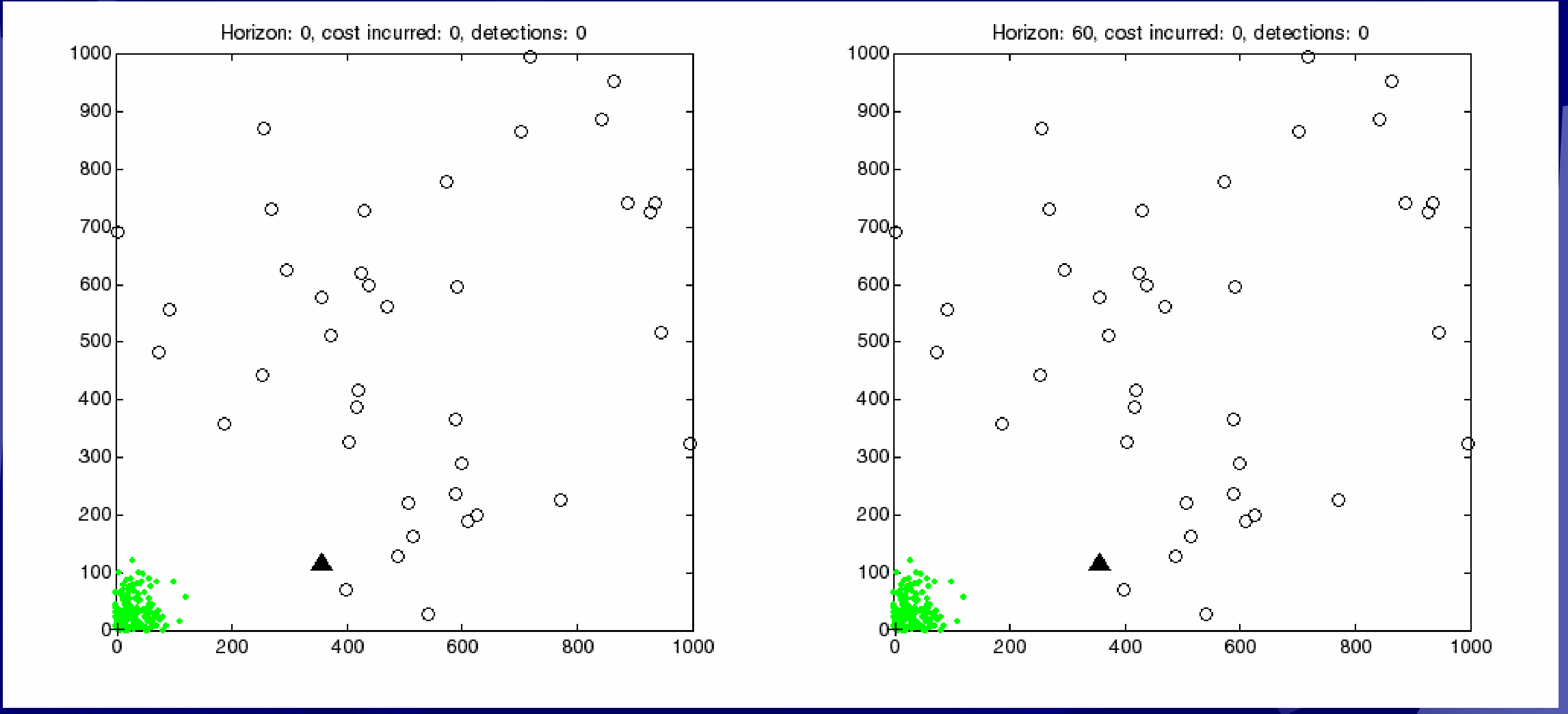
$$P = (I - R)^{-1} = I + R + R^2 + \ldots$$

- For any element of P, this sum corresponds to so-called "walk-sums"
  - Sums of products of elements of R corresponding to walks from one node to another
- BP computes ***strict subseries*** of these walk sums for the diagonal elements of P

Walk-sums, Part III

- Dynamic systems interpretation and questions:
  - BP performs this computation via a distributed algorithm with local dynamics at each node with minimal memory
    - Remember the most recent set of messages
  - Full walk-sums are realizable with local dynamics only of very high dimension in general
    - Dimensions that grow with graph size
  - There are ***many*** algorithms with increased memory that calculate larger subseries
    - E.g., include one more path
    - State or “node” augmentation (e.g., Kikuchi, GBP)
  - What are the subseries that are realizable with state dimensions that don’t depend on graph size?

Dealing with Limited Power: Sensor Tasking and ***Handoff***



So where are we going? - I

- Graphical models
  - New classes of algorithms
    - RCM++
    - Algorithms based on walk-sum interpretations and realization theory for graphical computations
    - Theoretical analysis and performance guarantees
  - Model estimation and approximation
    - Learning graphical structure
      - From data
      - From more complex models
  - An array of applications
    - “Bag of parts” models for object recognition (and ***maybe*** structural biology)
    - Fast surface reconstruction and visualization
    - …

So where are we going? - II

- Information science in the large
  - These problems are not problems in signal processing, computing, information theory
  - They are problems in ***all*** of these fields
- And we've just scratched the surface
  - Why should the graph of the phenomenon be the same as the sensing/communication network?
  - What if we send more complex messages with protocol bits (e.g. to overcome BP over-counting)
  - What if nodes develop protocols to ***request*** messages
    - In this case "no news" **IS** news…