Some mathematical and numerical challenges around ITER

TONUS, TOkamak NUmerical Simulation

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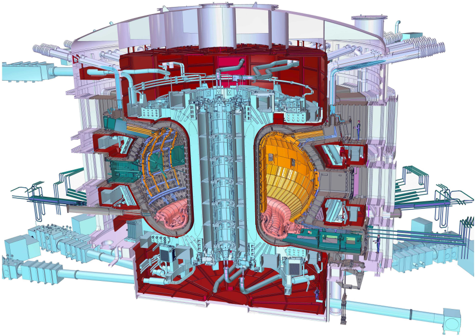
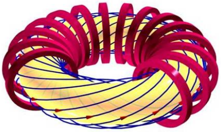
ITER project

International Thermonuclear Experimental Reactor, ITER project: thermonuclear fusion in a hot hydrogen plasma (more than 100 millions of °C). Energy of the future. 20 billions euros.

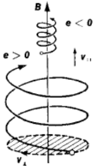


Tokamak

tokamak: magnetic plasma confinement in a torus. Poloidal coils $\Rightarrow$ toroidal field. Plasma current $\Rightarrow$ poloidal field $\Rightarrow$ plasma stability. Tamm-Sakharov in the 50's.



Strong magnetic field, few collisions: gyrokinetic turbulence.



Vlasov-Maxwell model

- Unknown: the distribution function $f(x, v, t)$. Number of ions at point x and time t having velocity v. The problem is time-dependent in a six-dimensional phase space.
- Vlasov equation with weak collisions

$$\partial_t f + v \cdot \nabla_x f + (E + v \times B) \cdot \nabla_v f = C(f) \simeq 0.$$

- Poisson equation (or Maxwell equations) on the electric potential Φ. The magnetic field is given.

$$\nabla \cdot E = \rho - \rho_0, \quad E = -\nabla \Phi.$$

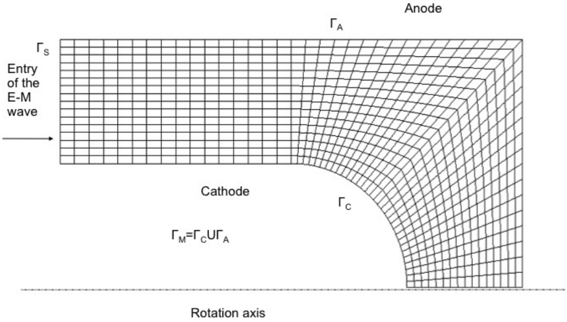
  The charge ρ is given by

$$\rho(x, t) = \int_v f(x, v, t) dv.$$

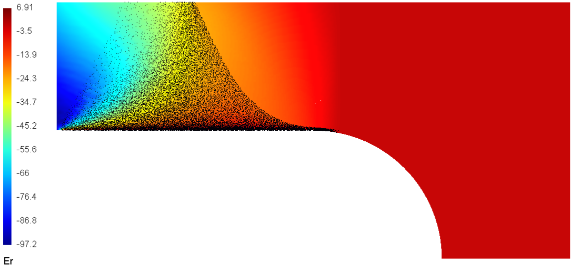
- Boundary conditions.

Vlasov-Maxwell simulation

Simplified case: 2D X-ray generator. Particle-In-Cell (Vlasov) Discontinuous Galerkin (Maxwell). GPU implementation (A. Crestetto)



Vlasov-Maxwell simulation



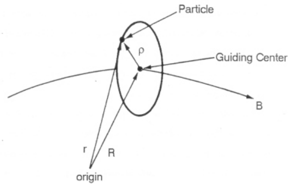
Tokamak simulation challenges

Solving the full Vlasov-Poisson or Vlasov-Maxwell model in a tokamak is too much expensive, even for supercomputers.

- Rigorous asymptotic simplifications or adapted mathematical representation of the solution.
- Adapted numerical schemes.
- Implementation in useful software.

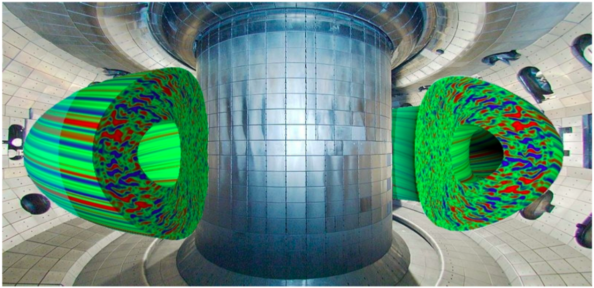
1) Gyrokinetic modeling

In the gyrokinetic model, the Vlasov equation is replaced by a transport equation of the guiding centers.



