Can wavelets help computers listen and focus their attention ?

An introduction to source separation with sparse decompositions

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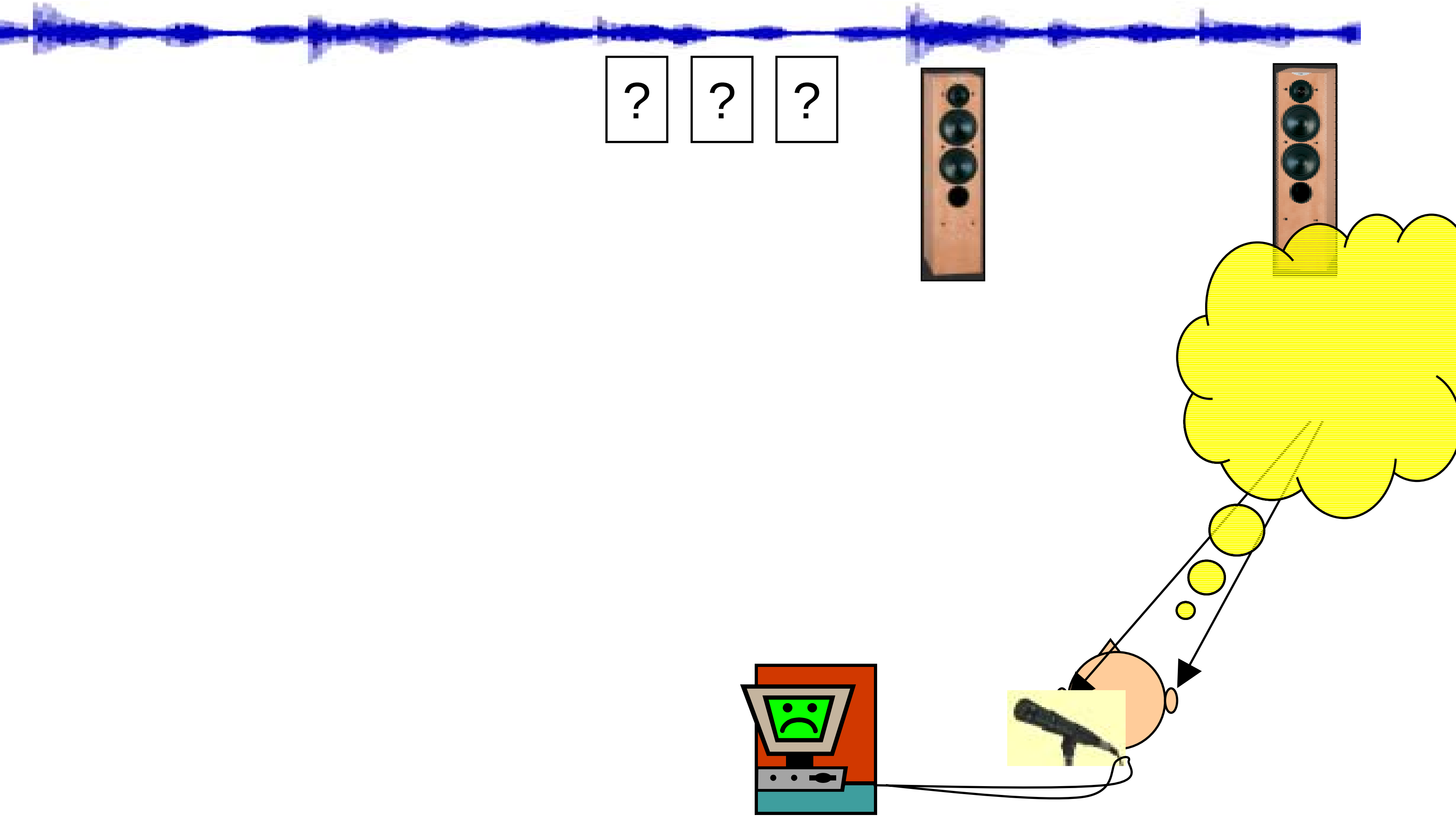
Sylvain Lesage
Alexey Ozerov

Morten Nielsen (Univ. Aalborg)
Pierre Vandergheynst (EPFL)

Prologue :
Listening cues



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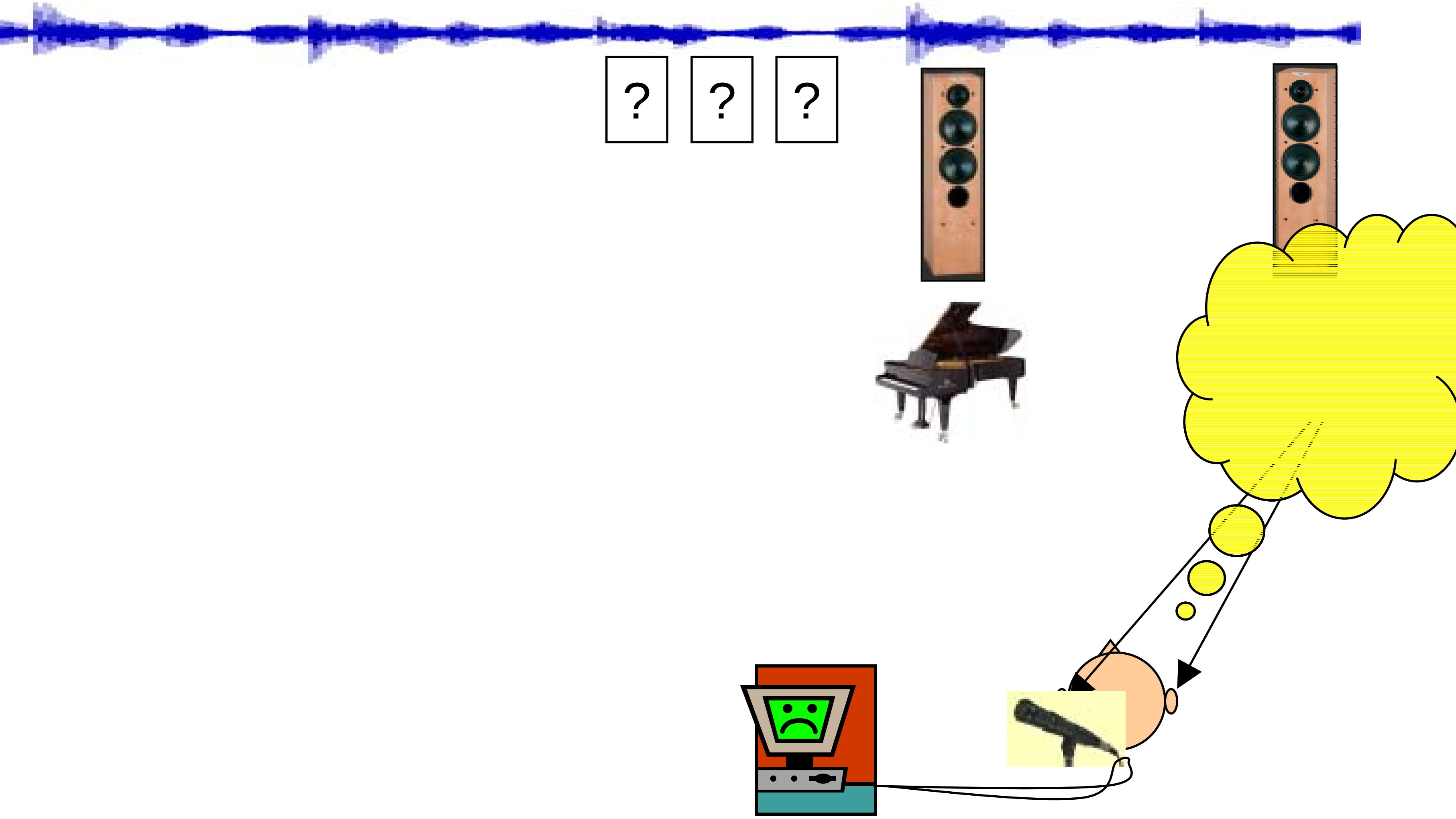

Prologue : Listening cues

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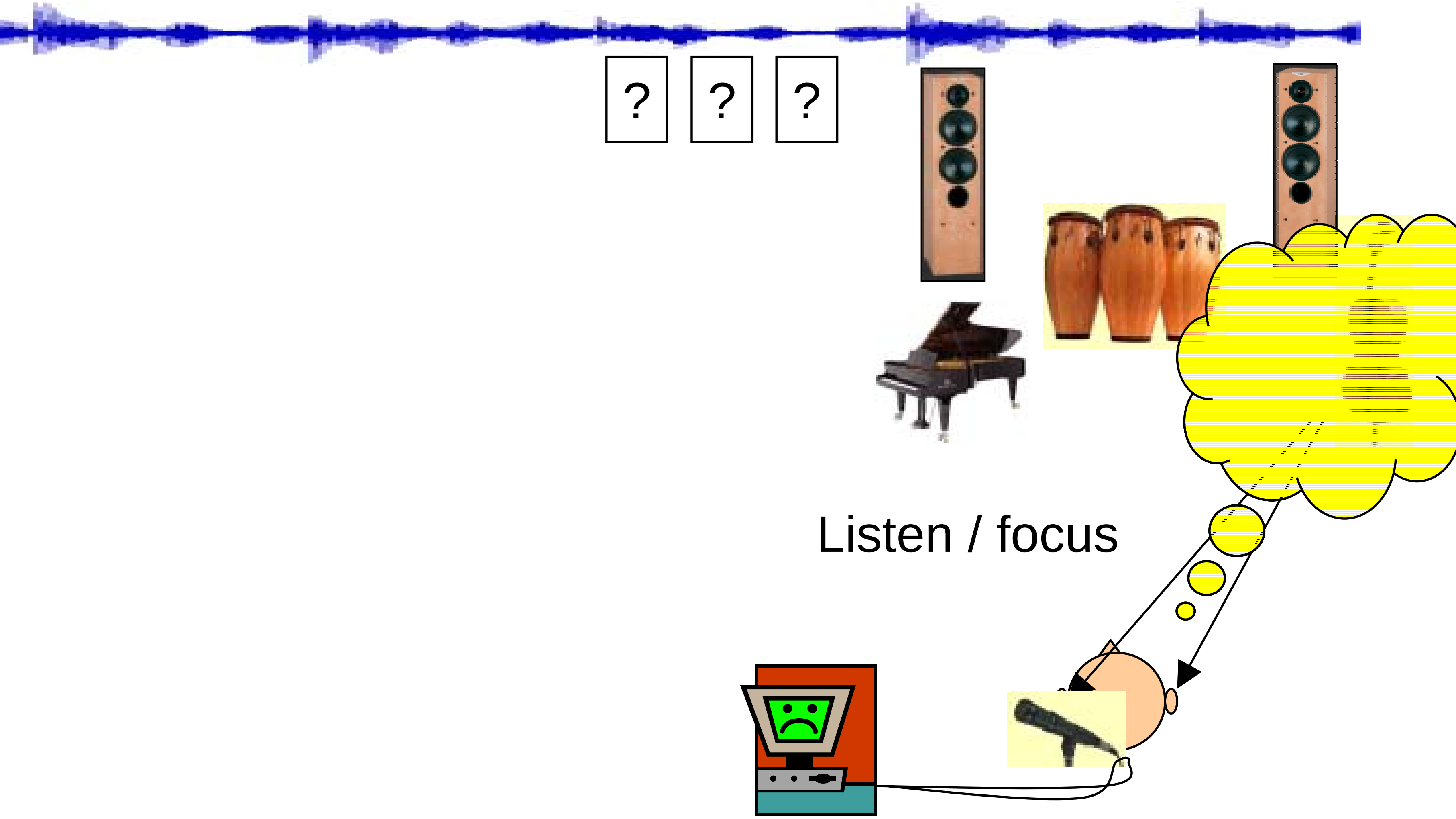


Prologue : Listening cues

- What is this ? ? ? ?
- Where is it ?
- Listening cues

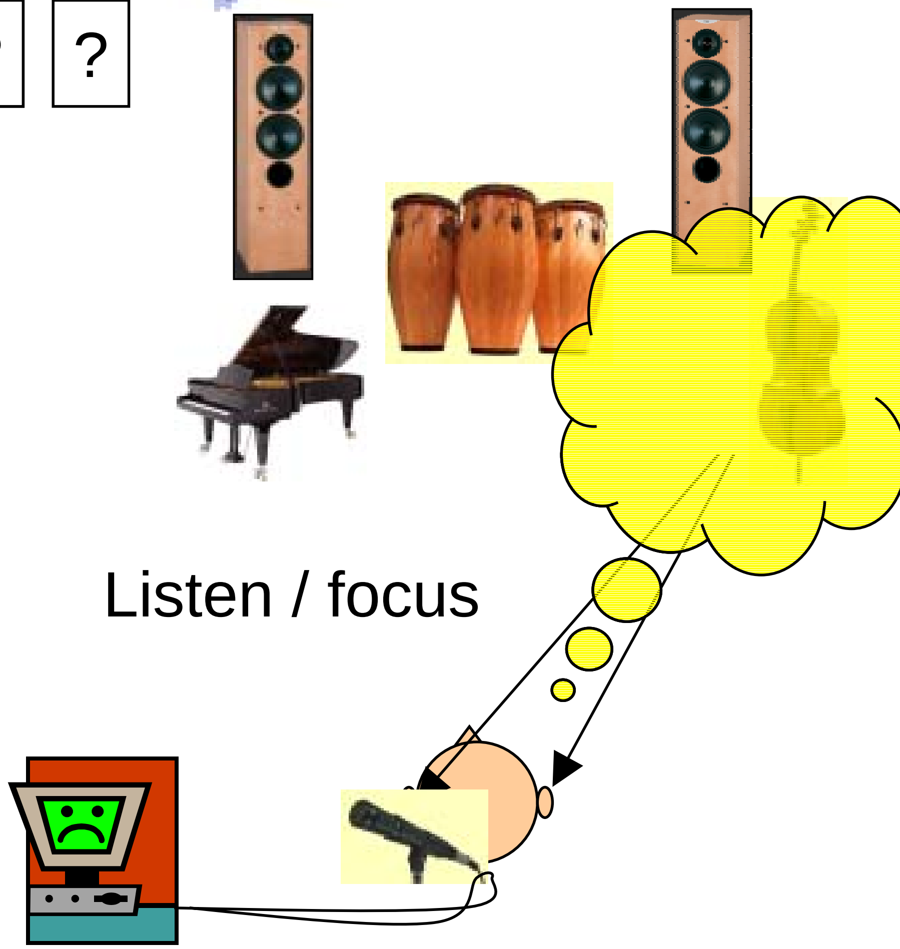
What : timbre

Pitch/frequency/patterns

Where : <u>direction</u>

Time difference

Intensity difference



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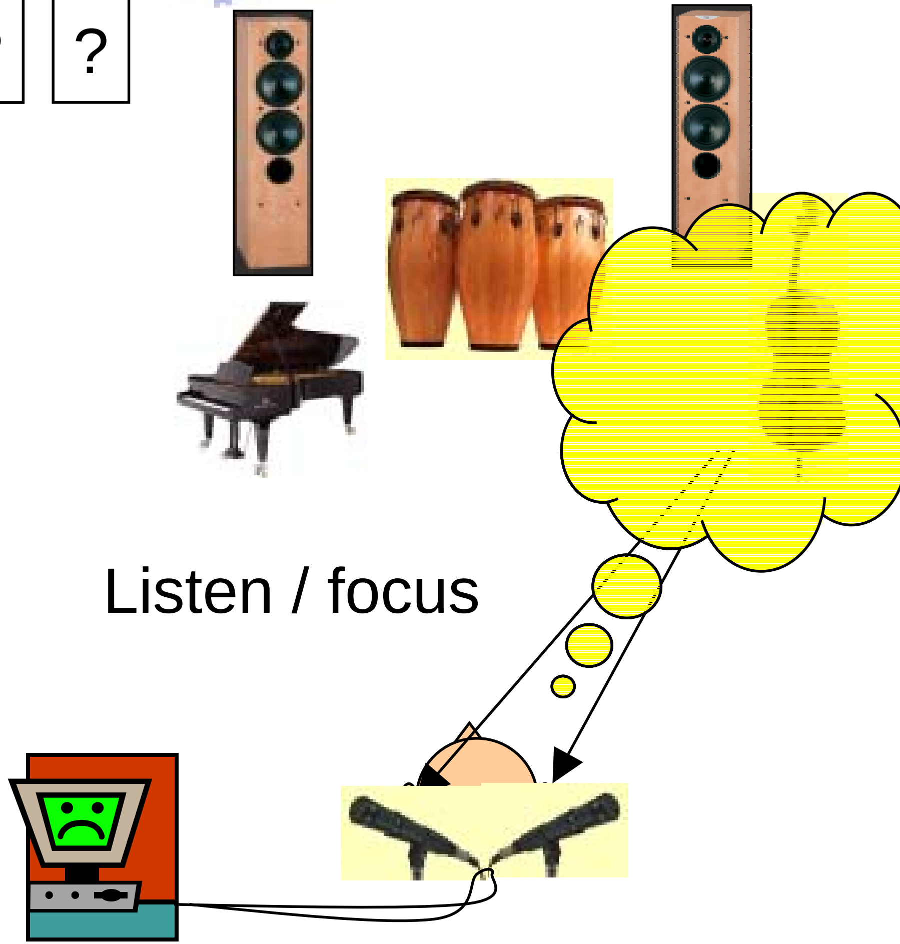
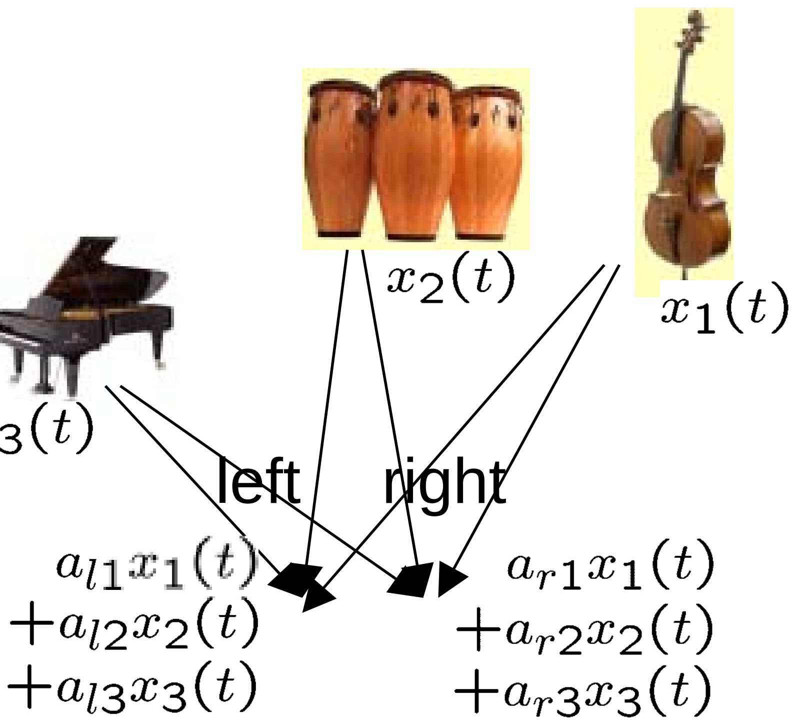


Roadmap

- Simplified acoustic model
- Linear algebra
- Time-frequency masking
- Conclusion and perspectives

A simplified acoustic model



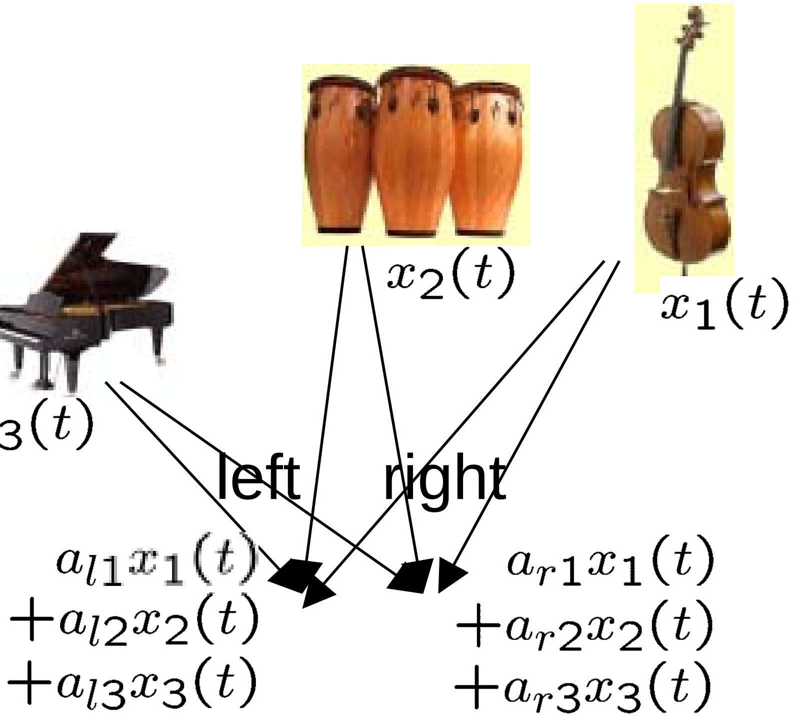
observations

$$\mathbf{b}(t) = \mathbf{A} \cdot \mathbf{x}(t)$$

mixing matrix

A simplified acoustic model

- Linear instantaneous mixture



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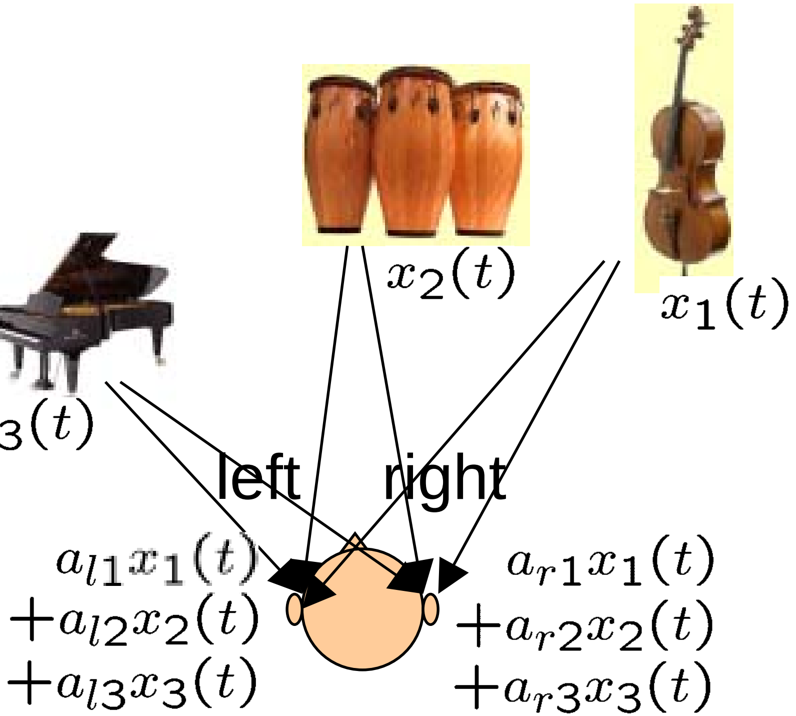

