Sparse Signal Processing

Parcimonie en Traitement du Signal

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Two inverse problems in audio processing

- **Source localization**
  - ✓ S. Nam
- **Audio inpainting**
  - ✓ A. Adler, N. Bertin, V. Emiya, M. Elad, C.Guichaoua, M. Jafari, M. Plumbley

small-project.eu

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Source localization

with S. Nam

Localization with few microphones

- **Possible goals**
  - ✓ **localize** emitting sources
  - ✓ **reconstruct** emitted signals
  - ✓ **extrapolate** acoustic field

- **Linear inverse problem**

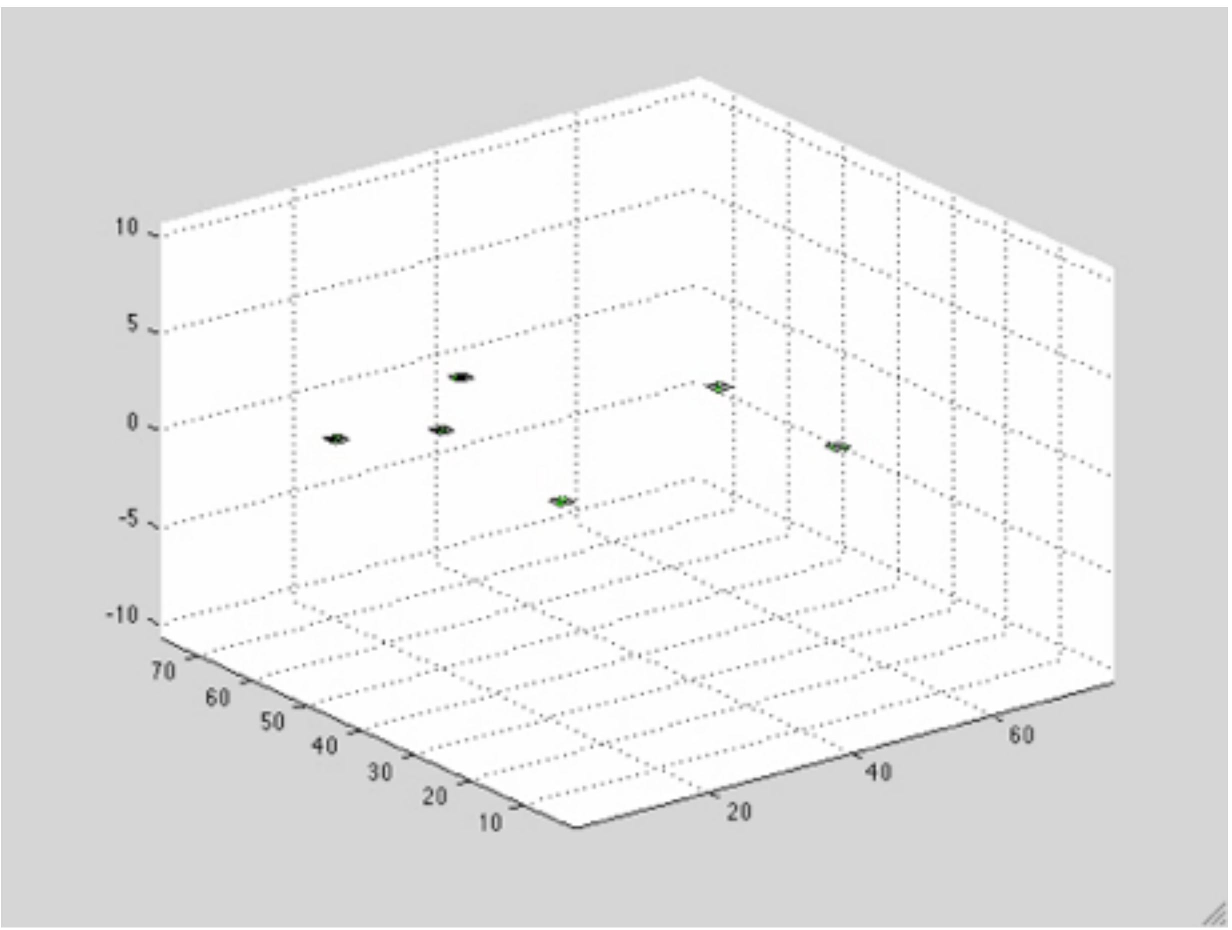
$$\mathbf{y} = \mathbf{M}\mathbf{x}$$

time-series recorded at sensors $\in \mathbb{R}^m$

(discretized) spatio-temporal acoustic field $\in \mathbb{R}^N$

- **Need a model**

Localization with few microphones



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Physics-driven design of model