The Poisson equation is replaced by the quasineutrality equation for the electric potential. Acting fields are obtained by gyroaveraging the real fields on the particles trajectories. Theoretical justification of such models.

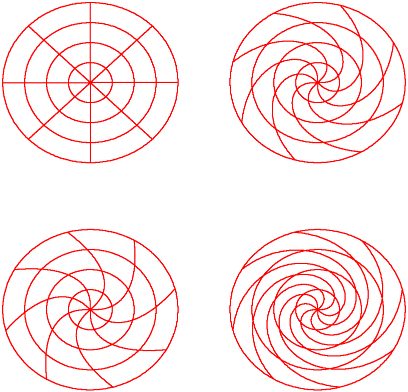
2) Field anisotropy in tokamak simulations



Fluctuations and anisotropy of the electric field (CERF project flyer, A. Koniges LBNL).

3) Aligned meshes

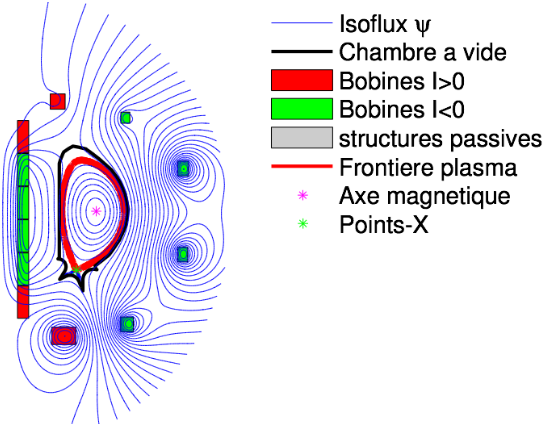
(G. Latu)



Stretching of the mesh along the toroidal direction.

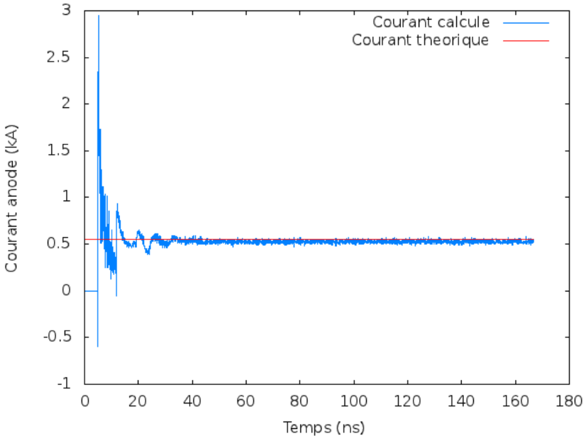
4) Real magnetic configuration

Coupling with magnetic solver, control.



(J. Blum, G. Selig)

5) Precise numerical method



The PIC methods are noisy. Eulerian or Semi-Lagrangian approaches are more reliable, while more expensive.

6) High Performance Computing (HPC)

- GYSELA, CEA gyrokinetic simulation tool. Recent 500,000 cores run.
- electron effects simulation possible within 10 years.
- Exascale computing. C2S@EXA IPL (S. Lanteri). Peta=10^{15}, Exa=10^{18}. Exaflops=10^{18} operations/second. 1 GPU$\simeq 10^{12}$ flops.
- Hot subject: Co-design for Exascale Research in Fusion CERF project in the US. Physicists, mathematicians, computers scientists of several institutions working on tokamak simulations "=" Fusion IPL + C2S@EXA IPL.

SeLaLib

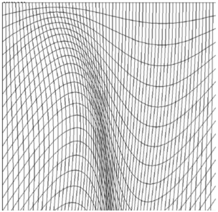
Transport equation, characteristic method, directional splitting, parallel transposition.
R. Blanchard, E. Chacon-Golcher, P. Navaro, M. Mehrenberger, E. Sonnendrücker

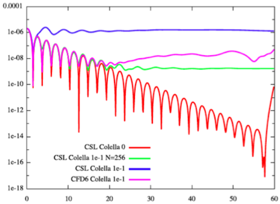
- SeLaLib is an acronym for Semi-Lagrangian Library. It is supported by an Inria ADT.
- It is a collection of tools, written in Fortran 2003, for solving plasma physics problems.
- As of June 2013 it contains: high order semi-Lagrangian solvers, Poisson FFT-based solvers, parallel tools.
- "Anteroom" of GYSELA...