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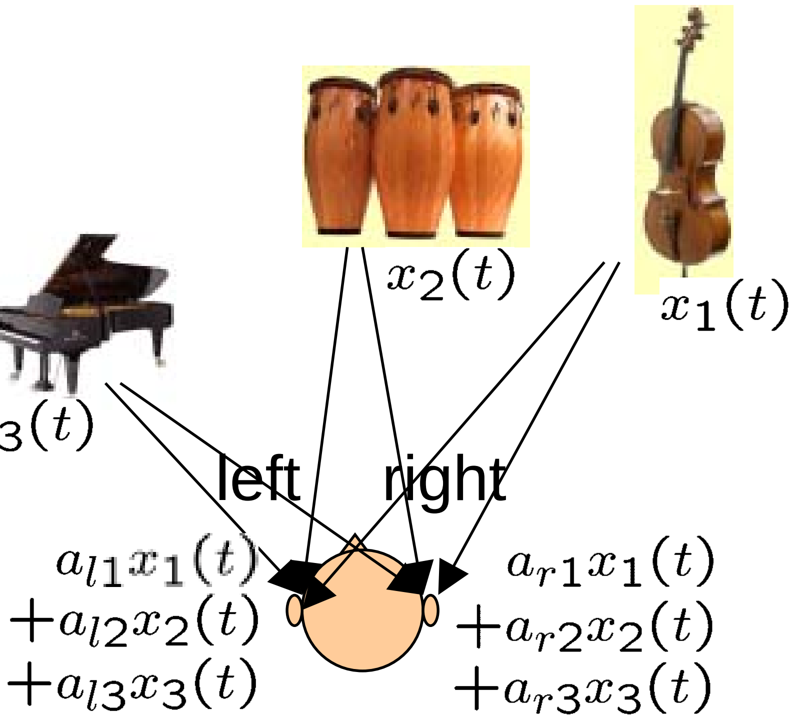

observations

$$\mathbf{b}(t) = \mathbf{A} \cdot \mathbf{x}(t)$$

mixing matrix

$$\begin{bmatrix} b_{\text{left}}(t) \\ b_{\text{right}}(t) \end{bmatrix} = \begin{bmatrix} a_{l1} & a_{l2} & a_{l3} \\ a_{r1} & a_{r2} & a_{r3} \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

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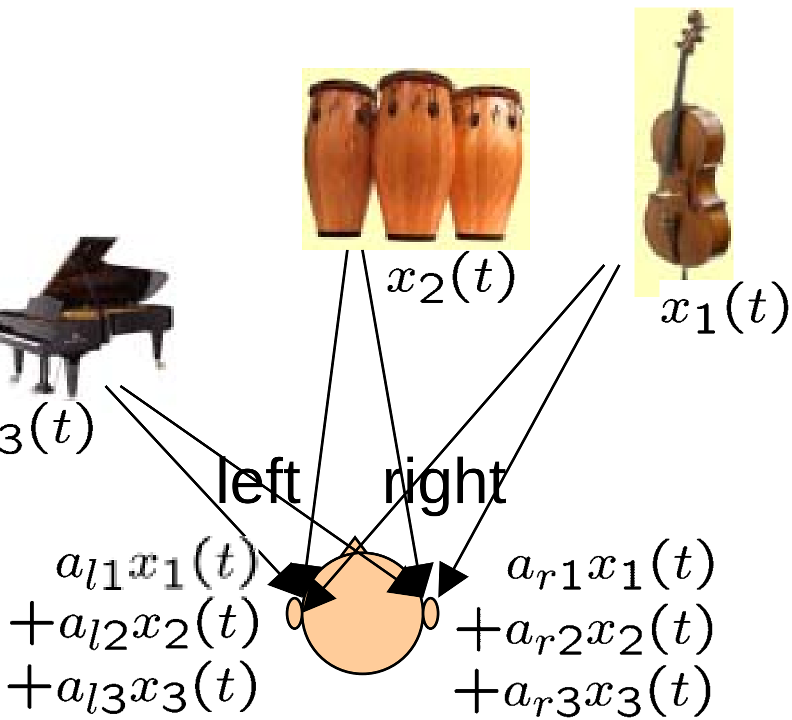

observations — sources

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Linear inverse problems

$\mathbf{A}$

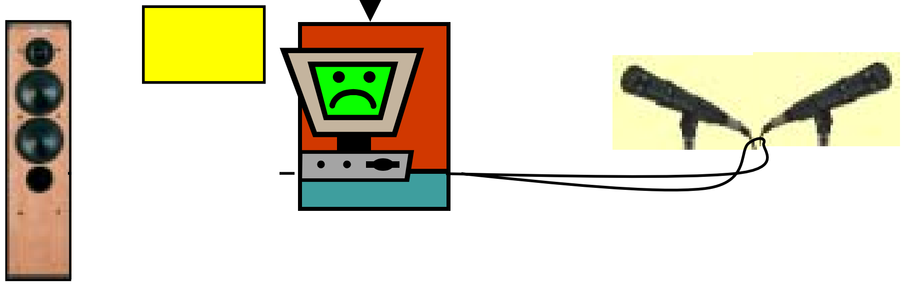
$$\mathbf{A}x = \mathbf{b}$$

Choose one solution = use *a priori* knowledge) on the sources

Example: minimize energy

Infinitely many solutions

$$x = \mathbf{A}^{\dagger}\mathbf{b}$$



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Linear inverse problems

- An approach to focus on each instrument
  1. Estimate $\mathbf{A}$
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Over-determined

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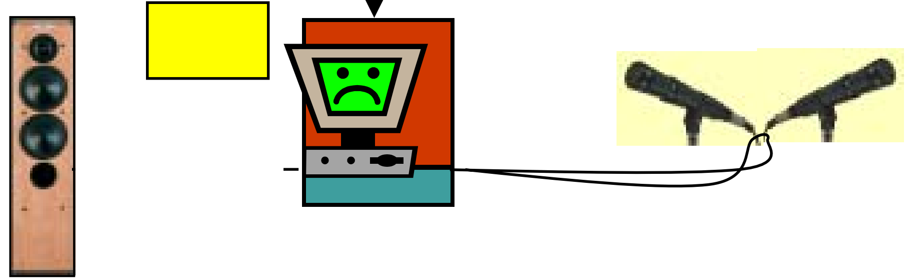

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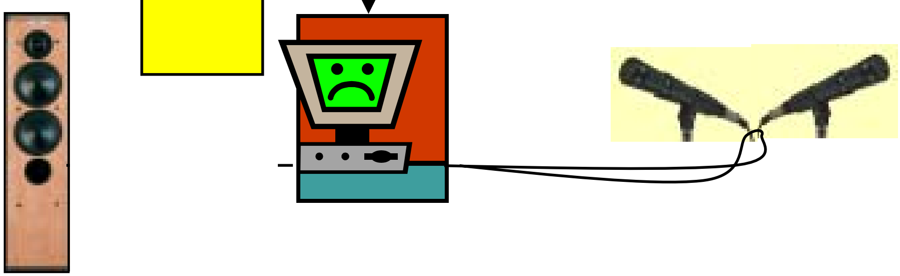

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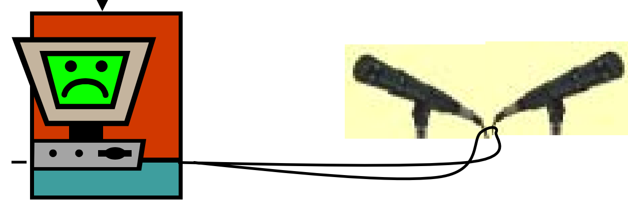

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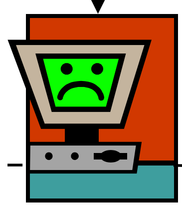

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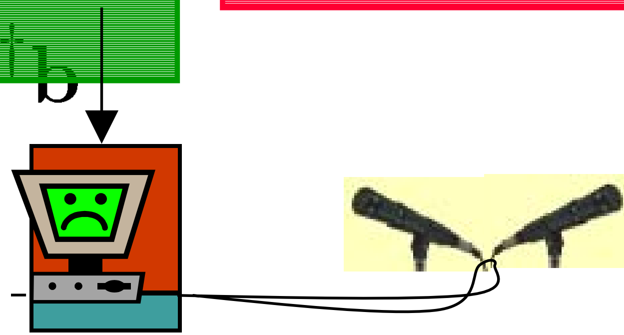

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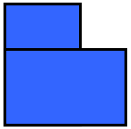

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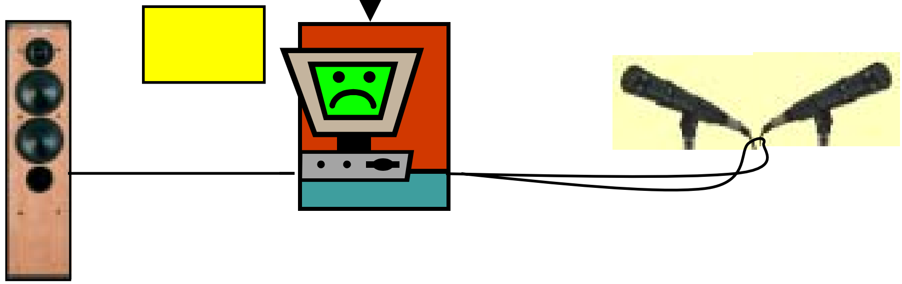

Linear inverse problems

- An approach to focus on each instrument

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Choose one solution = use *a priori* knowledge on the sources

Example: minimize energy

A unique solution?

$$\mathbf{A}x = \mathbf{b}$$

Infinitely many solutions

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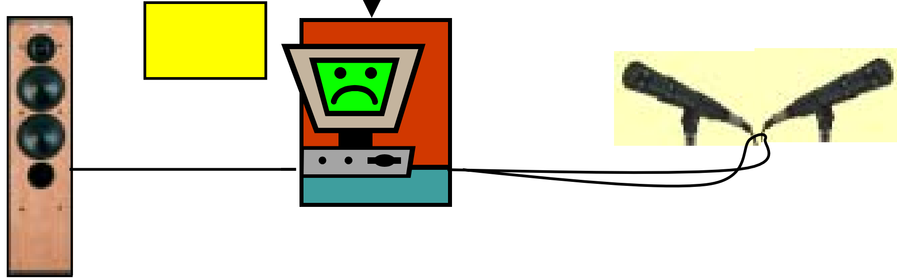
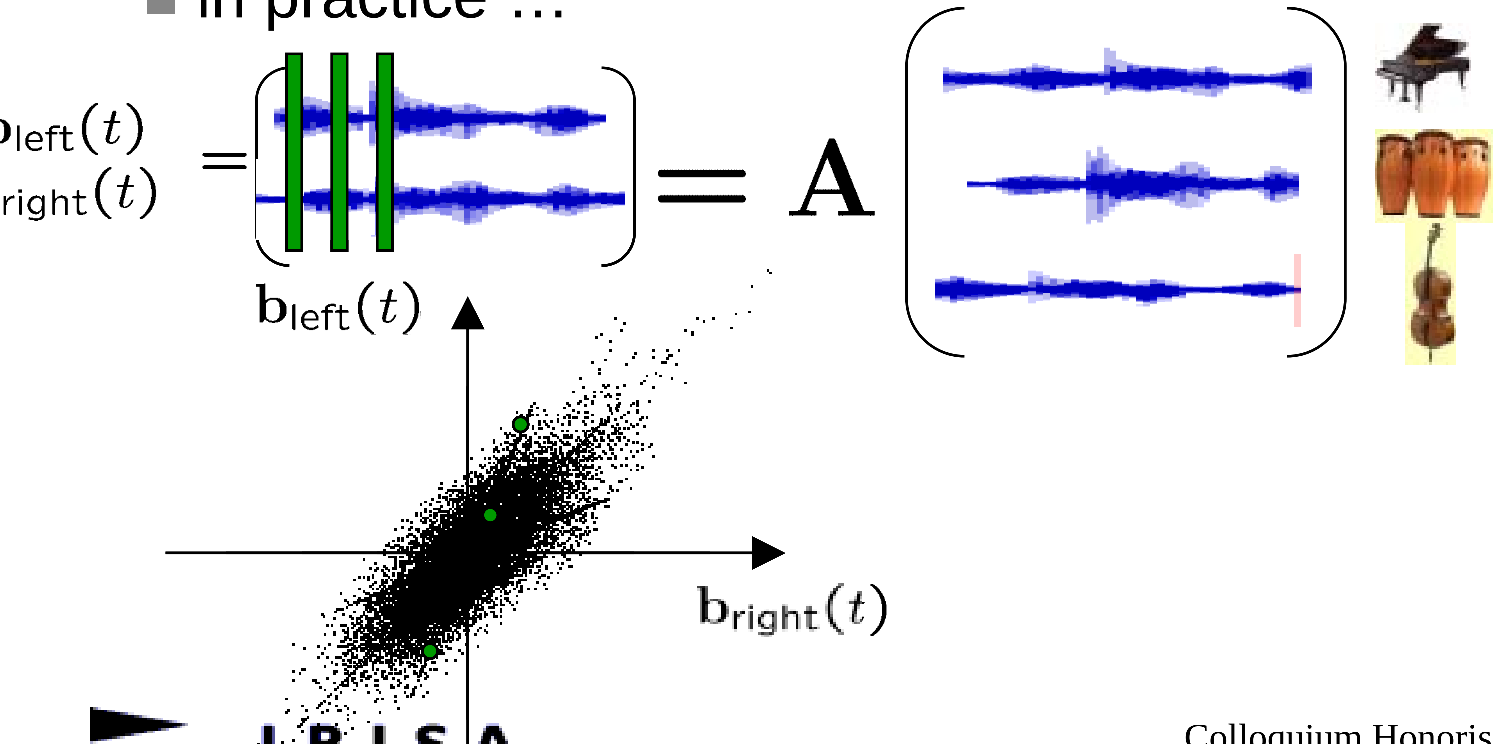


Agenda : find the directions invert the equations

- Simplified acoustic model? / Find directions?
  - if only one source is active …
  - in practice …



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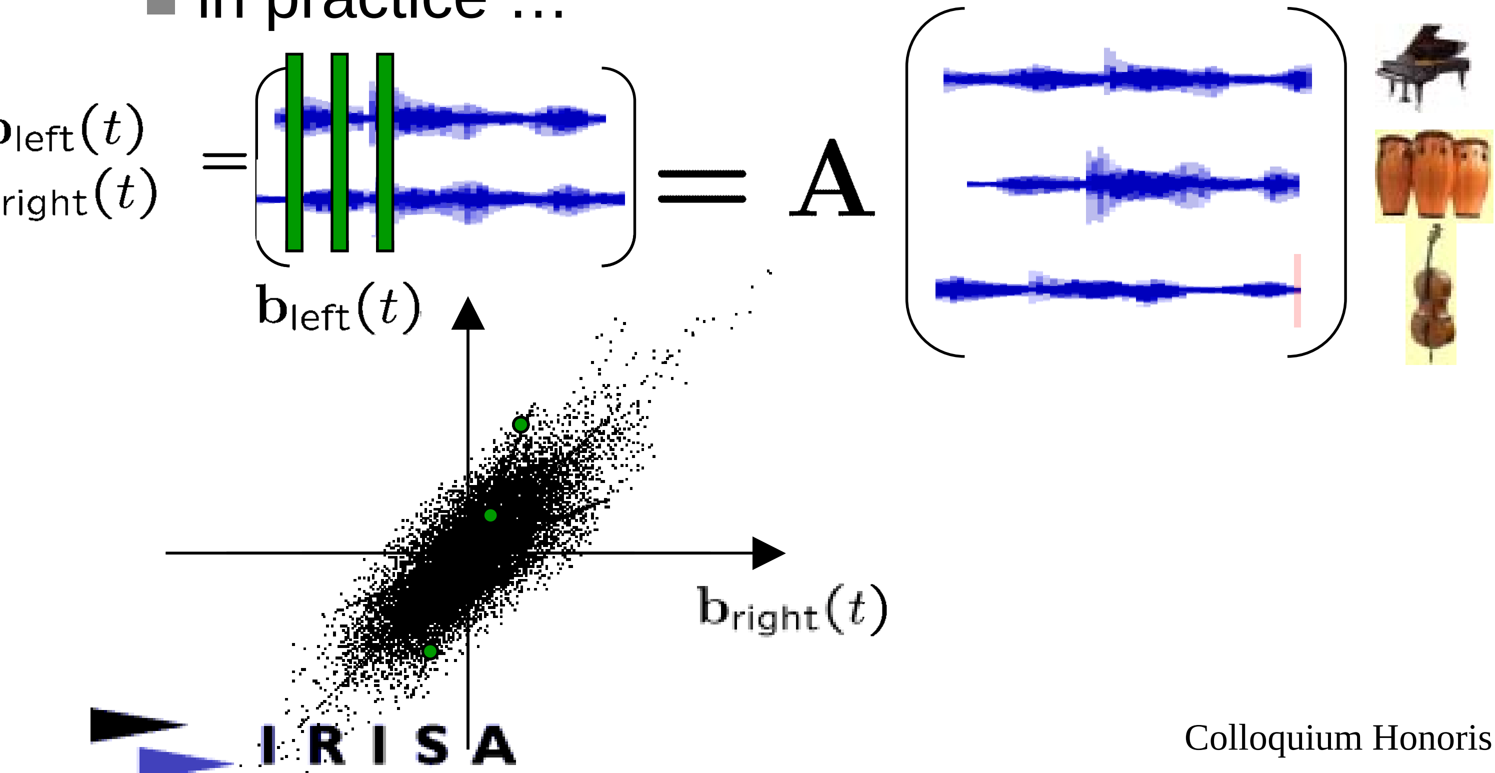