- **Pressure field** $p(\vec{r}, t)$

- **Wave equation on a domain**

$$(\Delta p - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}p)(\vec{r}, t) = s(\vec{r}, t), \ \vec{r} \in \dot{\mathcal{D}}$$

- **Boundary + initial conditions**, e.g.

$$\frac{\partial p}{\partial n}(\vec{r}, t) = 0, \ \vec{r} \in \partial\mathcal{D}$$

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Discretization

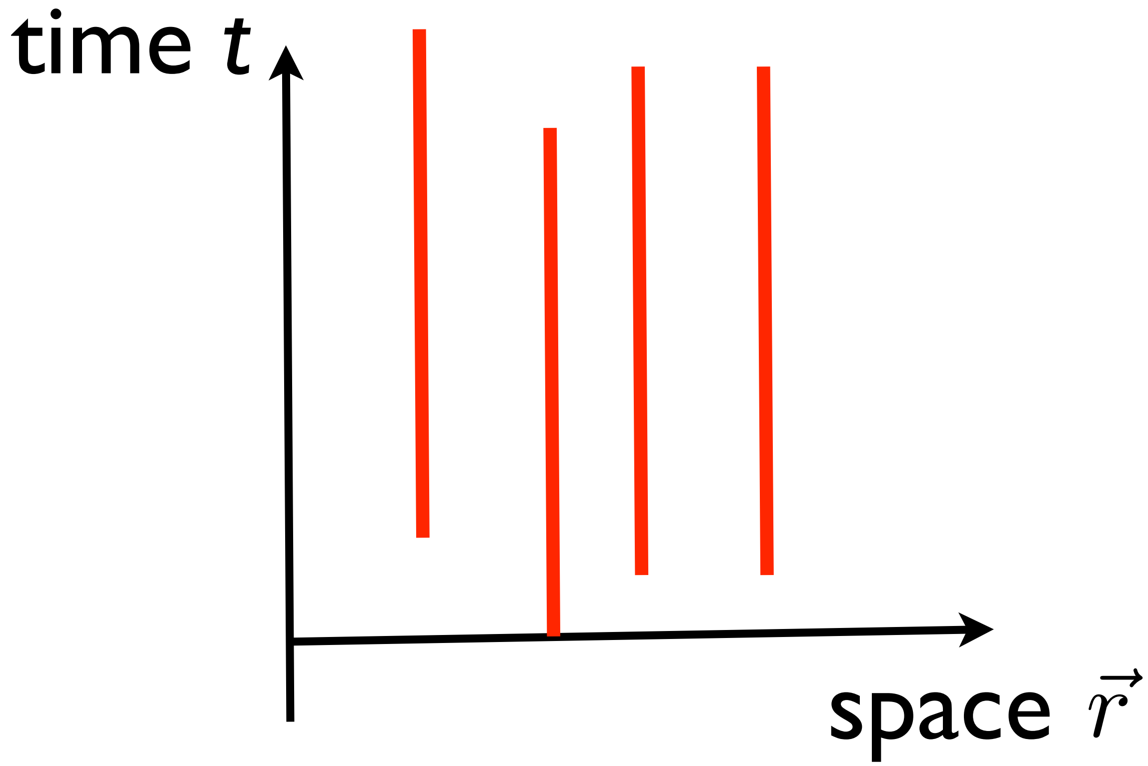
$\mathbf{x}$

$\mathbf{\Omega x} = \mathbf{z}$

sources & boundaries

Group sparse source model

- **Few non-moving sources = spatially sparse**



$\mathbf{z}_{\vec{r},t}$

lundi 12 novembre 12

Group sparse regularization

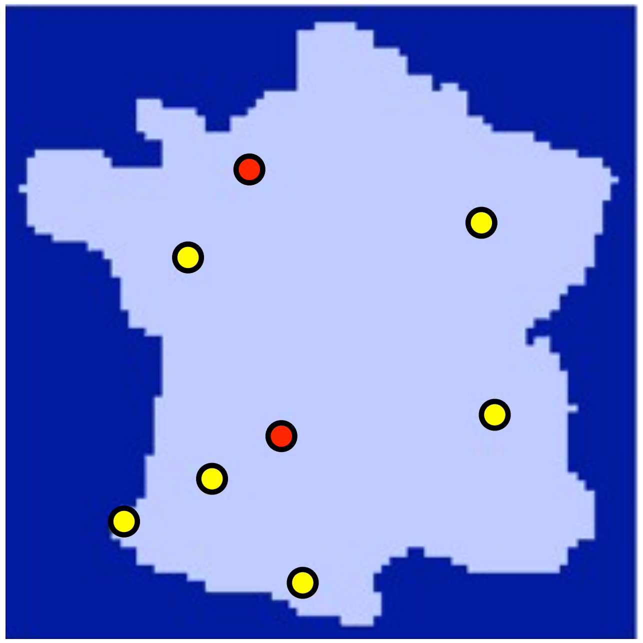
- **Inverse problem** $\mathbf{y} = \mathbf{M}\mathbf{x}$
- **Regularization with mixed norm**