Semi-Lagrangian solvers

M. Mehrenberger

- Conservative scheme (P. Glanc)
- High order scheme (C. Steiner)
- Vlasov on curvilinear meshes (A. Hamiaz)

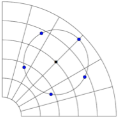
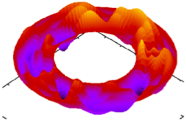
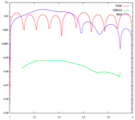
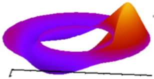
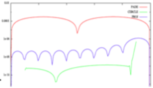




Gyroaverage operator

M. Mehrenberger

- Padé approximation on curvilinear meshes
- Quadrature on circle (C. Steiner)

Software: CLAC

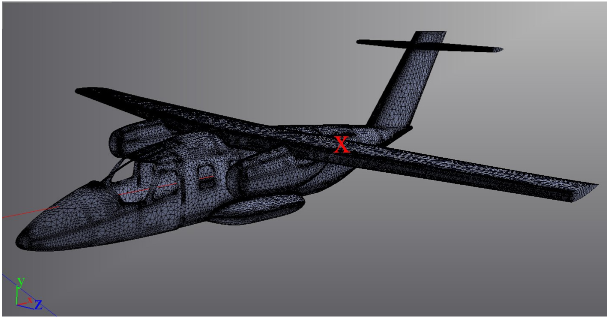
Hyperbolic system of conservation laws, reduced modeling, fine grain OpenCL parallelism, coarse grain MPI sub-domain parallelism.

PH, M. Massaro, T. Strub

- CLAC is an acronym for Conservation Laws Approximation on many Cores.
- It is a generic 3D Discontinuous Galerkin (DG) solver for system of hyperbolic conservation laws: Maxwell, MHD, waterbags, reduced Vlasov, *etc.*
- It is based on OpenCL and MPI and thus runs on clusters of GPU's. It allows arbitrary order and non conforming meshes.
- It is developed with the company AxesSim in Strasbourg.
- Collaboration with the CAMUS team for multicore optimizations.
- It is being rewritten for better memory access.

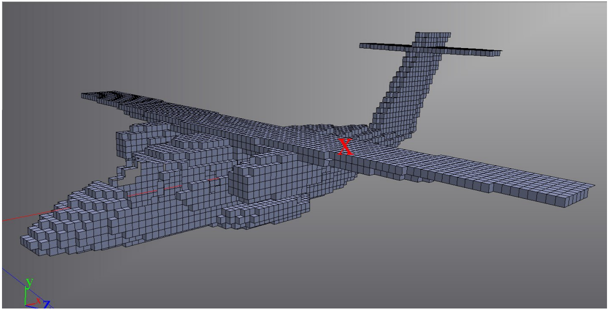
CLAC

Example: Maxwell simulation (T. Strub)



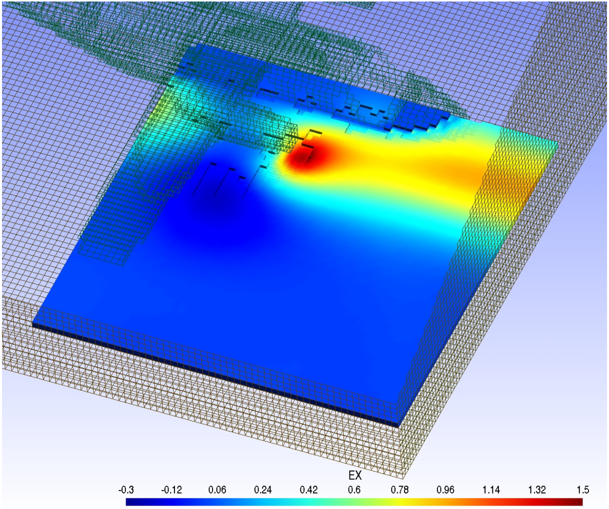
CAD mesh

CLAC



FDTD mesh (provisional)

CLAC



Electric field

CLAC

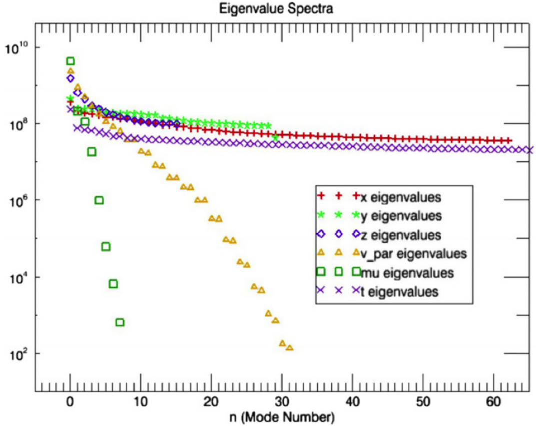
100 millions of unknowns, second order approximation, 50 time steps.

Arch.	time (s)	“speedup”
1x core Intel Xeon 2.3 GHz	5980	1
12x cores Intel Xeon 2.3 GHz	389	15 (?)
1x GPU AMD Radeon HD7970	95	63
4x GPU AMD Radeon HD7970	42	142

- “speedups” have to interpreted with care. The classical MPI implementation of ONERA is also efficient.
- MPI communications 10%, GPU-CPU transfers 30%
- we are currently reorganizing CLAC.