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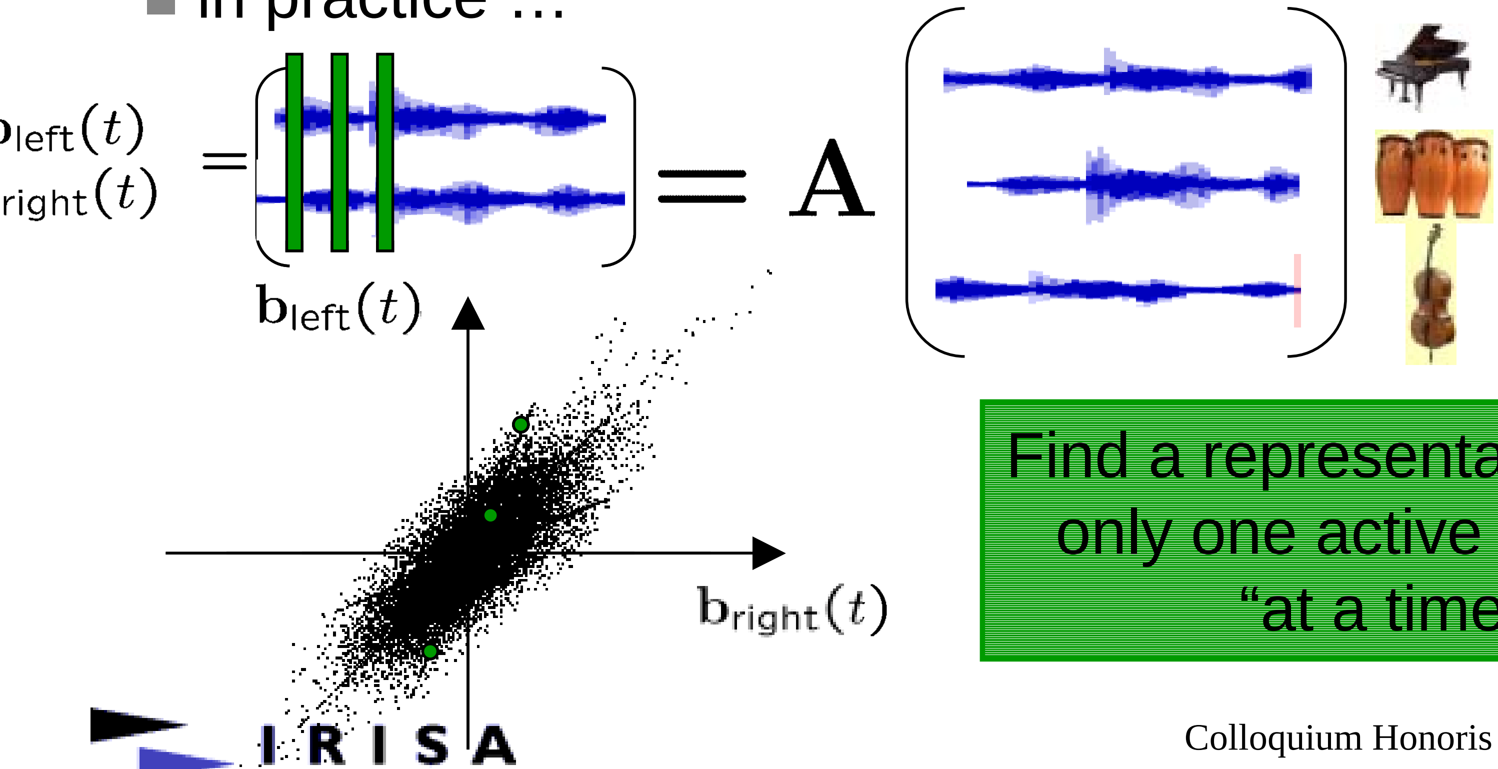
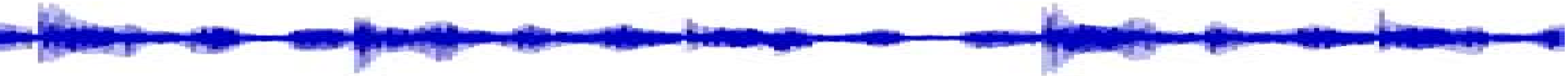
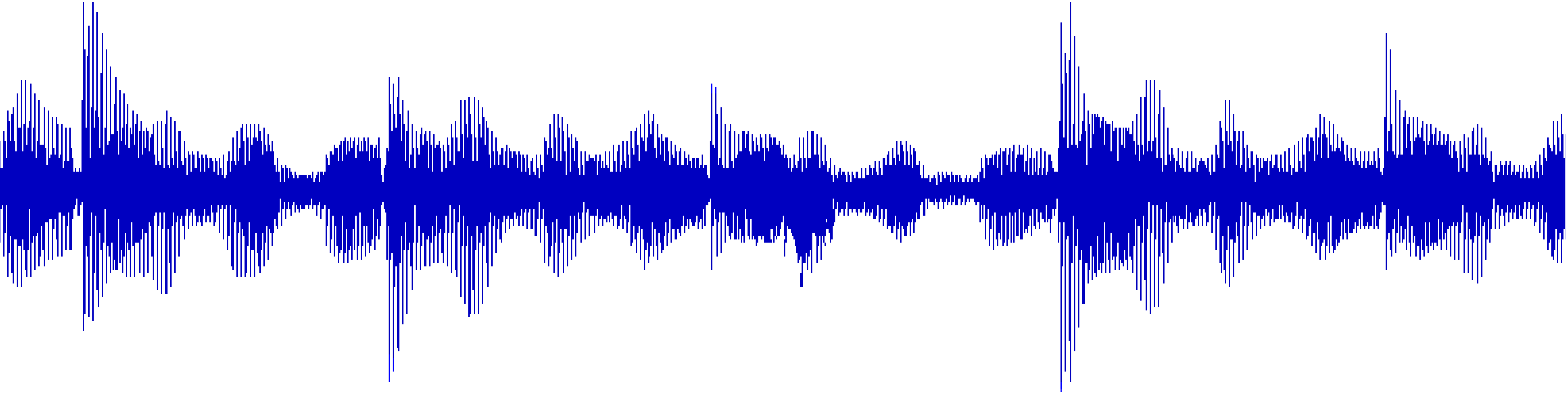


Find a representation with only one active source "at a time"

Time-frequency representation





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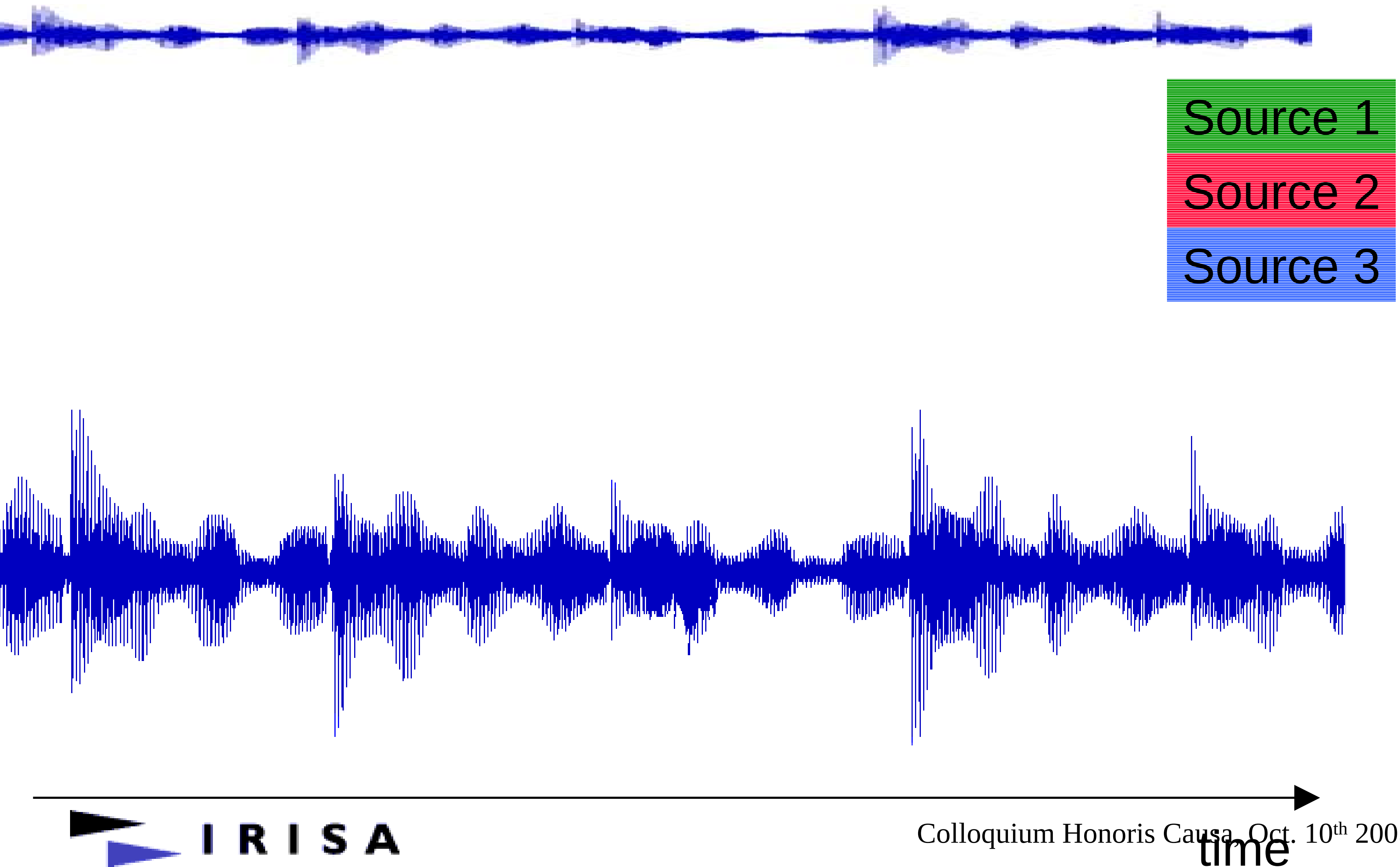
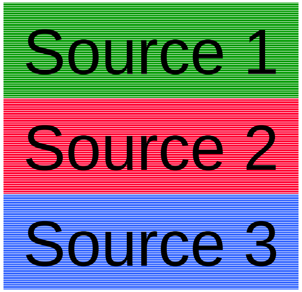
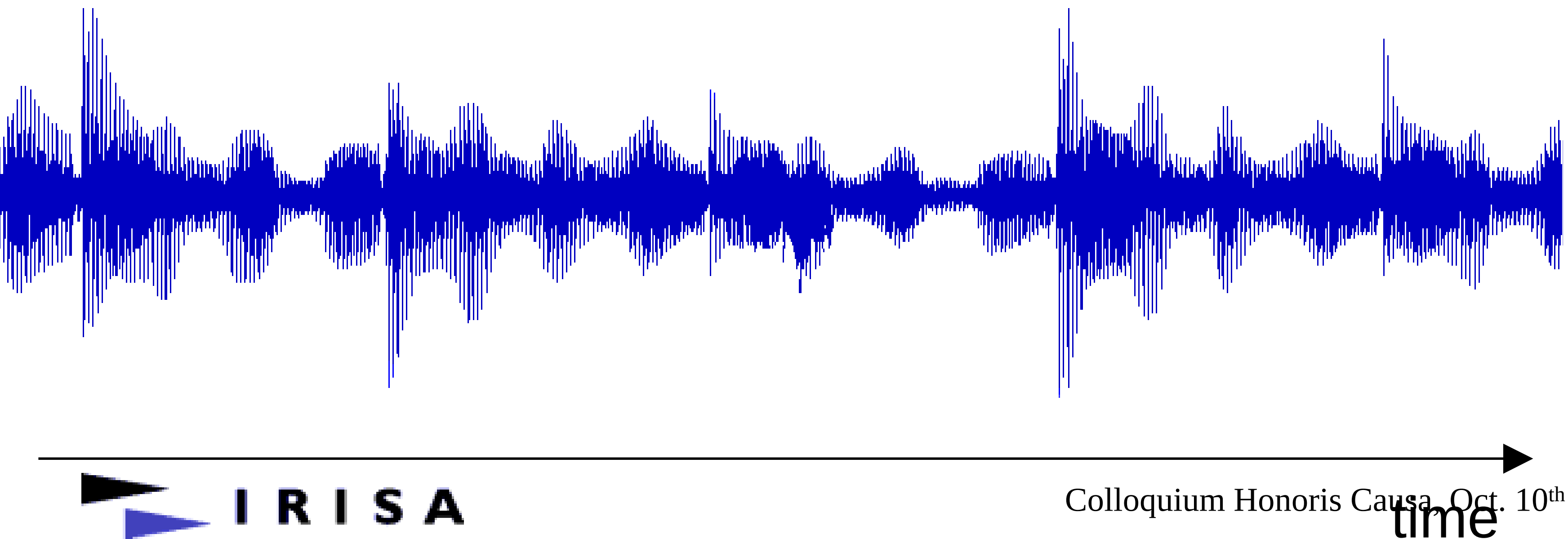


Time-frequency representation

- **Segmentation**





Time-frequency representation

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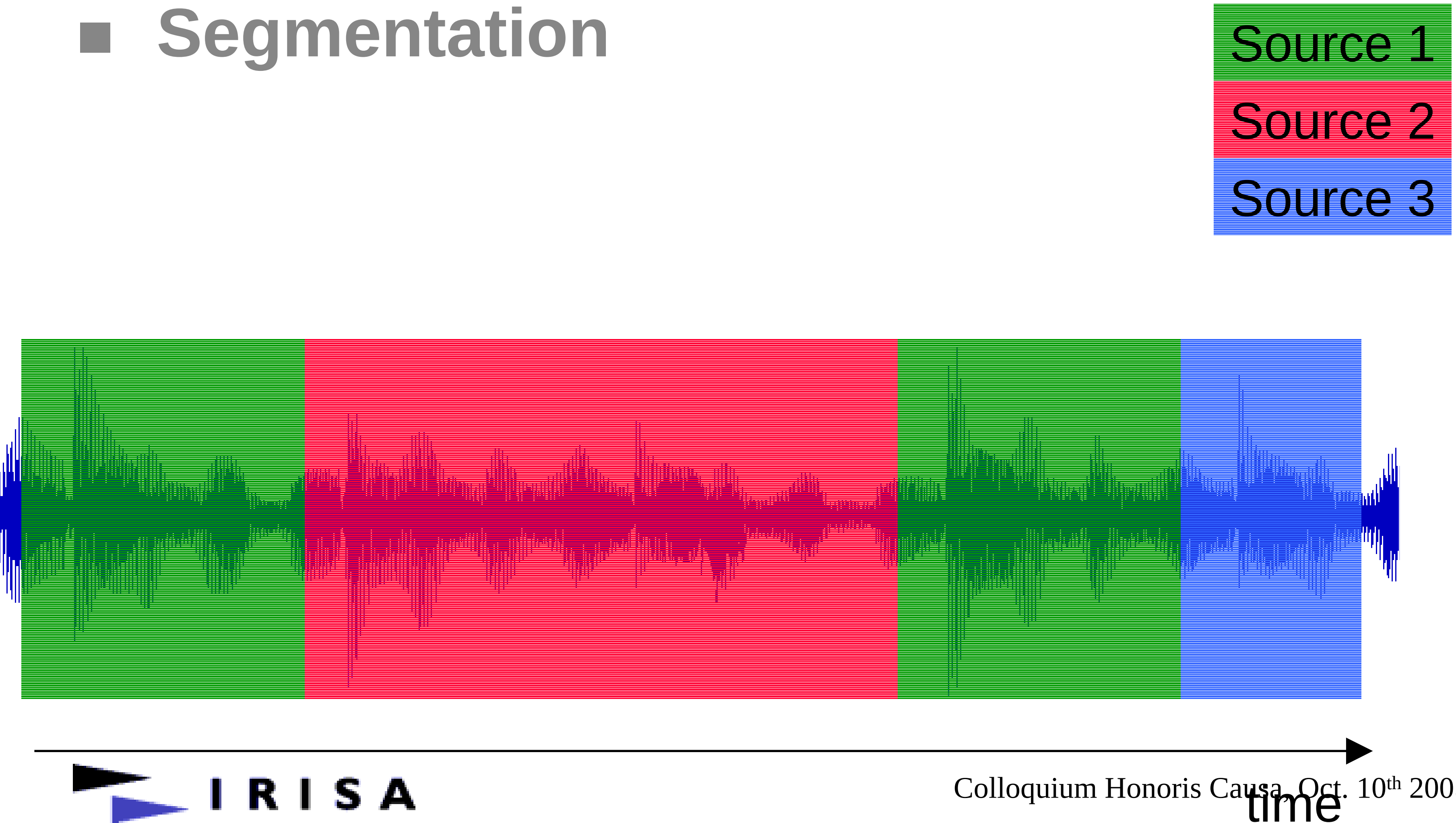
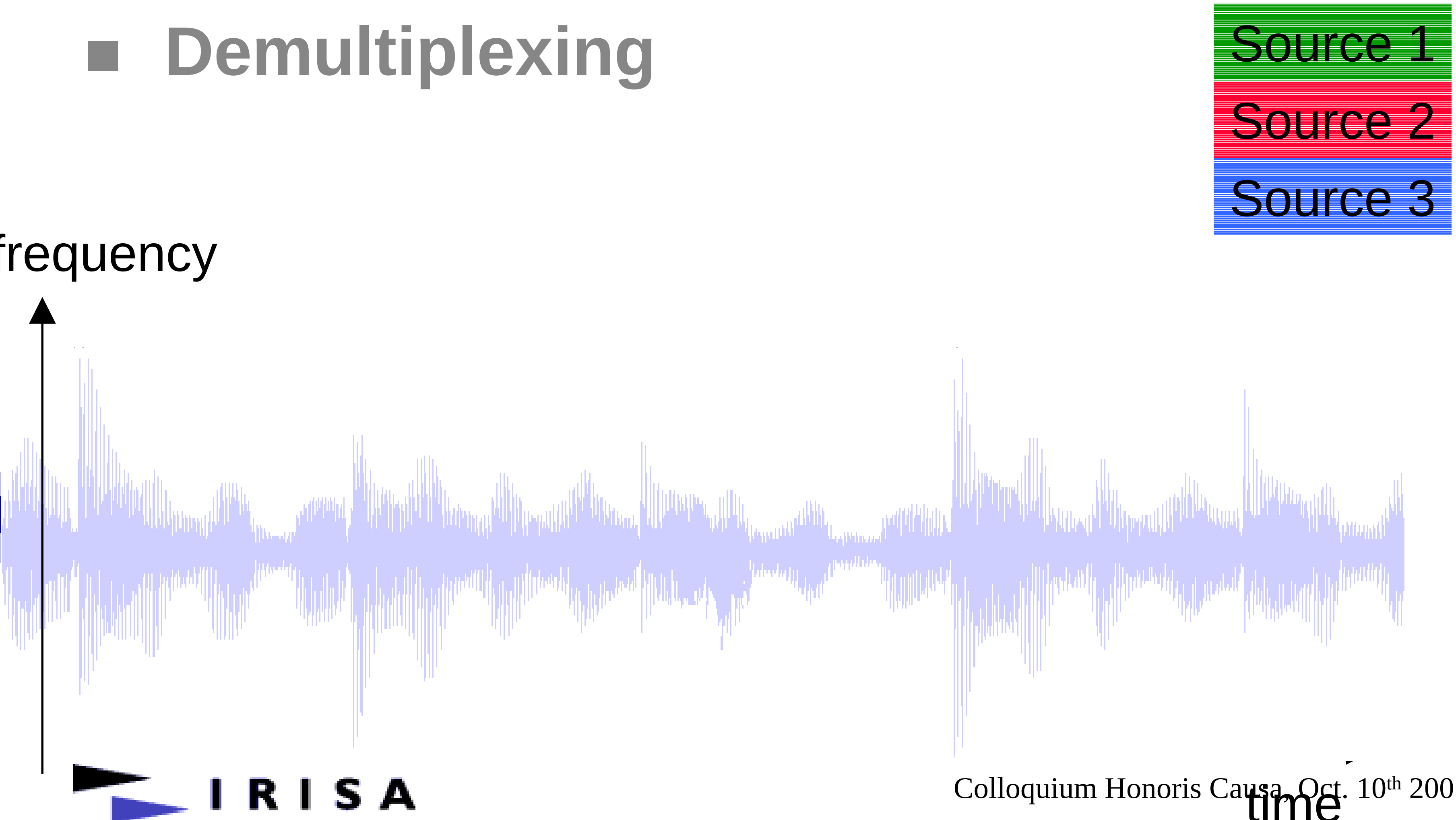