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{y} - \mathbf{M}\mathbf{x}\|_2^2 + \lambda\|\boldsymbol{\Omega}\mathbf{x}\|_{1,2}$$

  - **Convex optimization**: efficient & provably convergent algorithms
  - **Promotes group sparsity**, cf Kowalski & Torresani 2009, Eldar & Mishali 2009, Baraniuk & al 2010, Jenatton & al 2011

Sparse Field Reconstruction

- **Setting**
  - ✓ 2D+t vibrating plate 77x77
  - ✓ 2 sources, random location
  - ✓ 6 microphones, random location
  - ✓ known complex boundaries
  - ✓ ground truth generated with naive discretization



- **Results**

Ground truth

Sparse reconstruction

S. Nam and R. Gribonval. Physics-driven structured cosparse modeling for source localization, ICASSP 2012

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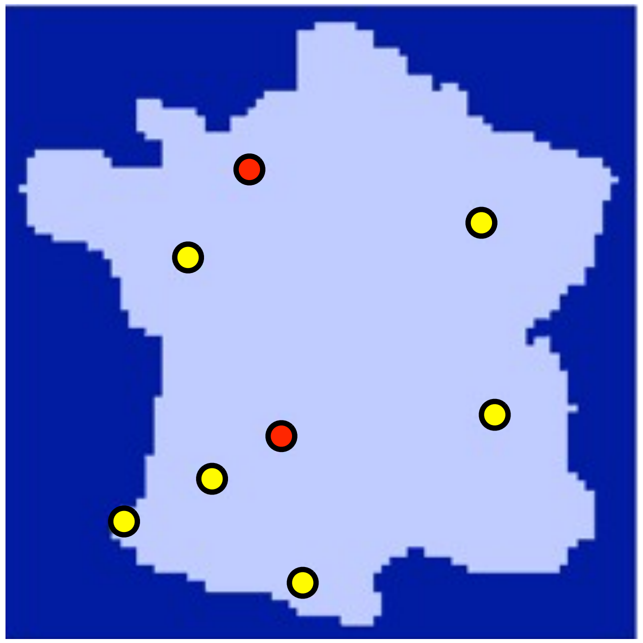