Gyrokinetic data analysis

Hatch et al. JCP 2012



Phase space turbulence. Smaller in the velocity directions.

Reduced models

We represent the distribution function as

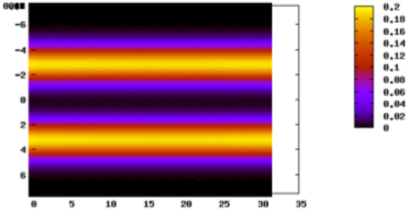
$$f(x,v,t) = M(\alpha(x,t),v),$$

where α is a vector of parameters that depends only on space and time. Generally, α is solution of a first order hyperbolic system. The model can be solved by an optimized solver for conservation laws. The following techniques enter this framework:

- waterbag approximations
- finite element approximations
- velocity-Fourier expansions (Eliasson)

Comparison of methods

Two-stream instabilitythes.



Reduced Vlasov: Lagrange + FV

(N. Pham)

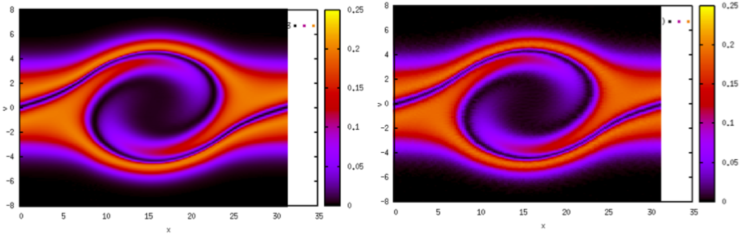


FIGURE 16. The distribution function of the two-stream test case at time $t = 25$. Left: reduced Vlasov-Poisson method. Right: the PIC method.

Reduced Vlasov: Lagrange + FV

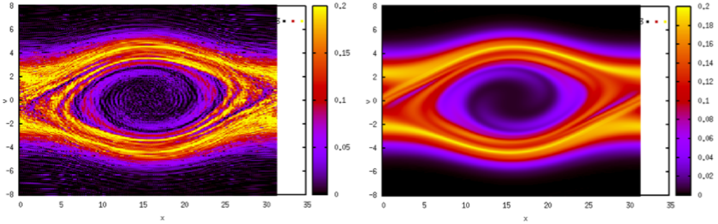
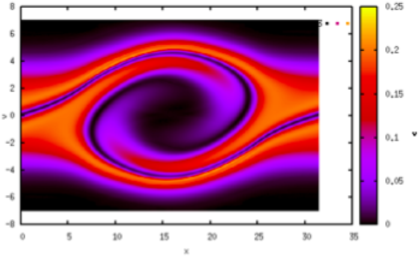


FIGURE 18. The distribution function of the two-stream test case at time $t = 50$ computed with the reduced Vlasov-Poisson method. Left: with the centered flux. Right: with the slightly upwinded flux.

Reduced Vlasov: Fourier

(N. Pham, L. Navoret)



Two-stream instability, Fourier-velocity method

Reduced Vlasov: waterbags

(A. Crestetto, N. Besse)

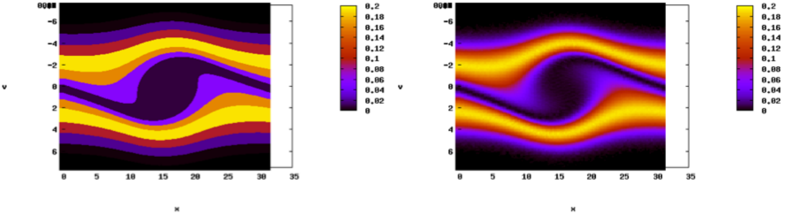


FIGURE 2.8 – Instabilité double faisceau - Fonction de distribution : modèle MWB (à gauche) comparé à la méthode PIC (à droite) aux temps $t = 0$, $t = 15$ et $t = 20$ (de haut en bas).

Ongoing works

- PIC AP solvers;
- 4D reduced Vlasov-Poisson solver;
- 4D drift-kinetic solver;
- Gyroaverage;
- SeLaLib (curvilinear meshes), CLAC (rewriting, memory optimizations).

Fusion IPL

Coordinated by J. Blum. Teams: Castor, Kaliffe, Tonus, LJLL, MIP, CEA IRFM.

- kinetic and gyrokinetic models: 5D and 6D simulations. SeLaLib
- fluid models, edge plasma turbulence, MHD instabilities. PlaTo.
- reduced fluid-kinetic models, collision models.
- Electromagnetic waves: reflectometry, RF heating.
- Control.