Time-frequency representation

- **Demultiplexing**



Time-frequency representation

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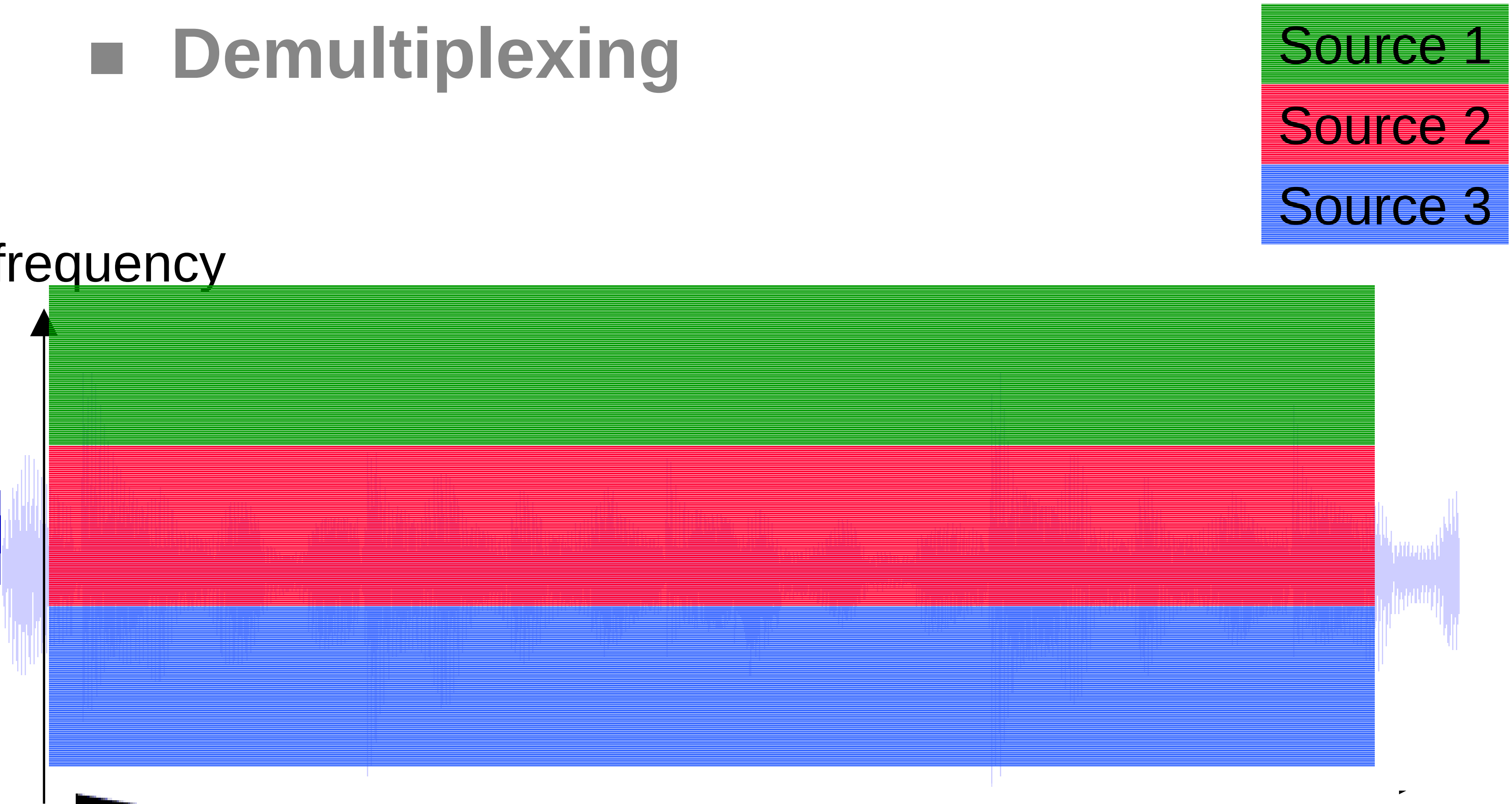
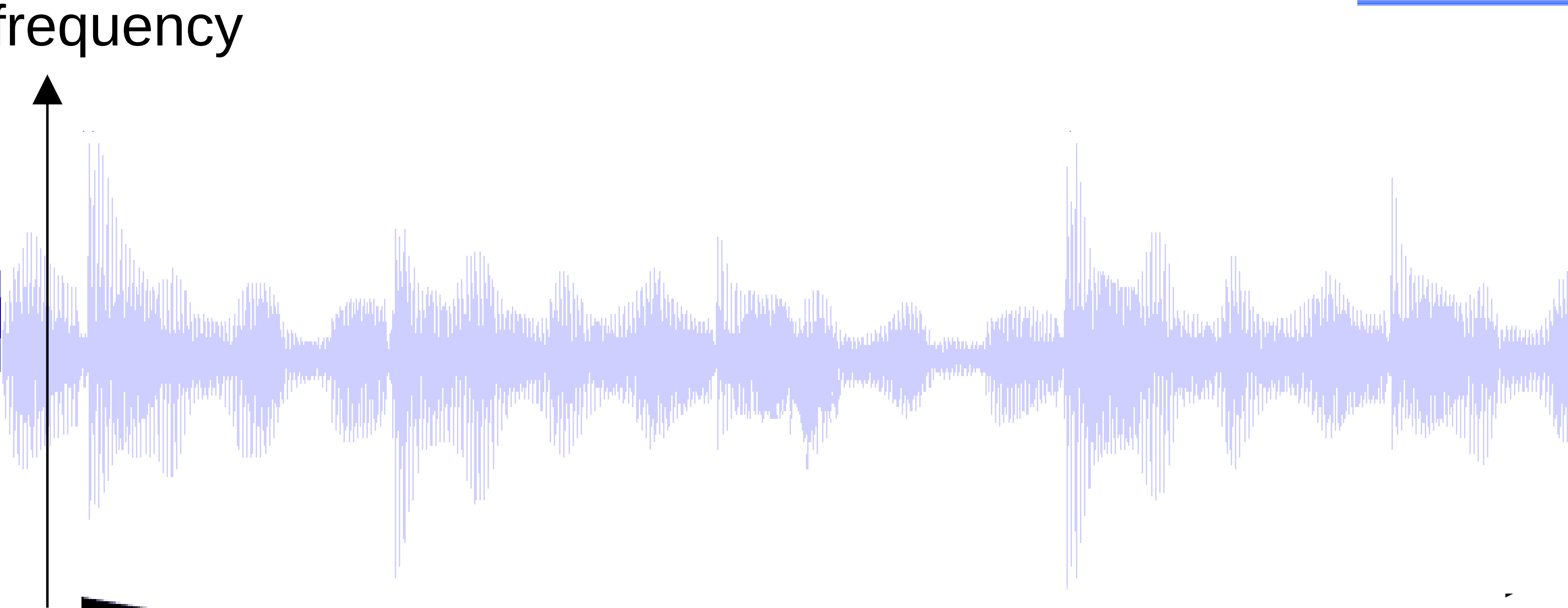


Time-frequency representation

- **Time-frequency masking**



Time-frequency representation

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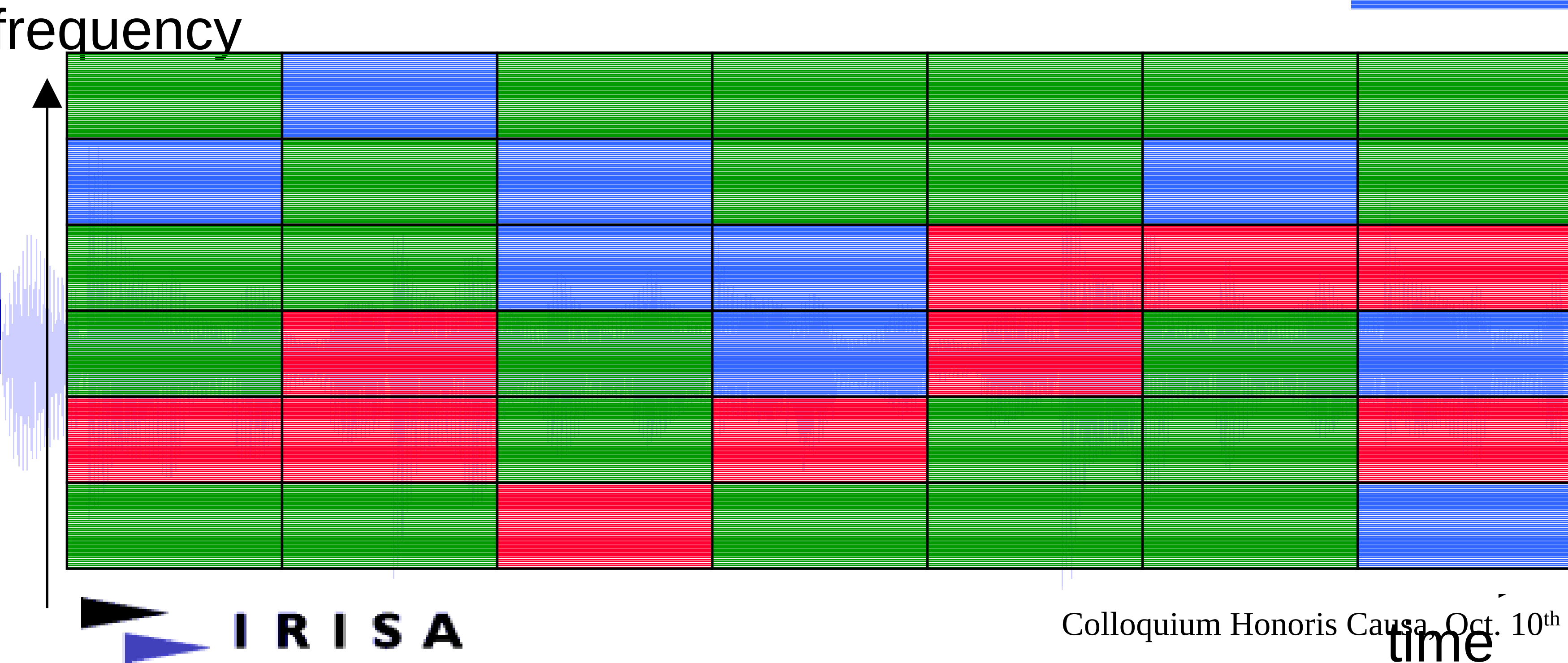
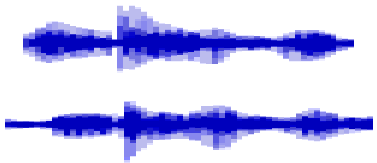
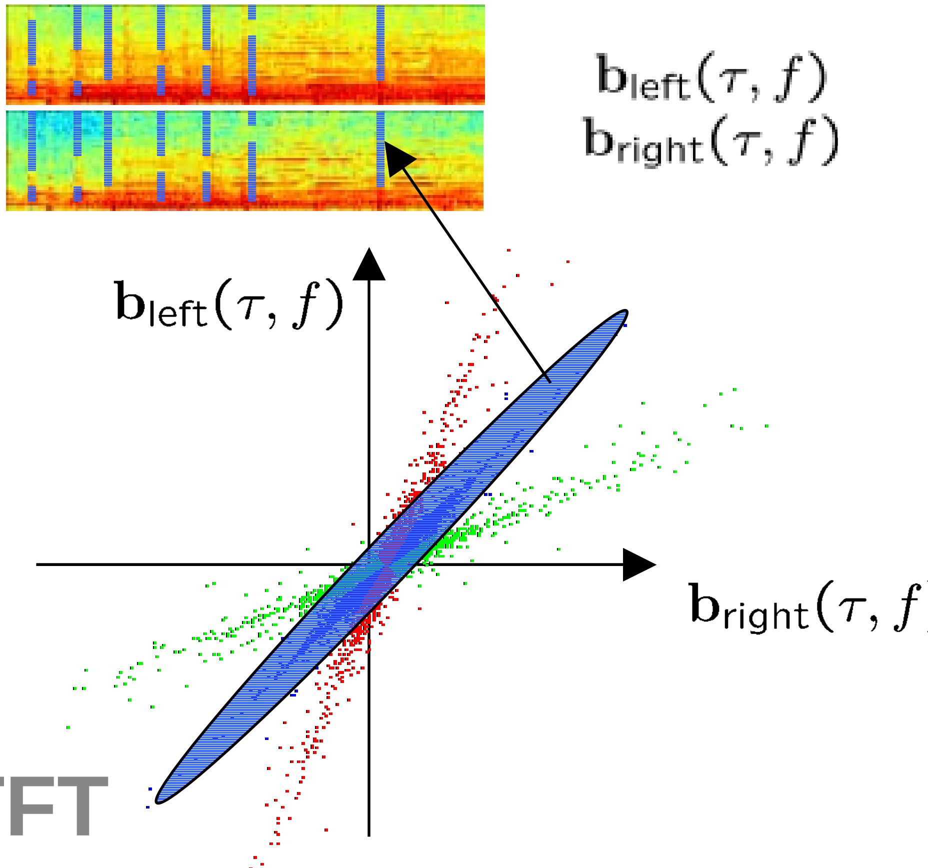


Agenda : find the directions invert the equations

- **Short Time Fourier Transform (STFT)**

$\mathrm{b}_{\mathrm{left}}(t)$
$\mathrm{b}_{\mathrm{right}}(t)$





- **Scatter plot**

- **Clustering**
  (Simon Arberet)

- **Mask + inverse STFT**

Agenda : find the directions invert the equations

- Short Time Fourier Transform (STFT)

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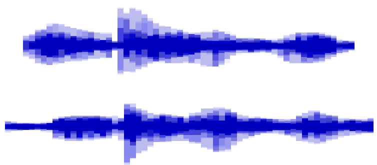
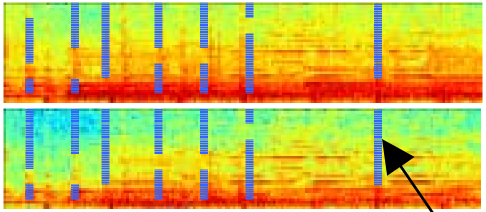
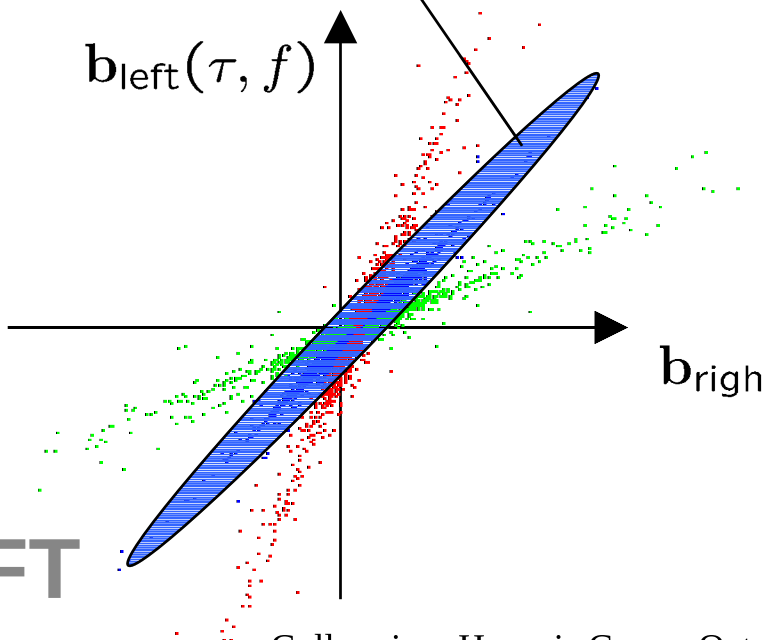




$\mathbf{b}_{\text{left}}(\tau, f)$
$\mathbf{b}_{\text{right}}(\tau, f)$

- Scatter plot



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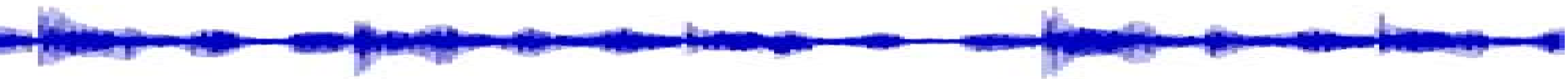

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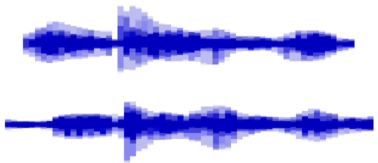

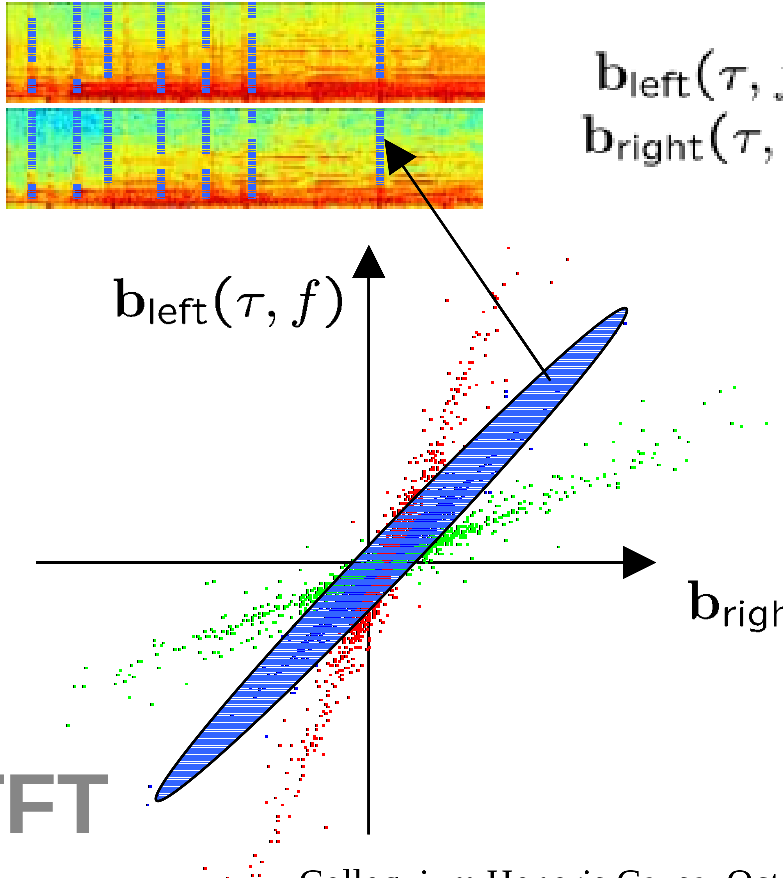

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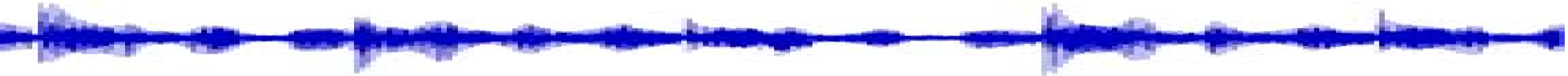

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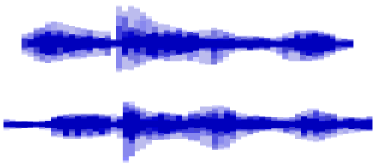

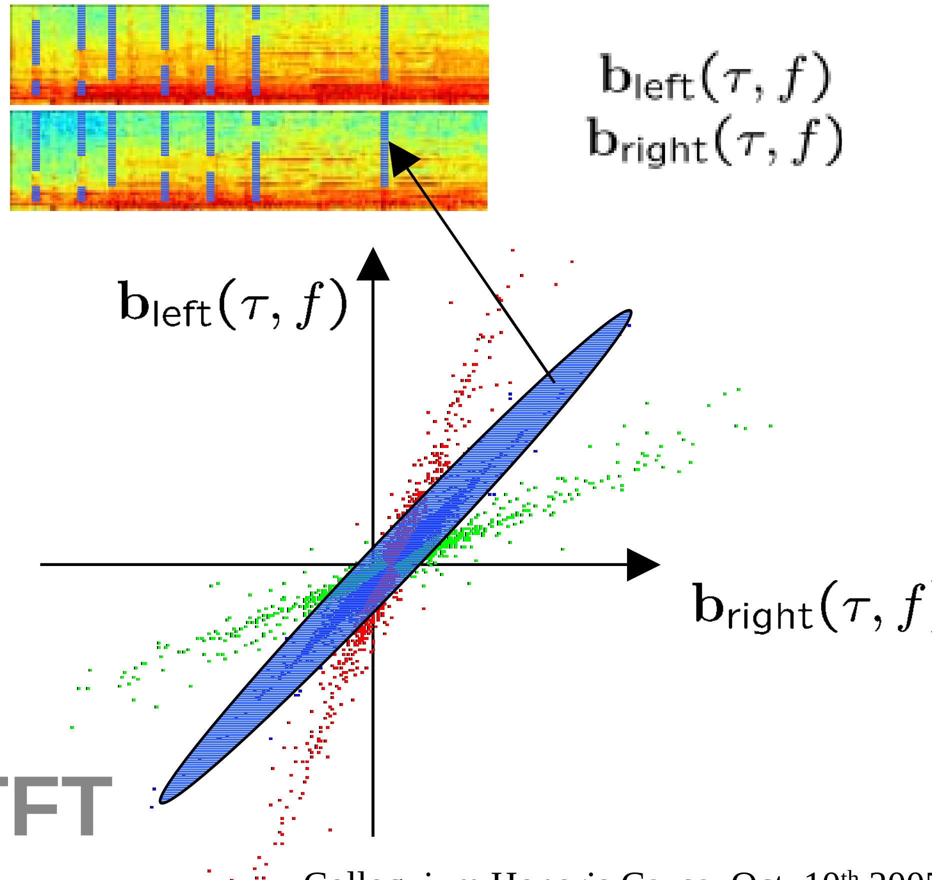

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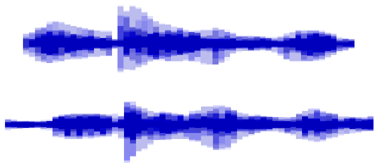

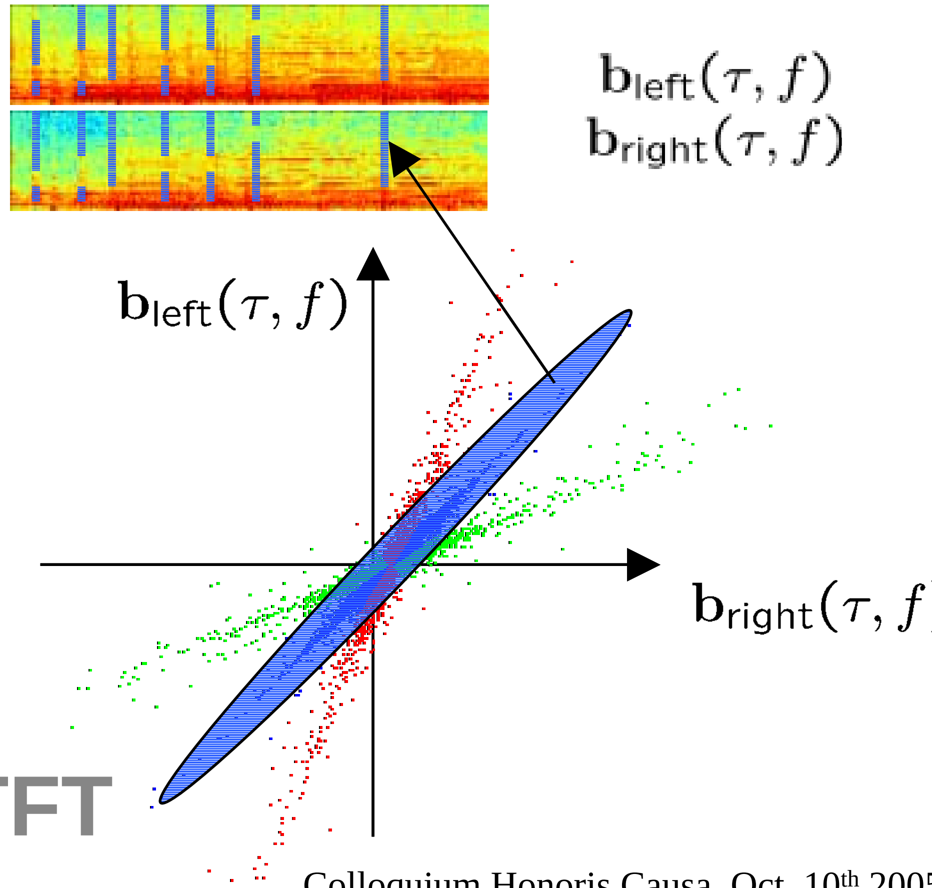

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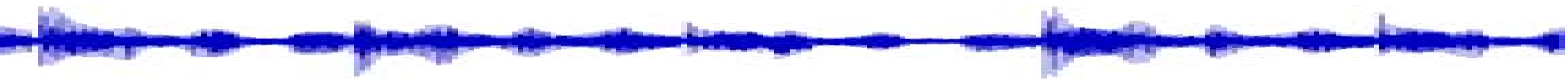