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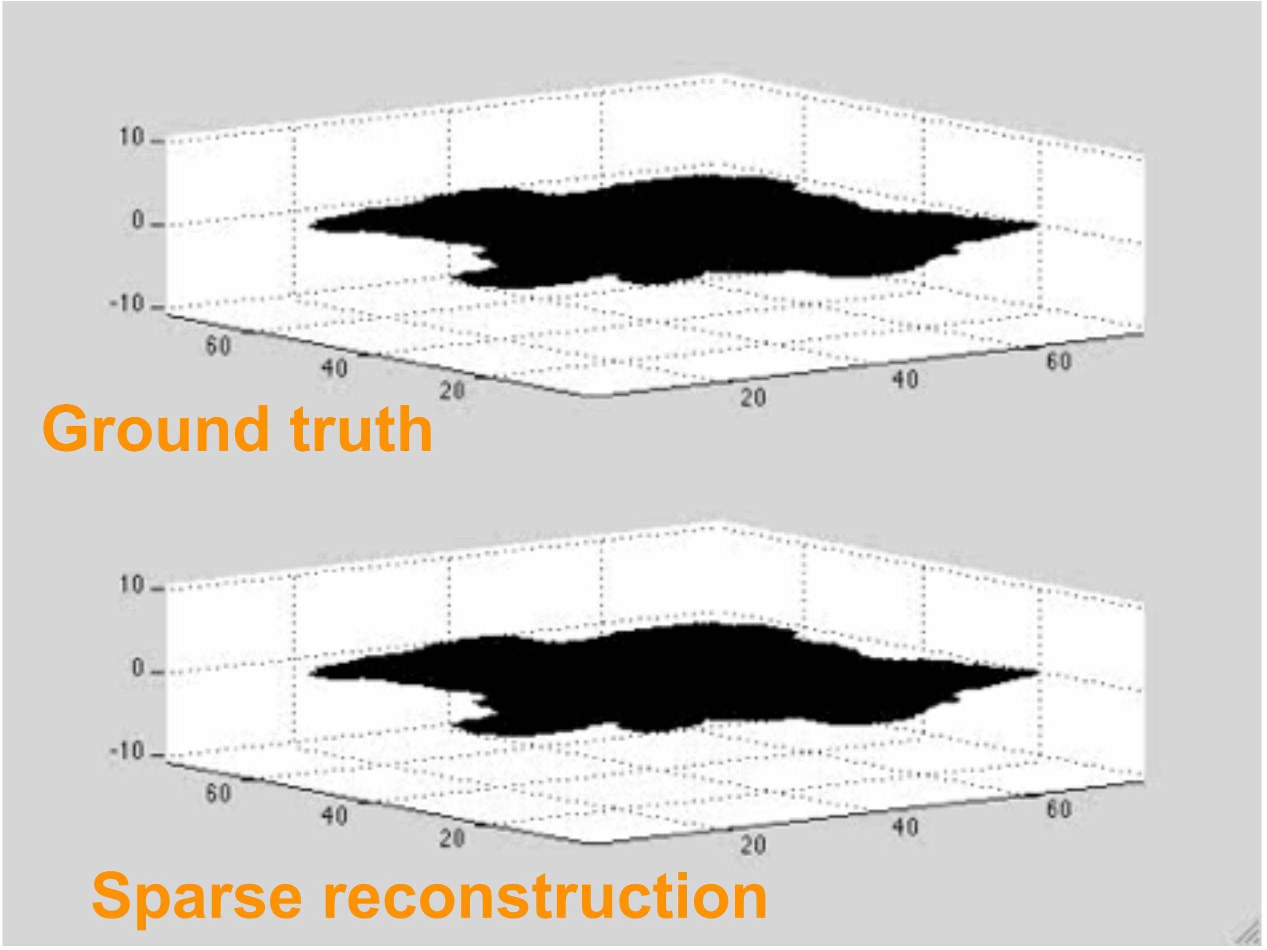