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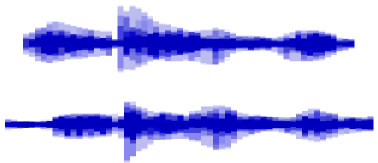

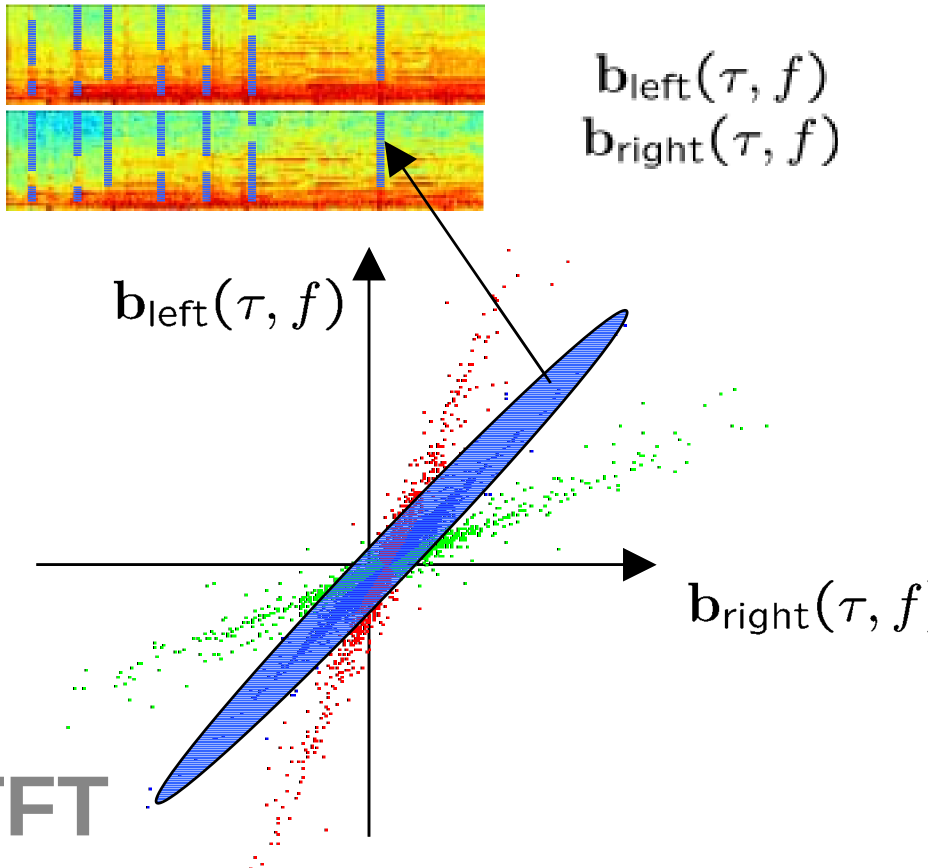

- Scatter plot
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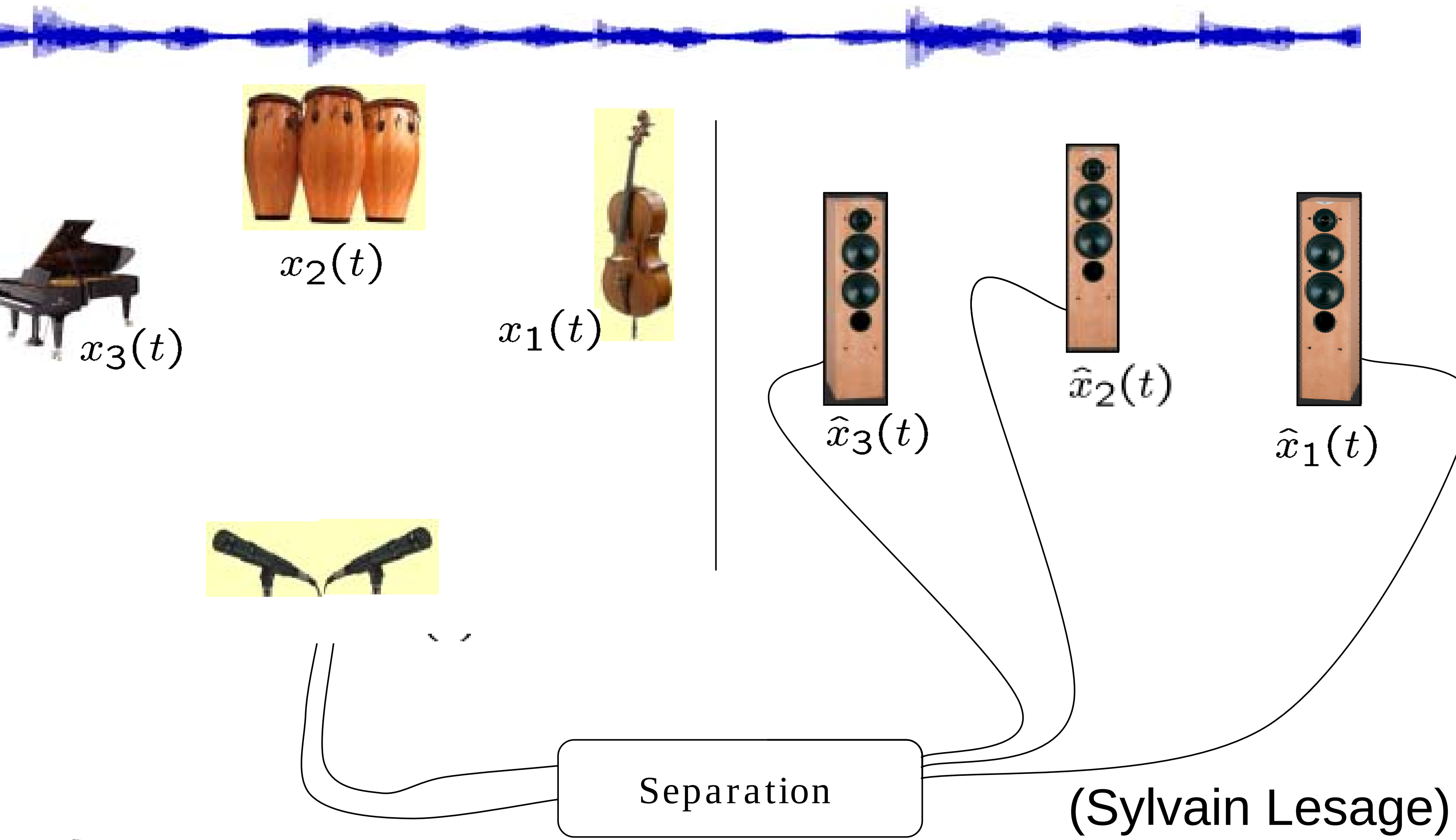
At last :
Stereophonic source separation



(Sylvain Lesage)

$$\hat{x}(t) = DUET(\mathbf{b}(t), \mathbf{A})$$

At last :
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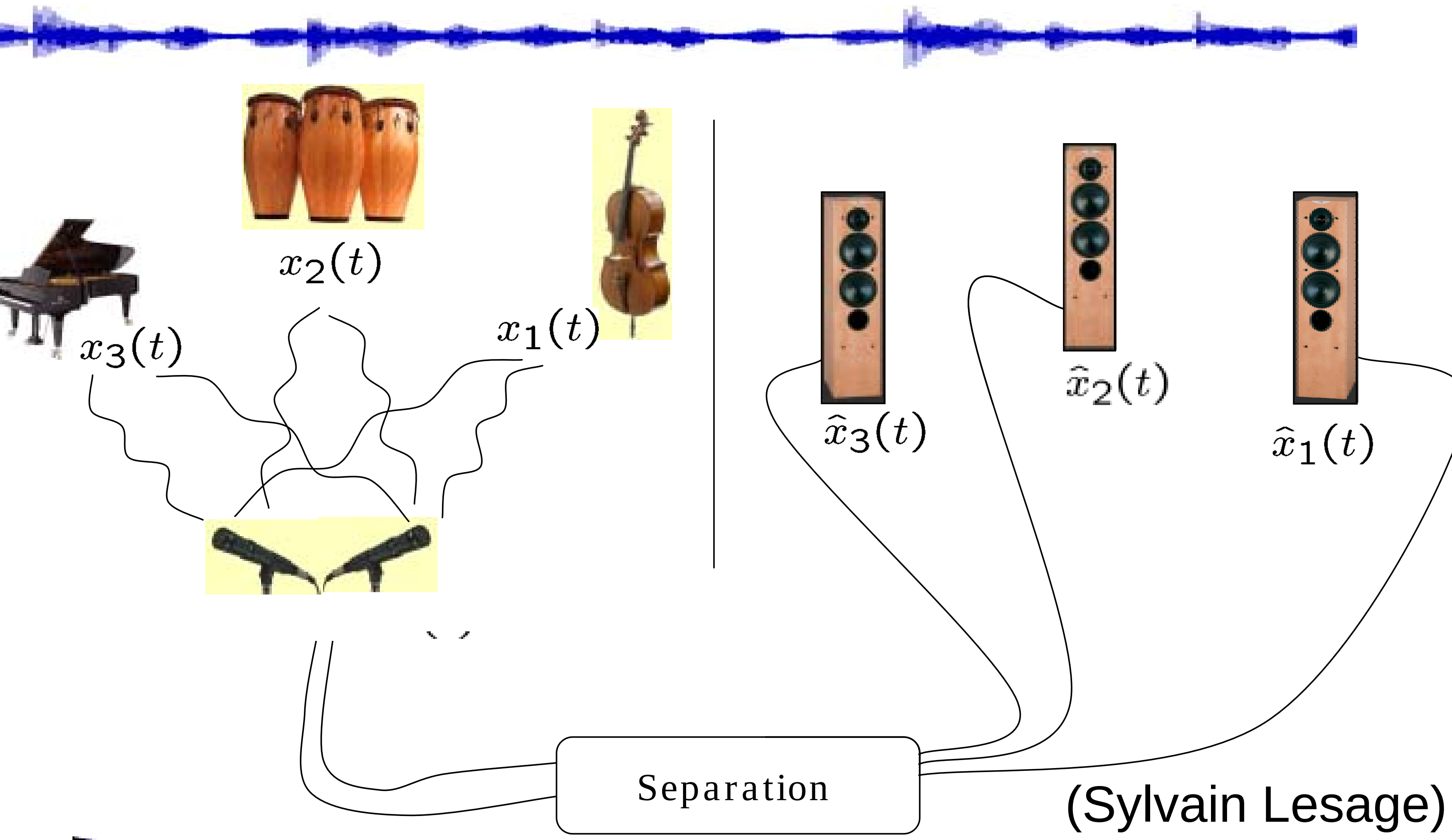

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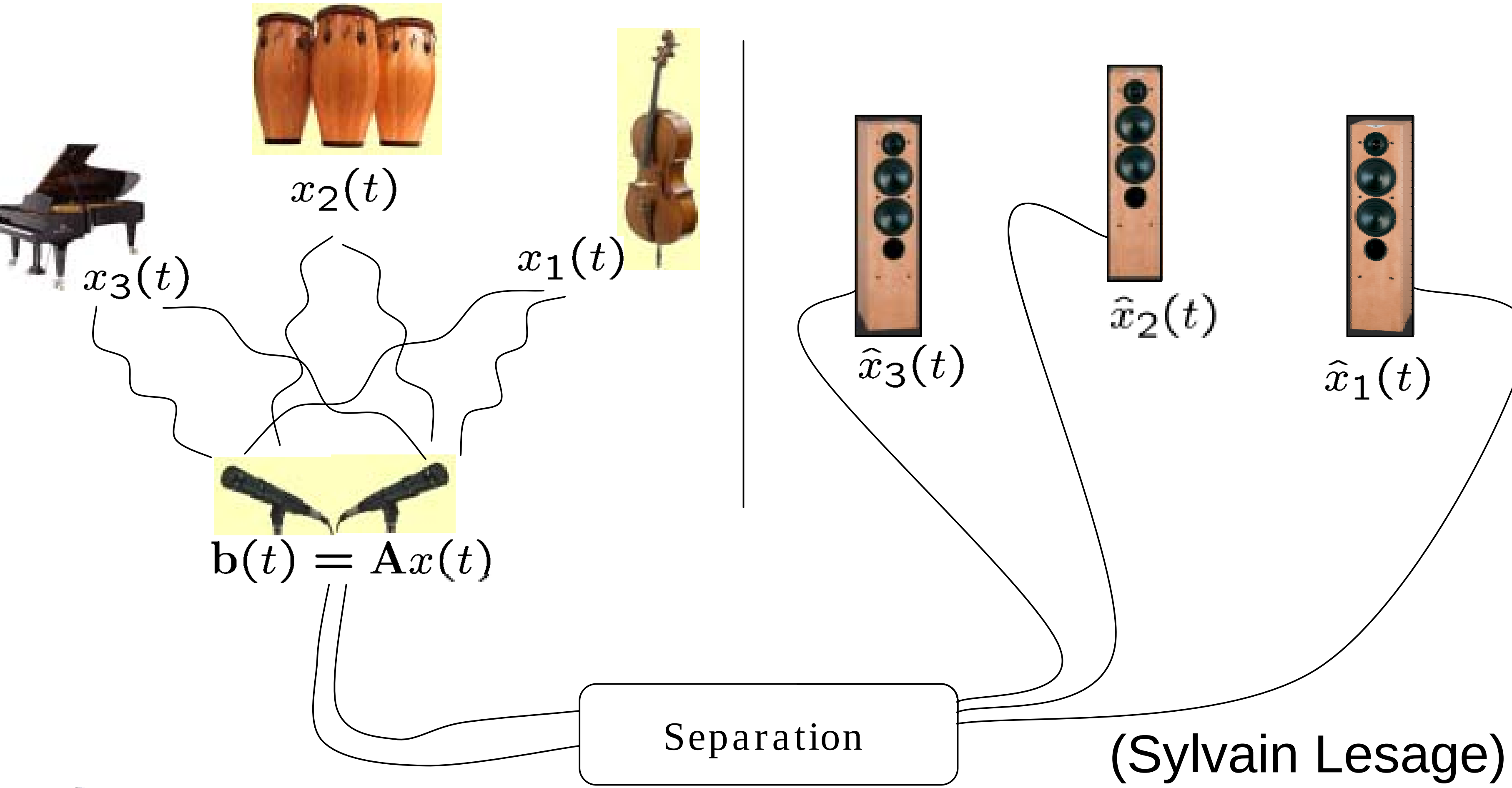
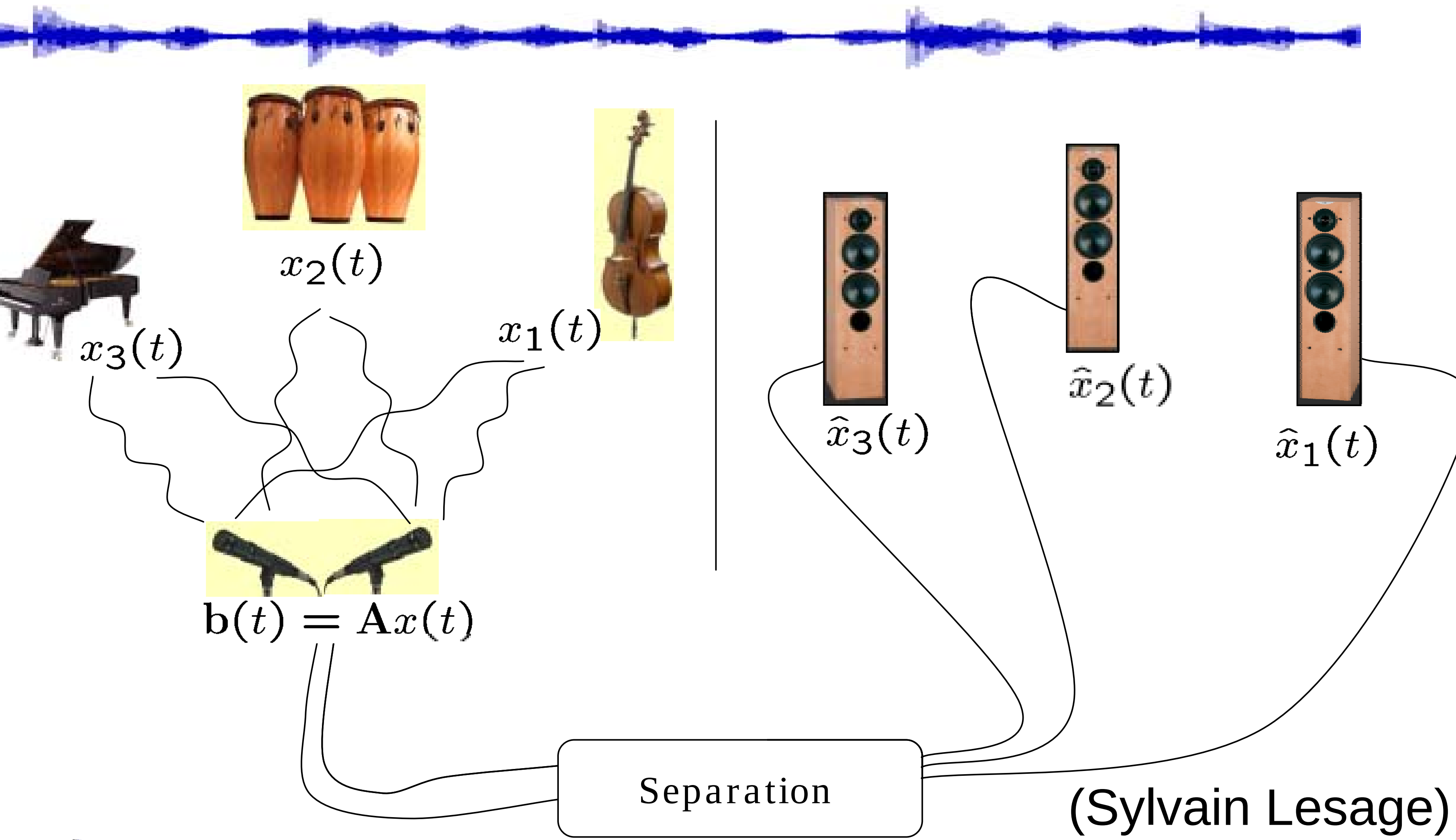


(Sylvain Lesage)

$\hat{x}(t) = D\dot{U}ET(\mathbf{b}(t), \mathbf{A})$

At last :
Stereophonic source separation



(Sylvain Lesage)

$$\hat{x}(t) = DUET(\mathbf{b}(t), \mathbf{A})$$

At last :
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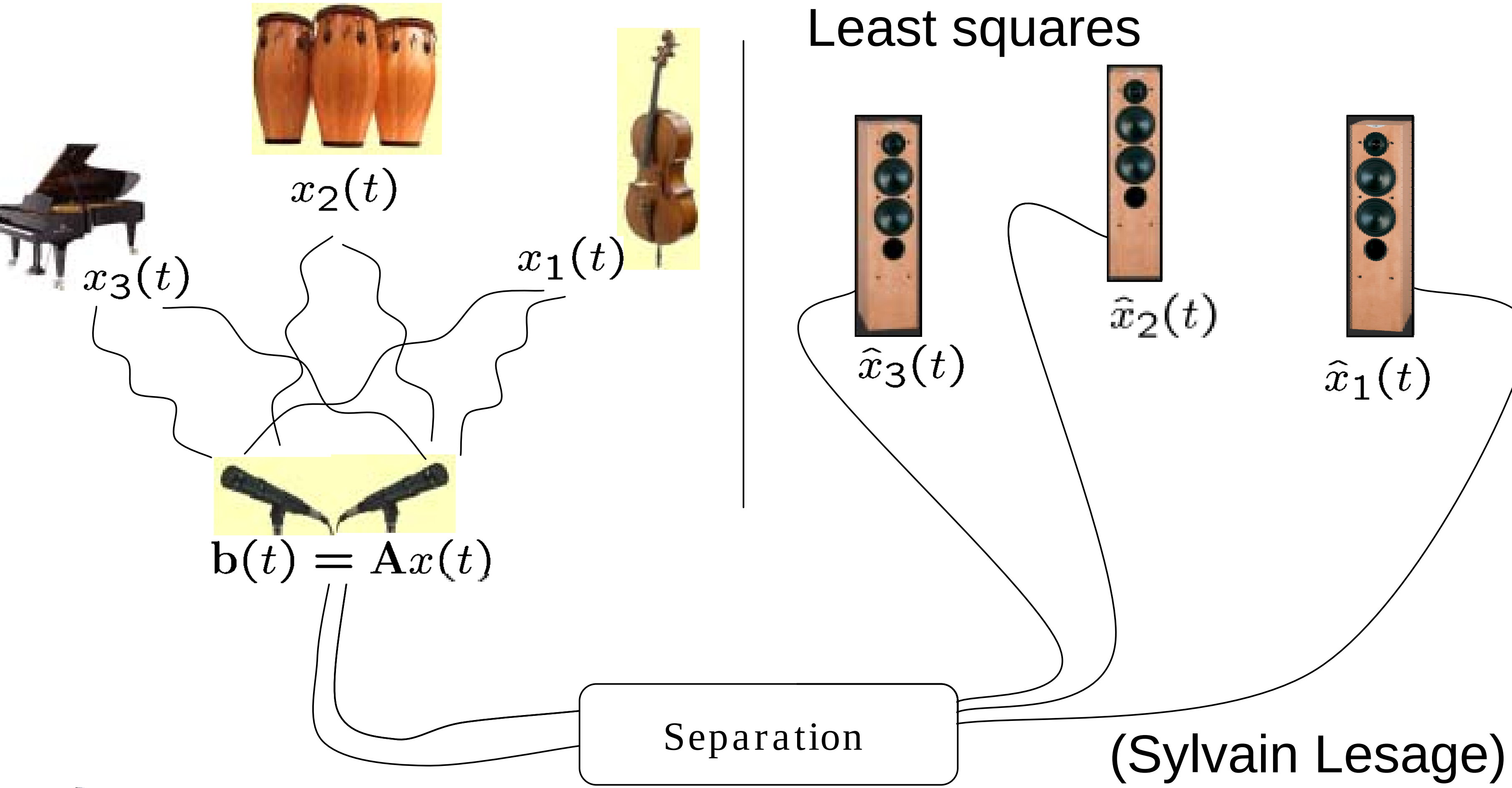

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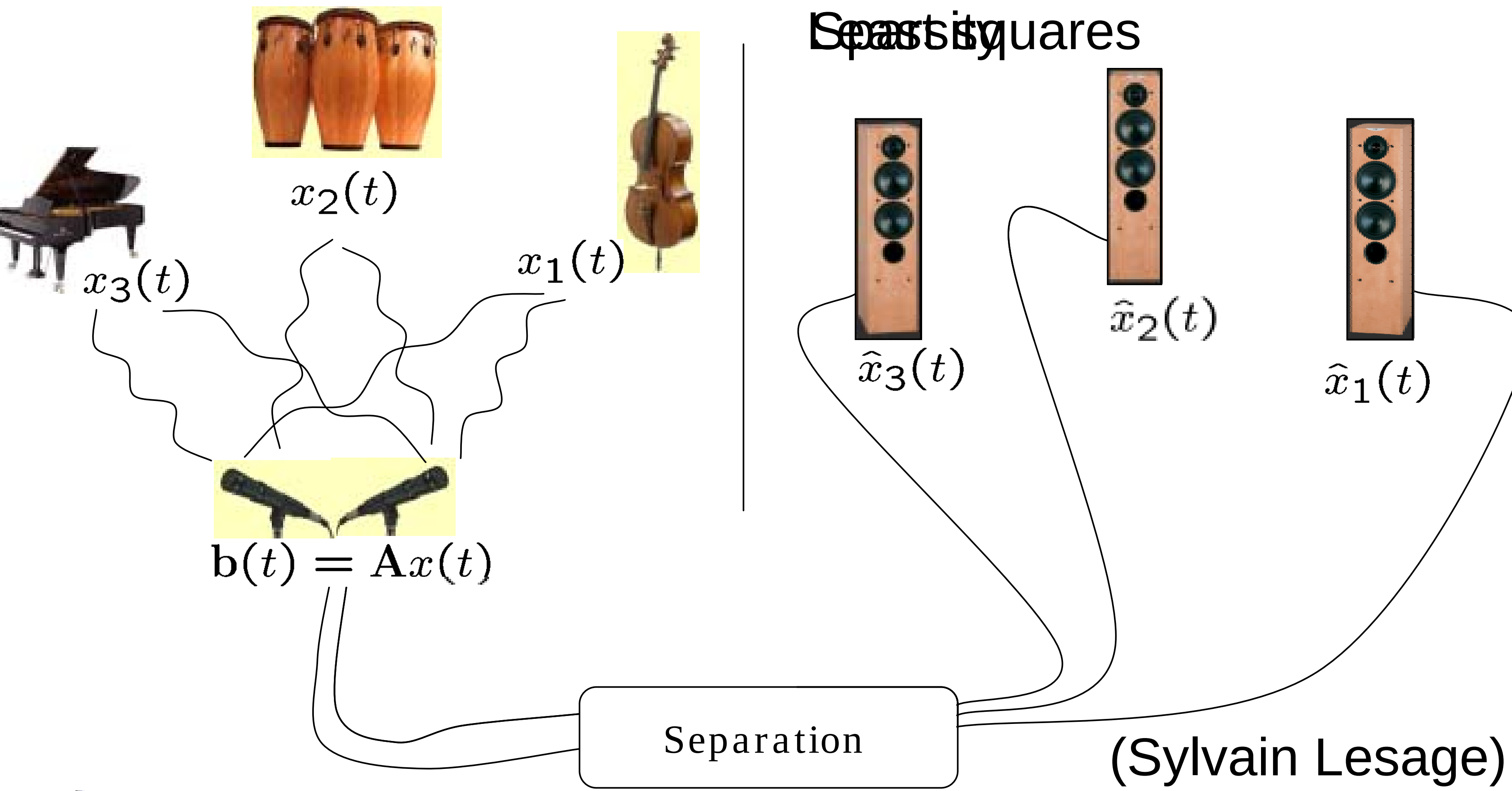
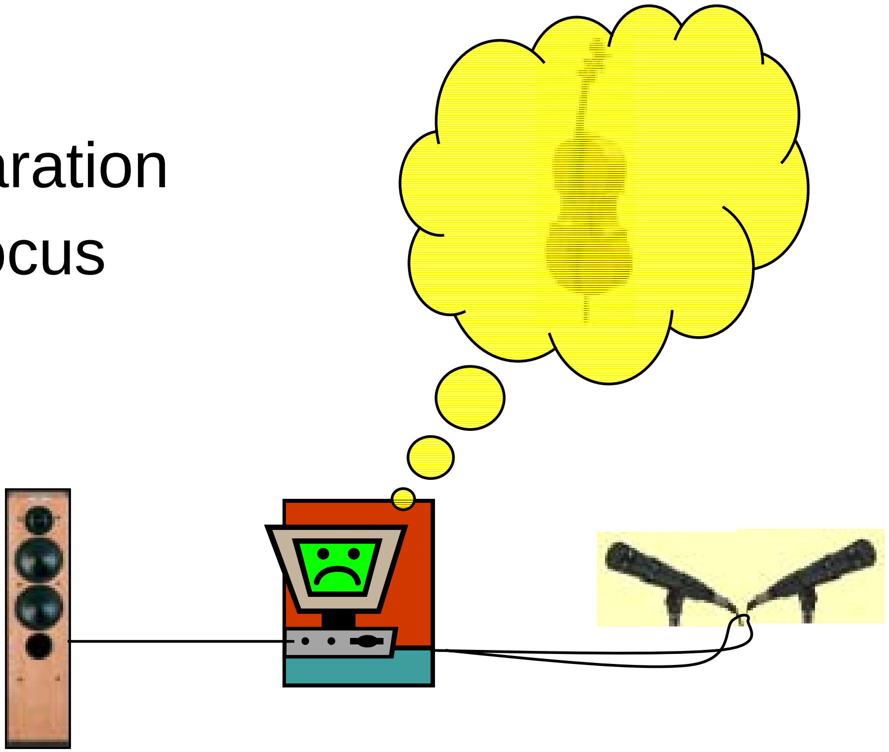


(Sylvain Lesage)

$$\hat{x}(t) = DUET(\mathrm{b}(t), \mathrm{A})$$

Conclusion

→ Source separation
Listen / focus



Conclusion

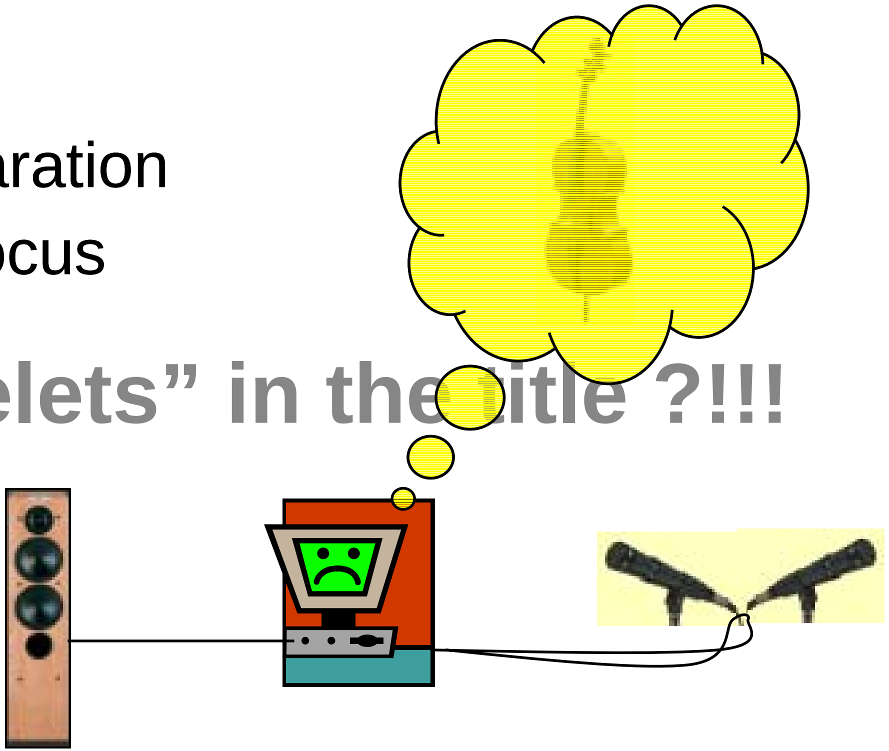
Time-frequency representation

+ clustering

Source separation

Listen / focus

… what about "wavelets" in the title ?!!!



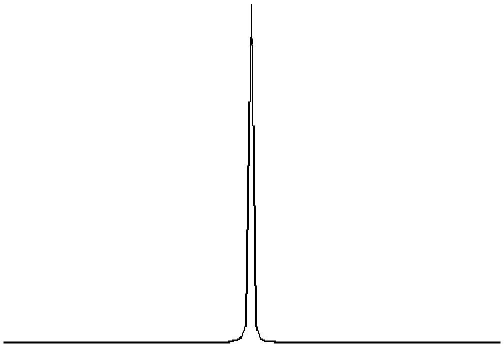
What about wavelets ?

$$x(t)$$

Many nonzero values
~Gaussian histogram

$$x(\tau, f)$$

Mostly zeroes = sparse
~Laplacian histogram



What about wavelets ?

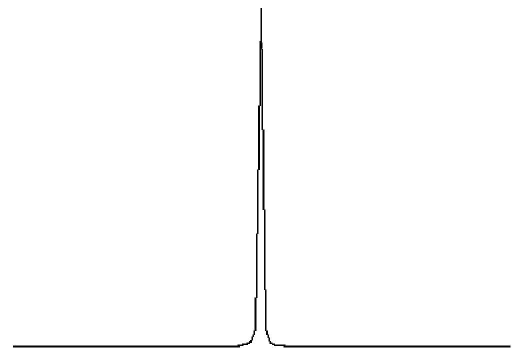
$$x(t) = \sum_{\tau,f} x(\tau, f) \cdot \varphi_{\tau,f}(t)$$

Ma

~Gaussian histogram

parse

~Laplacian histogram



What about wavelets ?

- Role of time-frequency representation

$$x(t) = \sum_{\tau,f} x(\tau,f) \cdot \varphi_{\tau,f}(t)$$

Ma

~Gaussian histogram

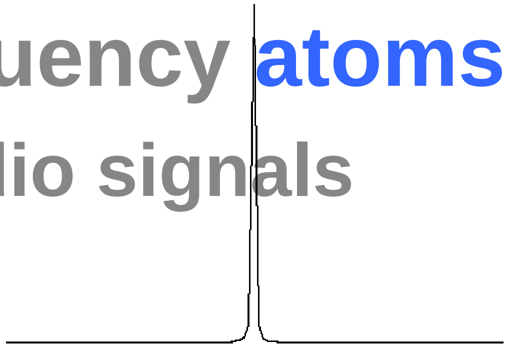
parse

~Laplacian histogram

Time-frequency **atoms**

Sparse coefficients for audio signals

Not for ECG, EEG, …



Some other possible waveforms

- Gabor atoms
- Dirac
- Local cosine basis
- Wavelets
- Wavelet packets
- Curvelets
- Bandelets
- Pointlets
- Brushlets
- Coiflets
- Ridgelets
- …

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Sparse decompositions with redundant dictionaries

$$\{\varphi_k(t)\}$$

$$x(t) = \sum_k c_k \cdot \varphi_k(t)$$

aster

Matching Pursuit

sparse

Not sparse

~Sparse ~efficient

Combinatorial

Sparse decompositions with redundant dictionaries

- **Redundant dictionary of waveforms**

$$\{\varphi_k(t)\}$$

- **Sparse representation**

$$x(t) = \sum_k c_k \cdot \varphi_k(t)$$

$$x = c\Phi$$

- **Redun ... ed**

too many solutions → **compute one**

aster | Matching Pursuit | sparse

Not sparse | ~Sparse ~efficient | Combinatorial

Sparse decompositions with redundant dictionaries

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faster — $\min \|c\|_2$ — Matching Pursuit — sparse

Not sparse ~Sparse ~efficient Combinatorial

Sparse decompositions with redundant dictionaries

- Redundant dictionary of waveforms

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aster | $\min \|c\|_2$ | Matching Pursuit | $\min \|c\|_0$ | sparse

Not sparse | ~Sparse ~efficient | Combinatorial

Sparse decompositions with redundant dictionaries

- **Redundant dictionary of waveforms**

$$\{\varphi_k(t)\}$$

- **Sparse representation**

$$x(t) = \sum_k c_k \cdot \varphi_k(t)$$

$$x = c\Phi$$

- **Redun ... ed**

too many solutions ➡ **compute one**

aster | $\min \|c\|_2$ | Matching Pursuit | $\min \|c\|_1$ | $\min \|c\|_0$ | sparse

⟵⟶

Not sparse | ~Sparse ~efficient | Combinatorial

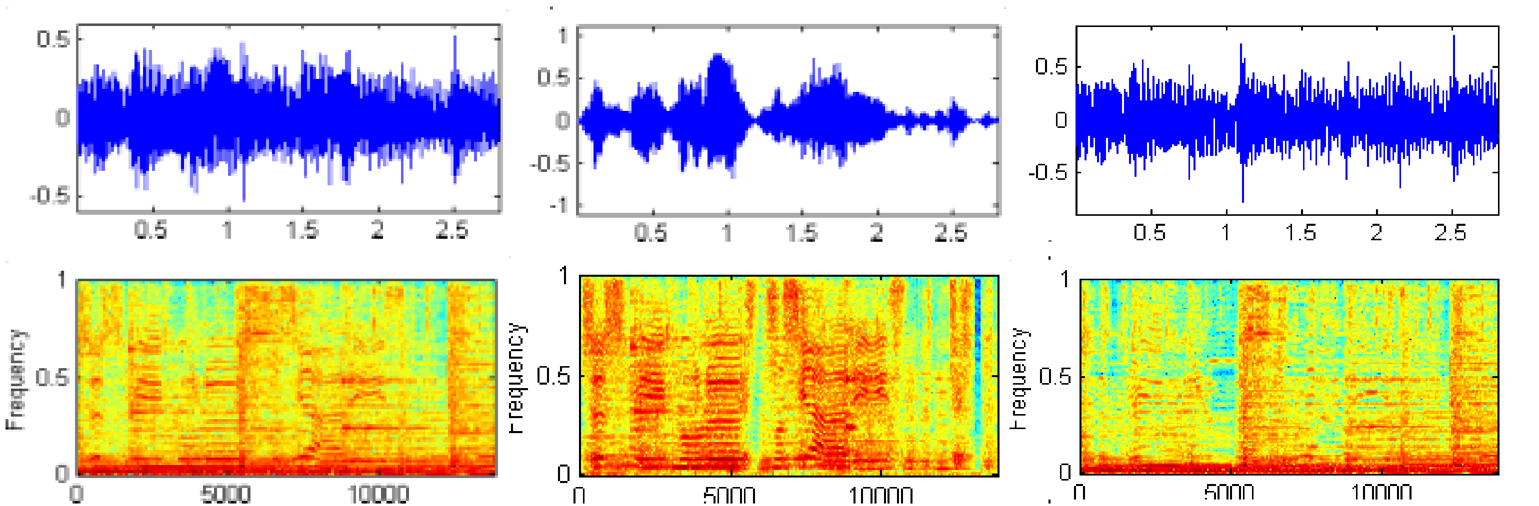
Conclusion

Conclusion

- Sparse representations help for source separation
- Some other applications
  - Low bitrate image / signal compression (JPEG,MP3)
  - Communications (MIMO) ?
- Main challenges
  - Adapted dictionaries
  - *(ECG, EEG, geology, financial data, …)*
  - Efficient decomposition algorithms
- What about computer audition ?
  - Simple acoustic model = approximate
    - → room acoustics = convolution
  - Sparsity = rough source model
    - → statistical source models

Beyond sparsity : Monaural source separation

Mixture	Estimated voice	Estimated music



Tools: -time-frequency representations

-statistical models

(Alexey Ozerov, FT R&D)

To go further
… beyond sparsity

Important dates :

$$b = \text{SPARS05} \approx \mathbf{A} \cdot [1, 1, 1, 1, 1, 0, \ldots]^T$$

June : abstracts

$$= [\text{Signal}, \text{Processing}, \text{Adaptive}, \text{Sparse}, \text{Structured}, \text{Representations}, \ldots$$

September : registration

http://spars05.irisa.fr

spars05@irisa.fr

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… beyond sparsity

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- Organization:
  - CNRS/MathSTIC “Sparse structured approximations of audio signals”

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To go further
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  Sparse Approximations in Signal and Image Processing
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  Greed is good: algorithmic results for sparse approximations, IEEE Trans. Inform. Th., Octobre 2004

- http://www.irisa.fr/metiss/gribonval/

To go further
Matching Pursuit Toolkit

Φ M

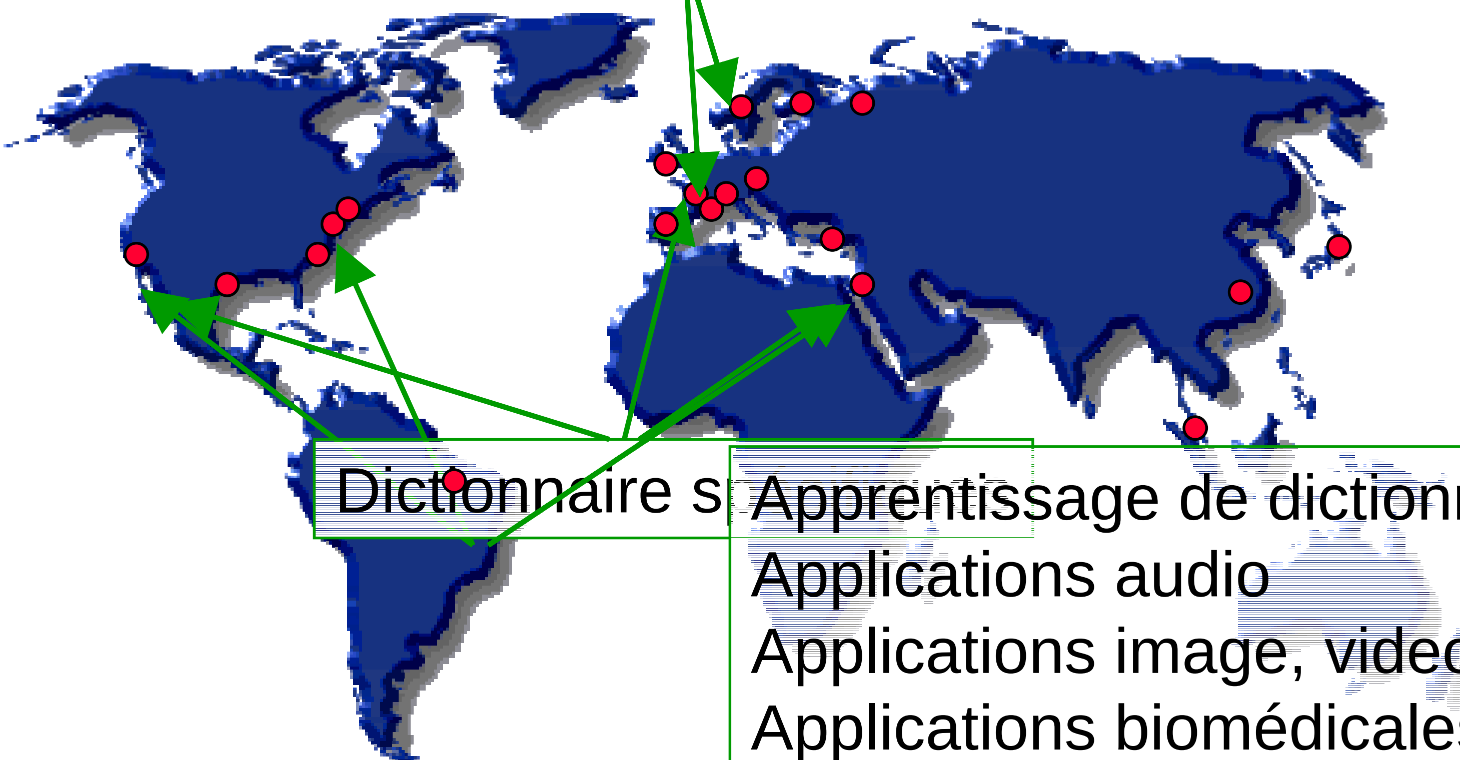
To go further
Matching Pursuit Toolkit

- **FAST** : the fastest available to our knowledge
  - 1hour @ 16kHz : 58 920 593 samples,
  - Φ 178 773 275 atoms, M 1 500 000 iterations (SNR = 30dB)
  - 20 minutes computation time
- State of the art :
  - **Multichannel** signal processing
  - Available *blocks* : Dirac, Gabor, Harmonic
- **Flexible** and standalone
  - Build your own applications with the library
  - Add your own blocks / atoms / transforms
  - Command line utilities
  - Flexible XML description of dictionaries
  - Independent of external interface (no Matlab, no LastWave)
- Multi-platform (Unix based), std C++, **open source**

sacha@irisa.fr, remi.gribonval@irisa.fr

To go further
Some ongoing extensions

Sparsity ⟶ structure

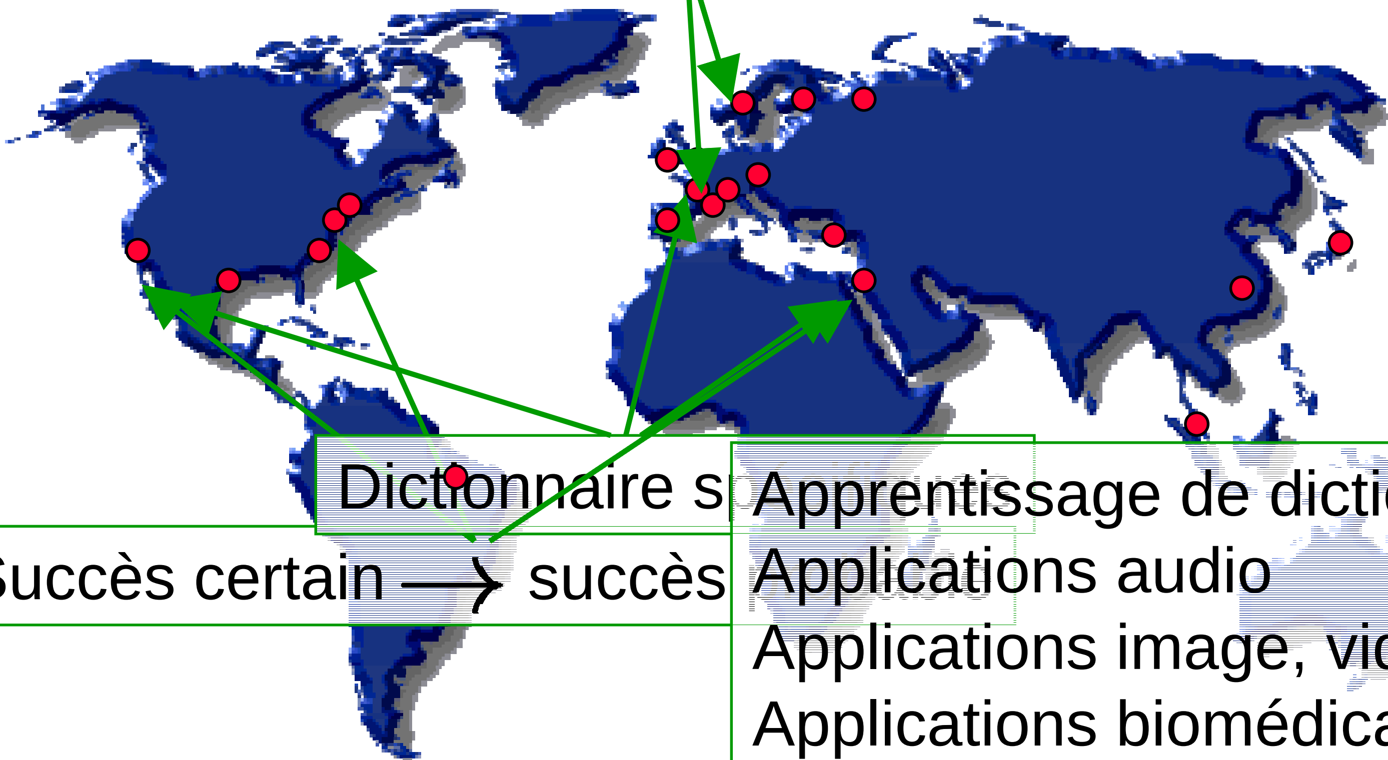


Dictionnaire s

Apprentissage de dictionnaire (ICA)
Applications audio
Applications image, video
Applications biomédicales (ECG,…
Applications télécom …

To go further
Some ongoing extensions

Sparsity ⟶ structure



Dictionnaire s

Succès certain ⟶ succès

Apprentissage de dictionnaire (ICA)
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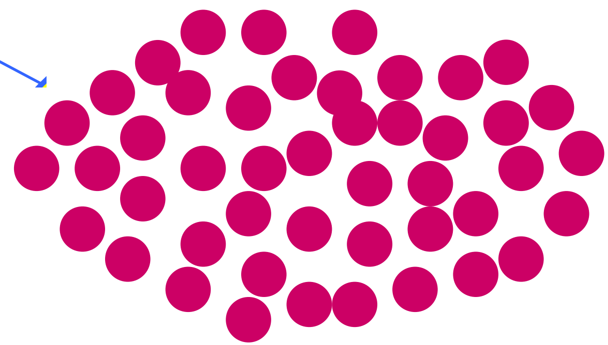
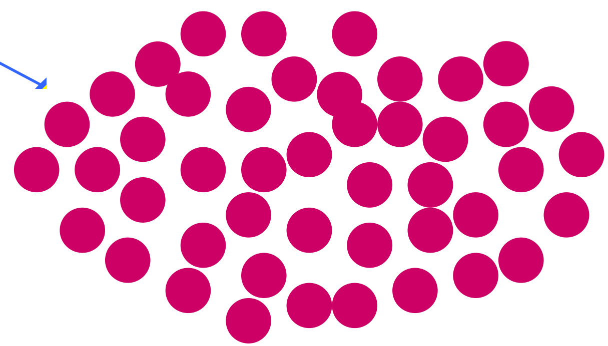
Time-frequency representation

- 1 mixture



Time-frequency representation

- 1 mixture
- 2 sources



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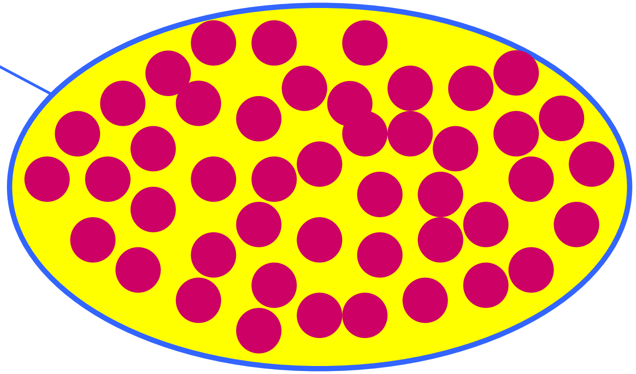
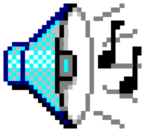
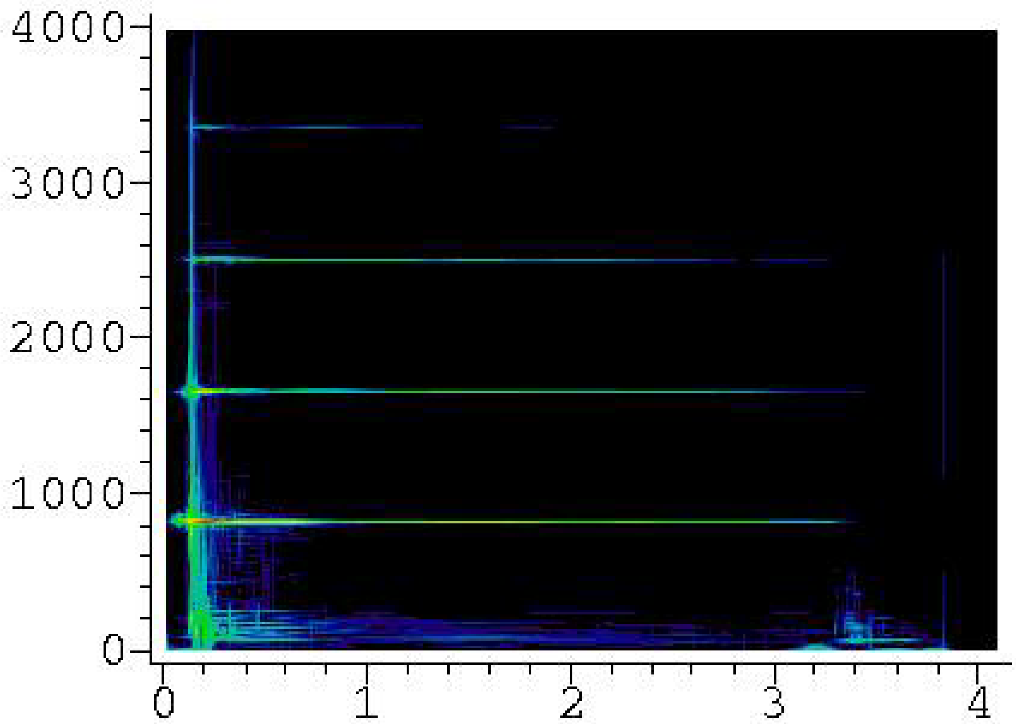


Exemple
Décomposition de son

- Une note de piano, 4 secondes à 8kHz





Exemple
Décomposition de son

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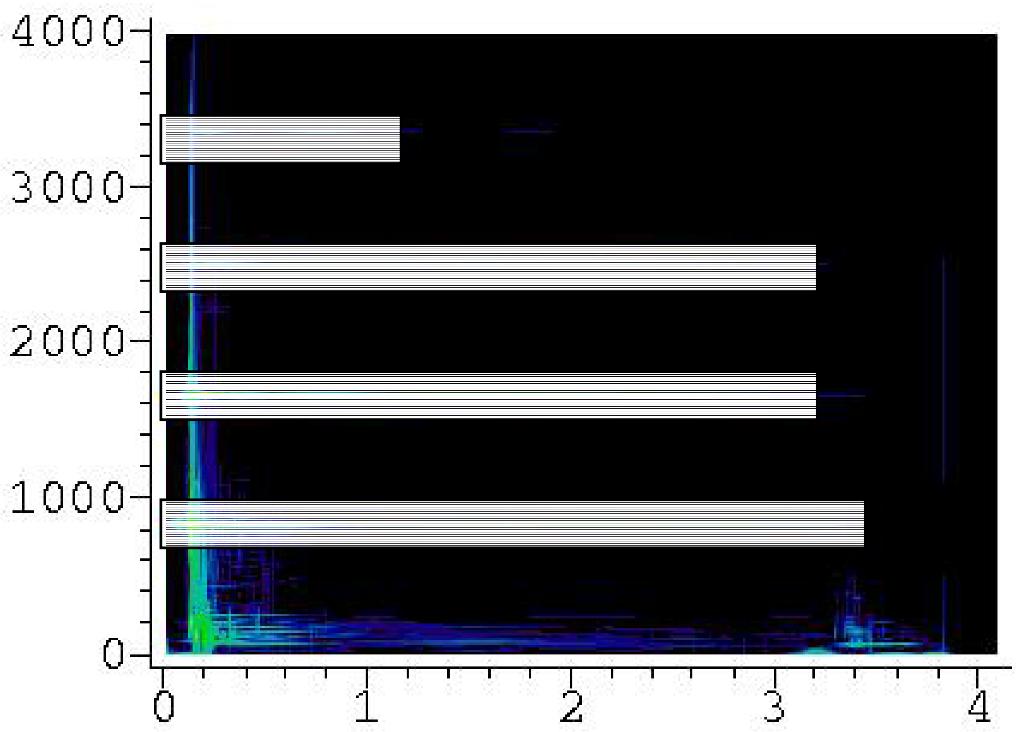
•Corde
= structure longues



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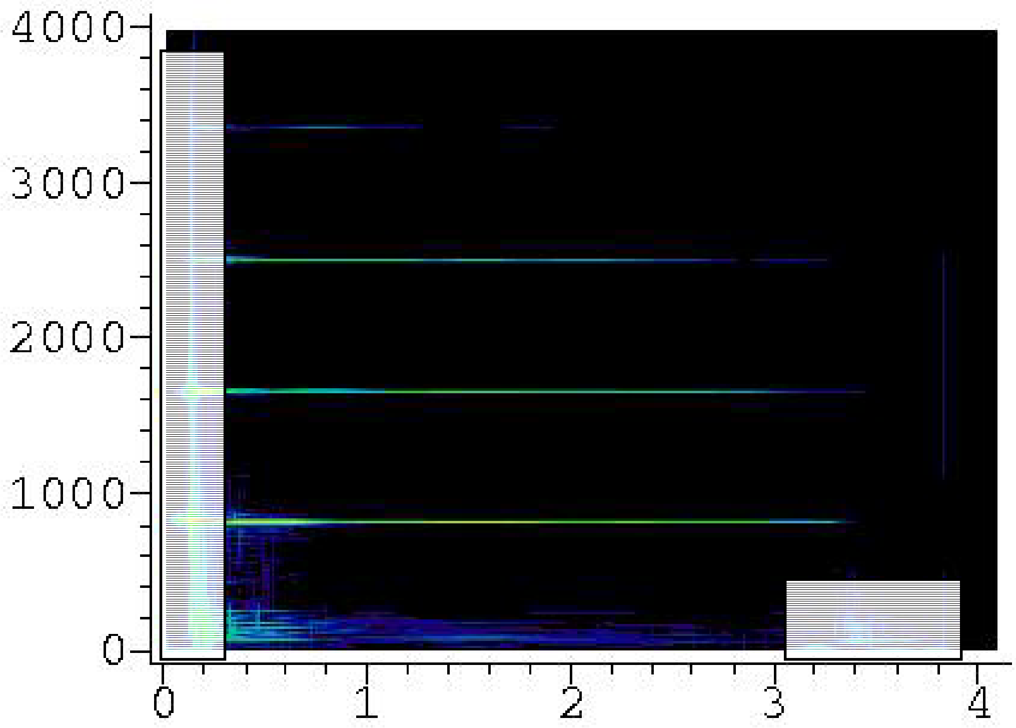
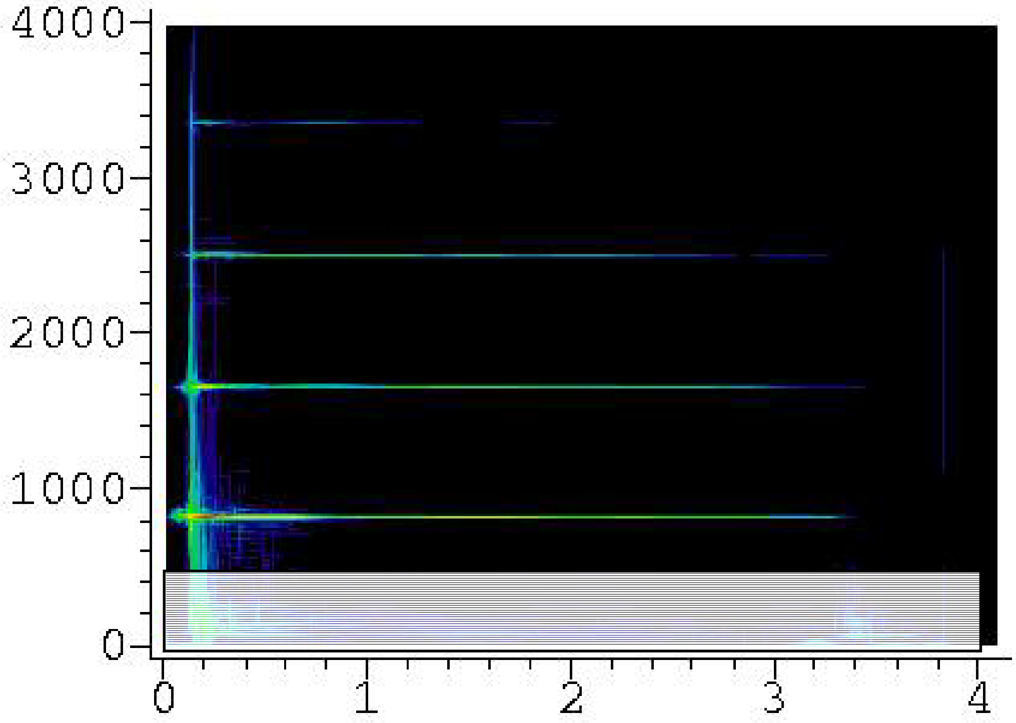


•Marteau
et étouffoir
=structures courtes

Exemple
Décomposition de son

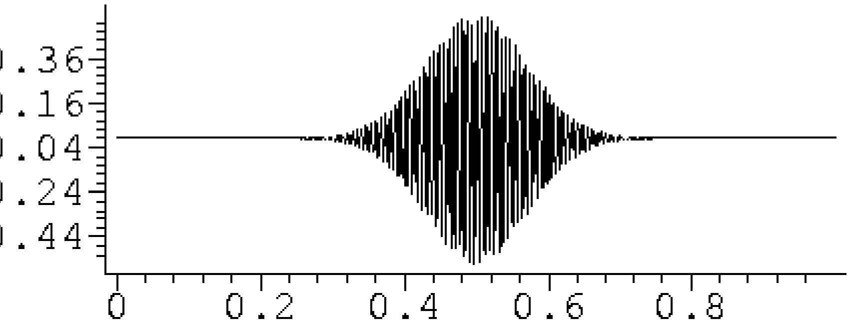
- **Une note de piano, 4 secondes à 8kHz**



•Table
=structures basse fréquence

Wavelets ?
Time-frequency atoms!

Time-frequency atom



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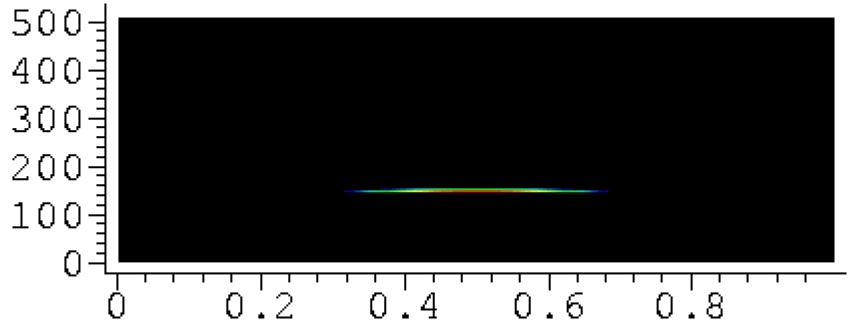

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Scale s, time t, frequency f

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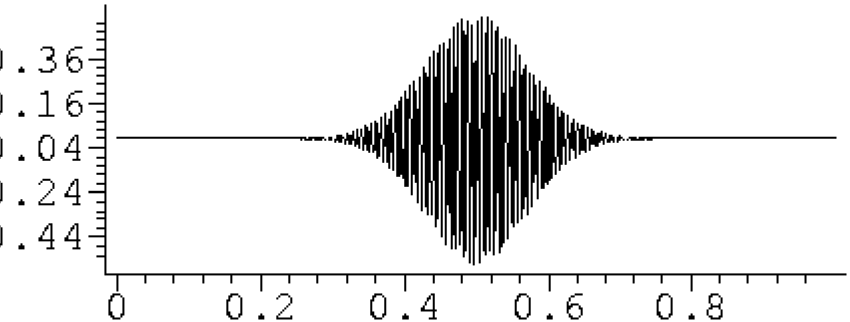

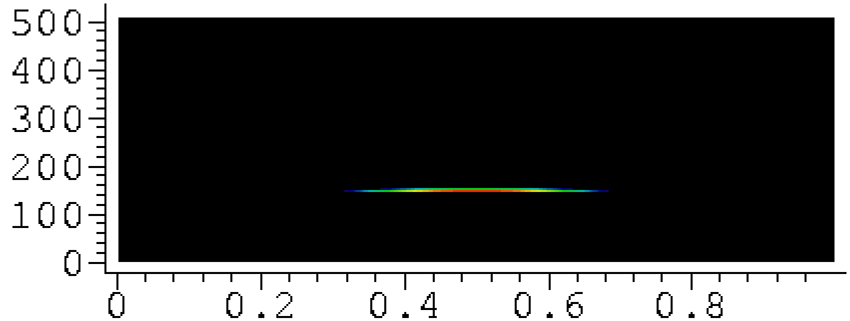

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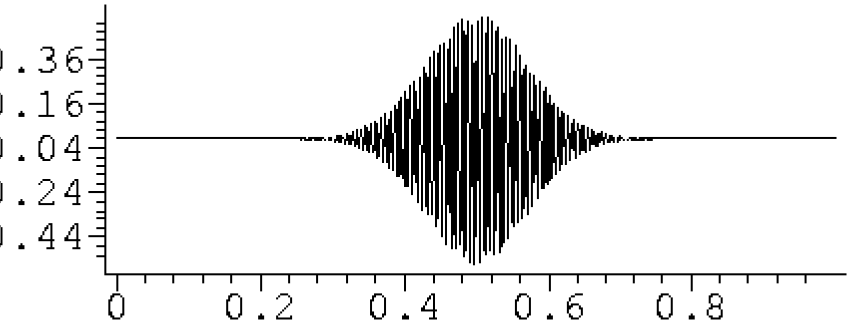

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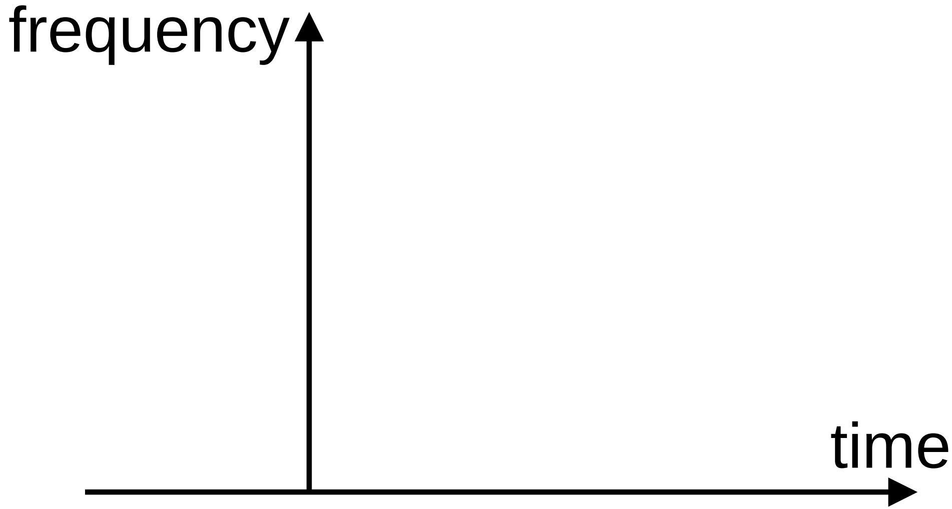

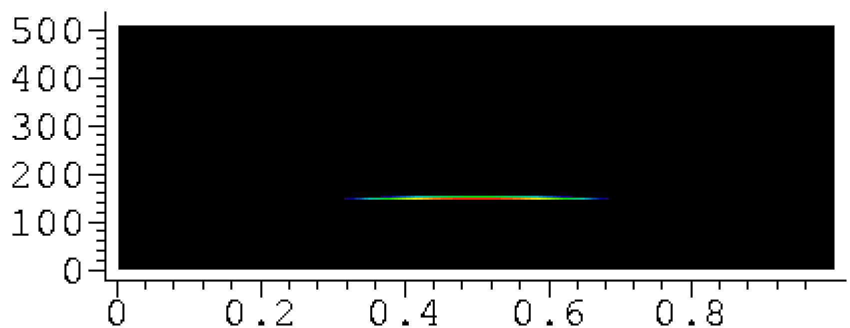

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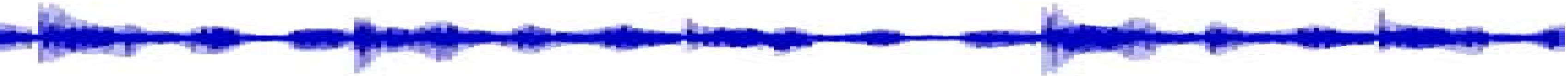

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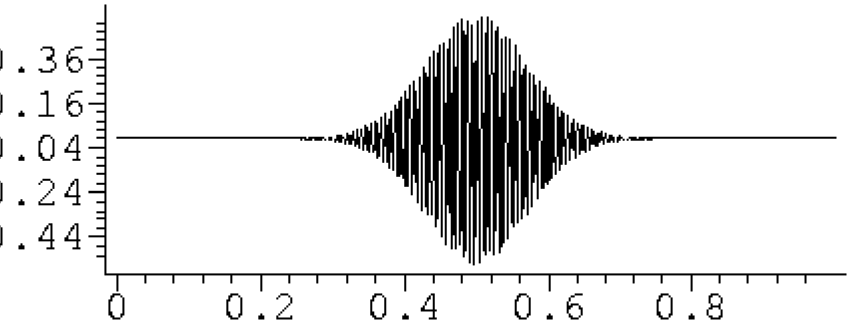

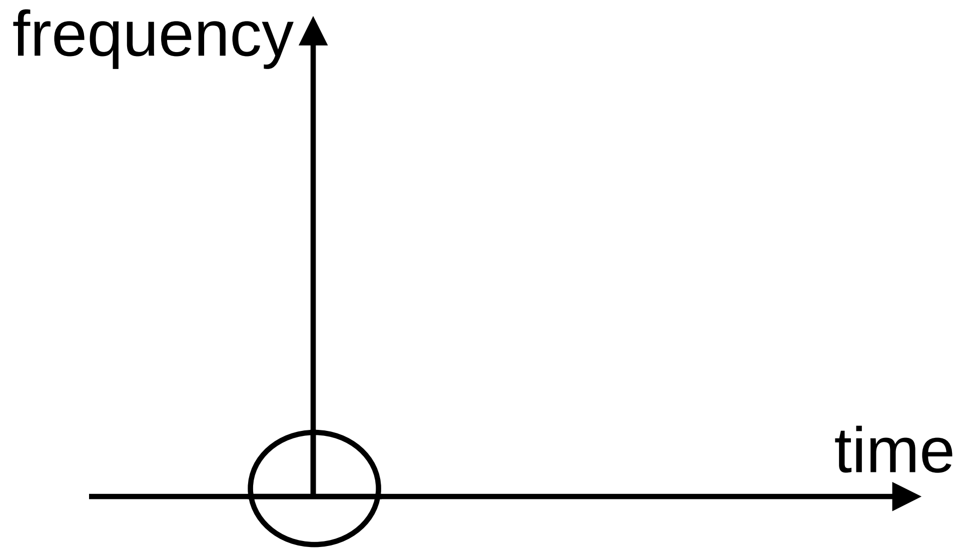

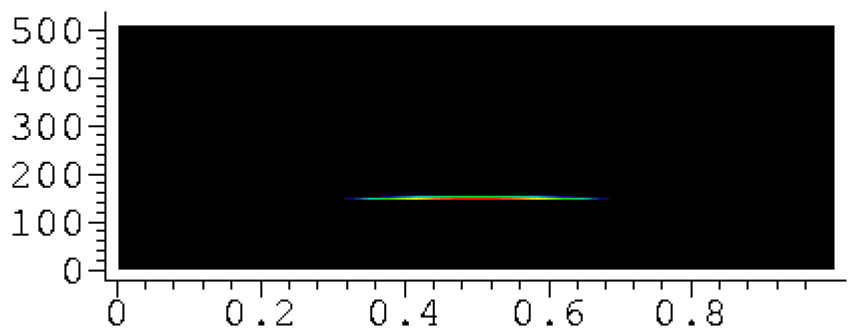

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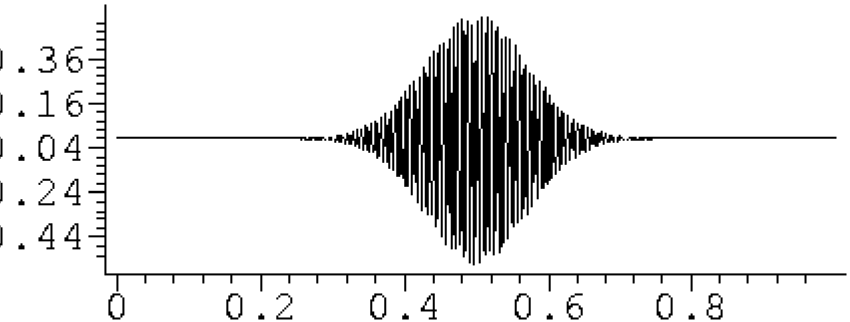

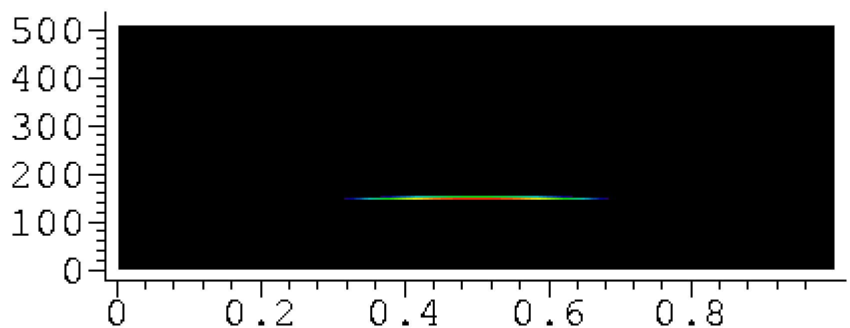

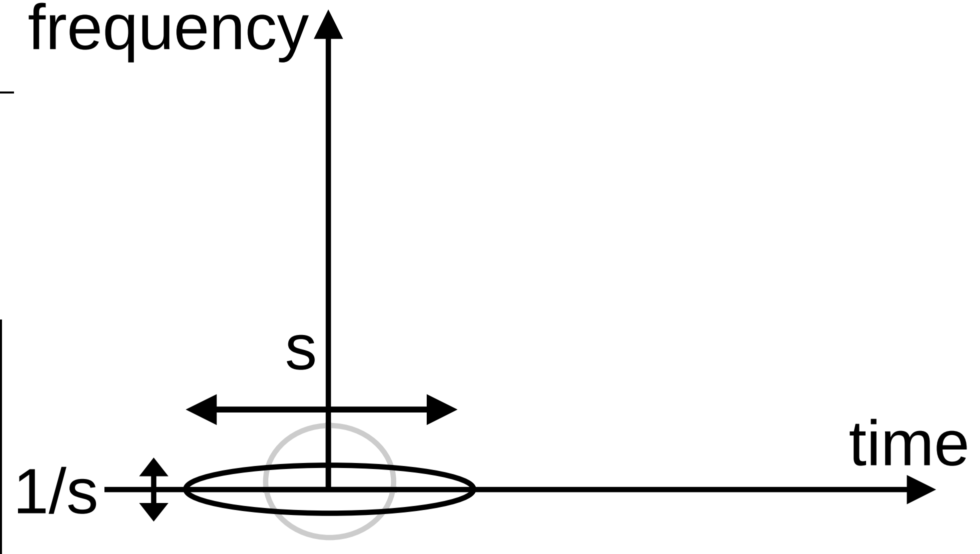

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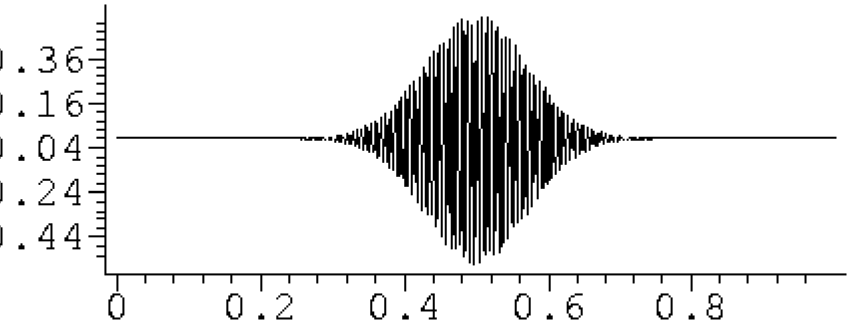

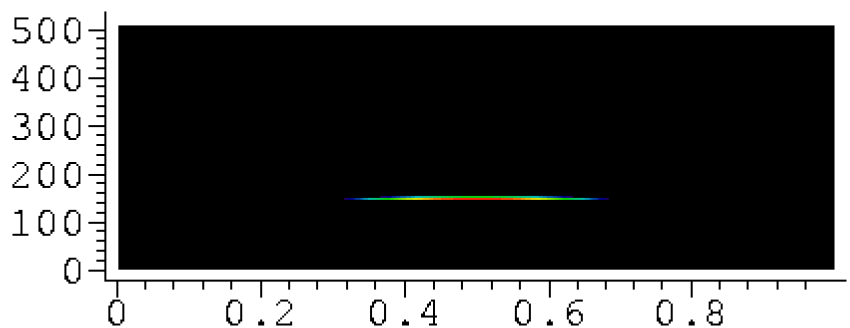

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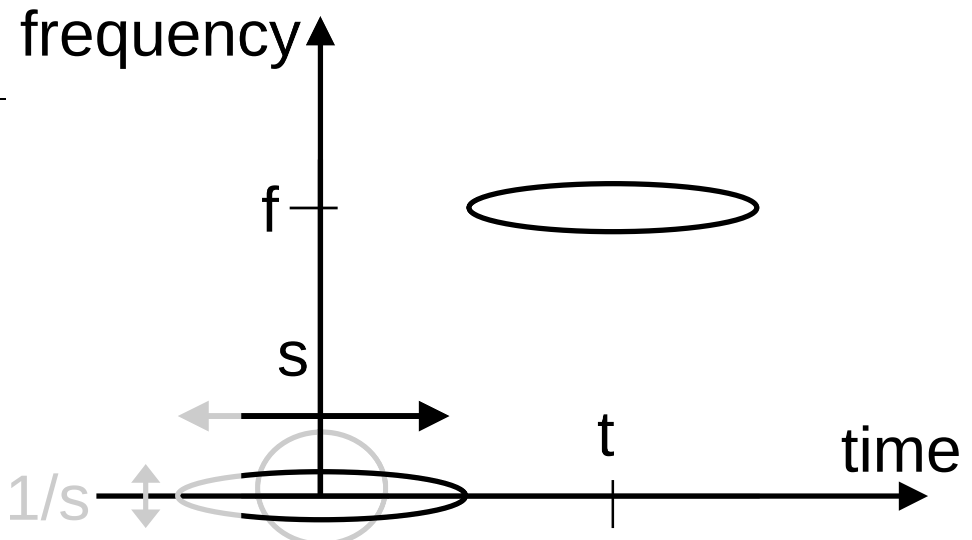

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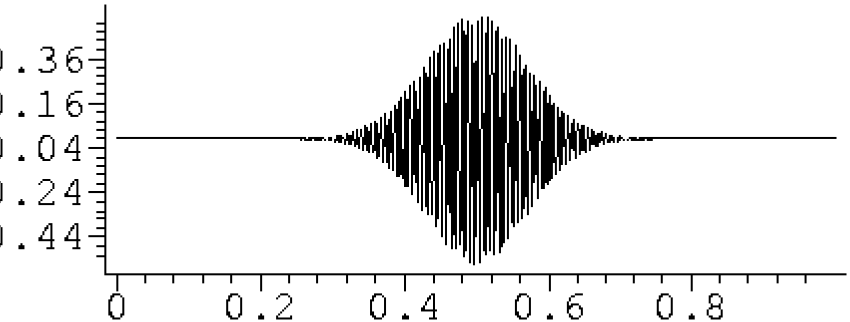

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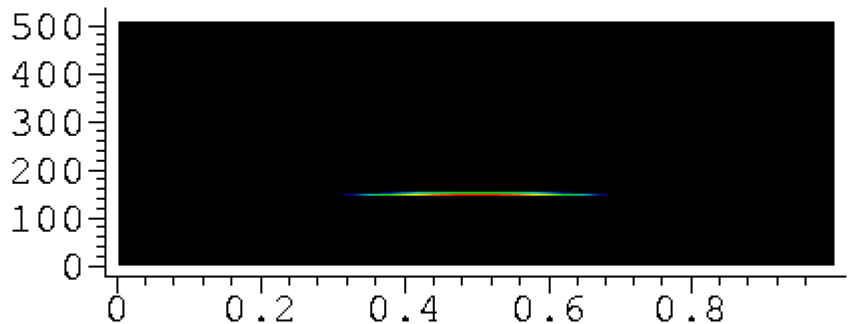

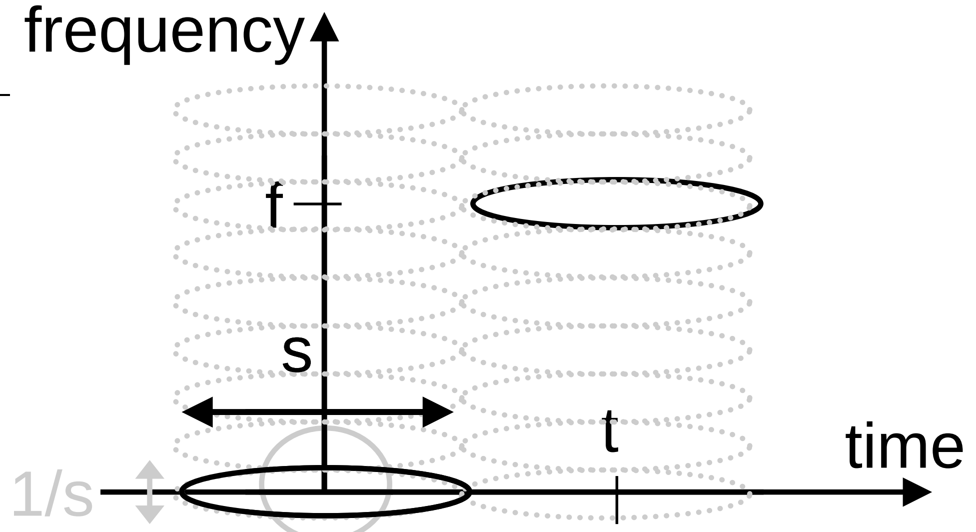

Wavelets ?
Time-frequency atoms!

Chirplets ncy atom

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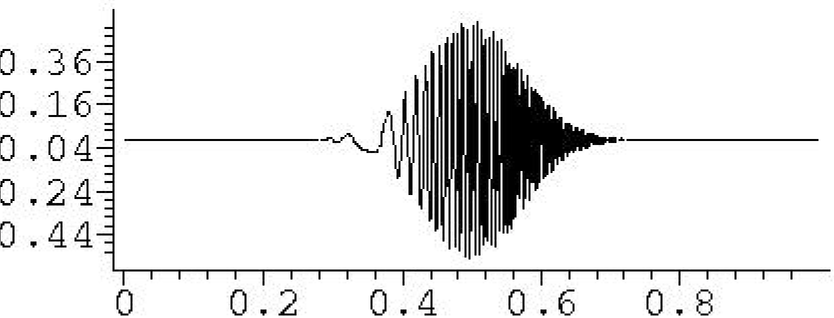

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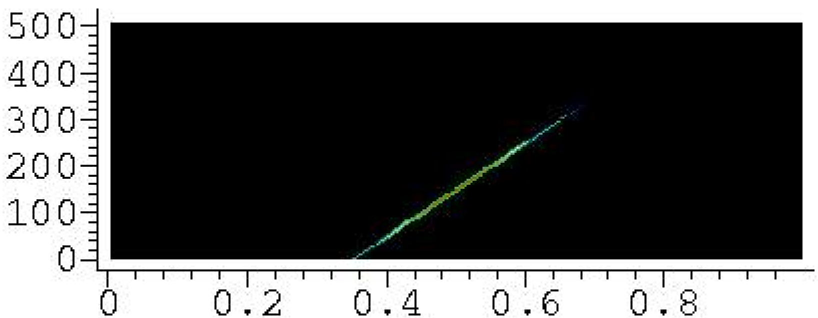

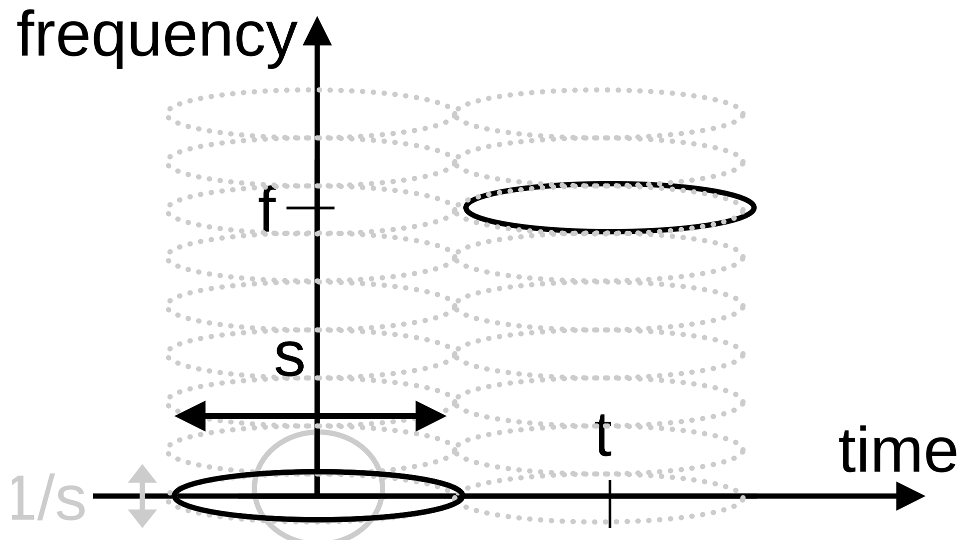

Wavelets ?
Time-frequency atoms!

Chirplets ncy atom

Scale s, time t, frequency f , chirprate c

$$\varphi_{s,\tau,f}(t) = \frac{1}{\sqrt{s}} w\left(\frac{t-\tau}{s}\right) e^{2i\pi f t} \;\; e^{i\pi c (t-\tau)^2}$$