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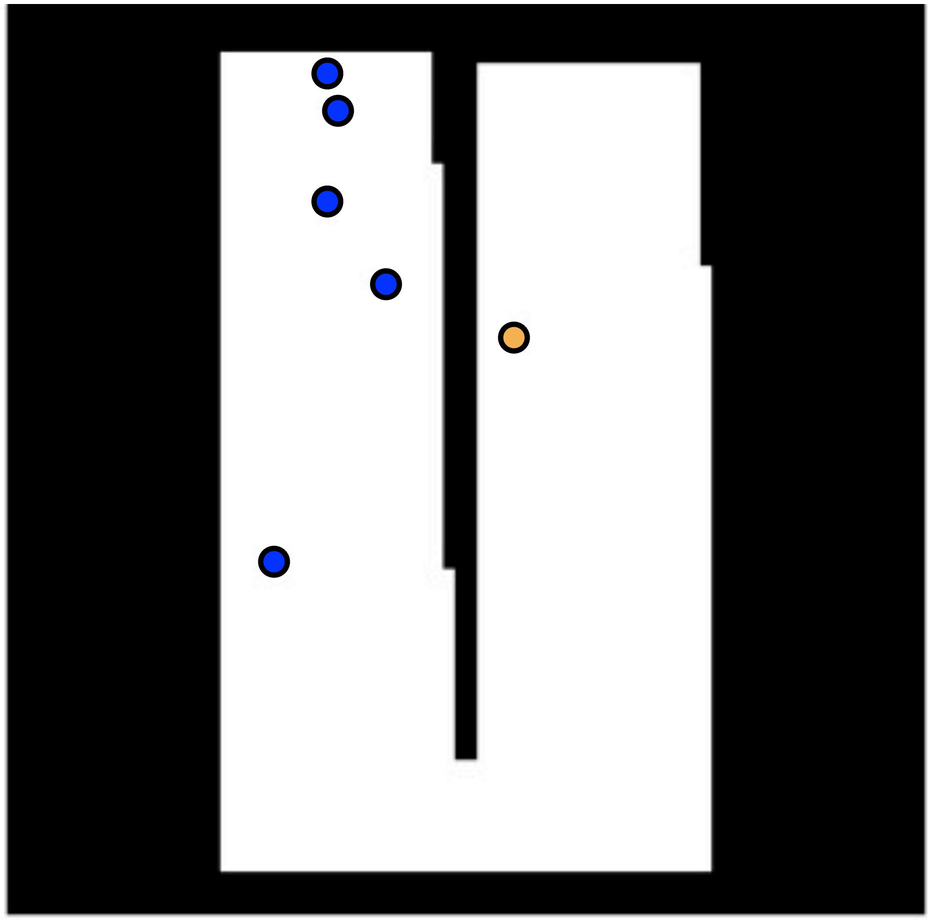
Localizing the source next door

- **Domain, Source and Microphones**



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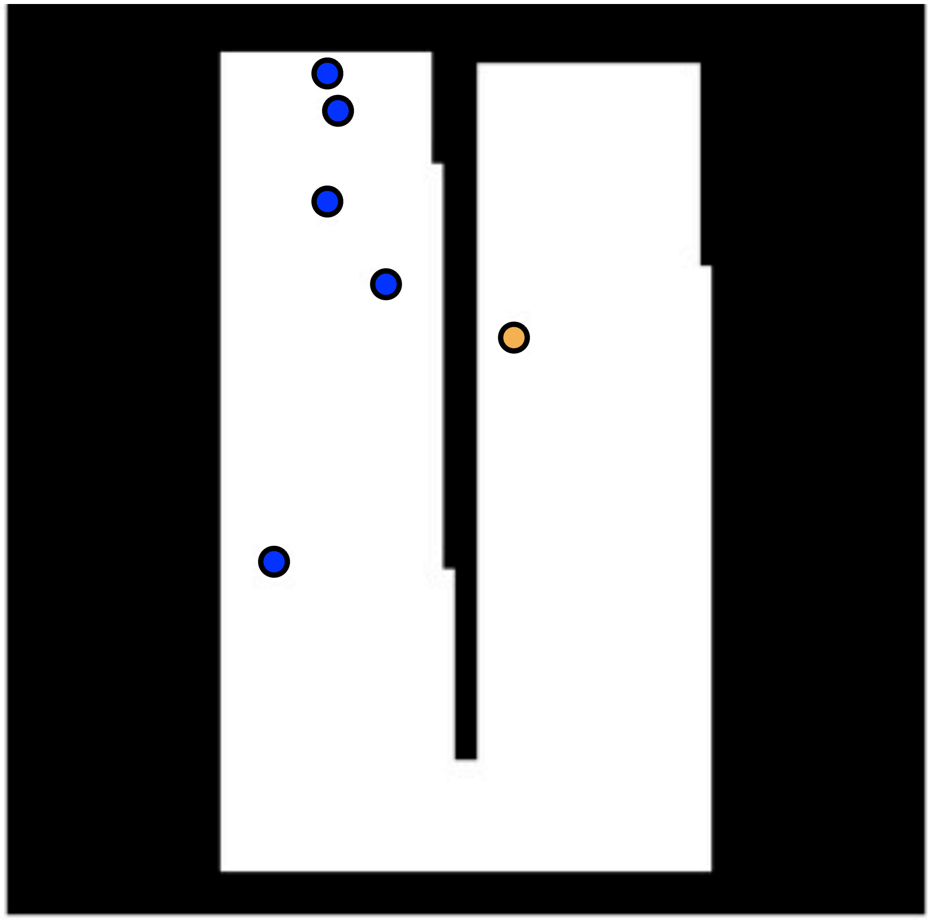

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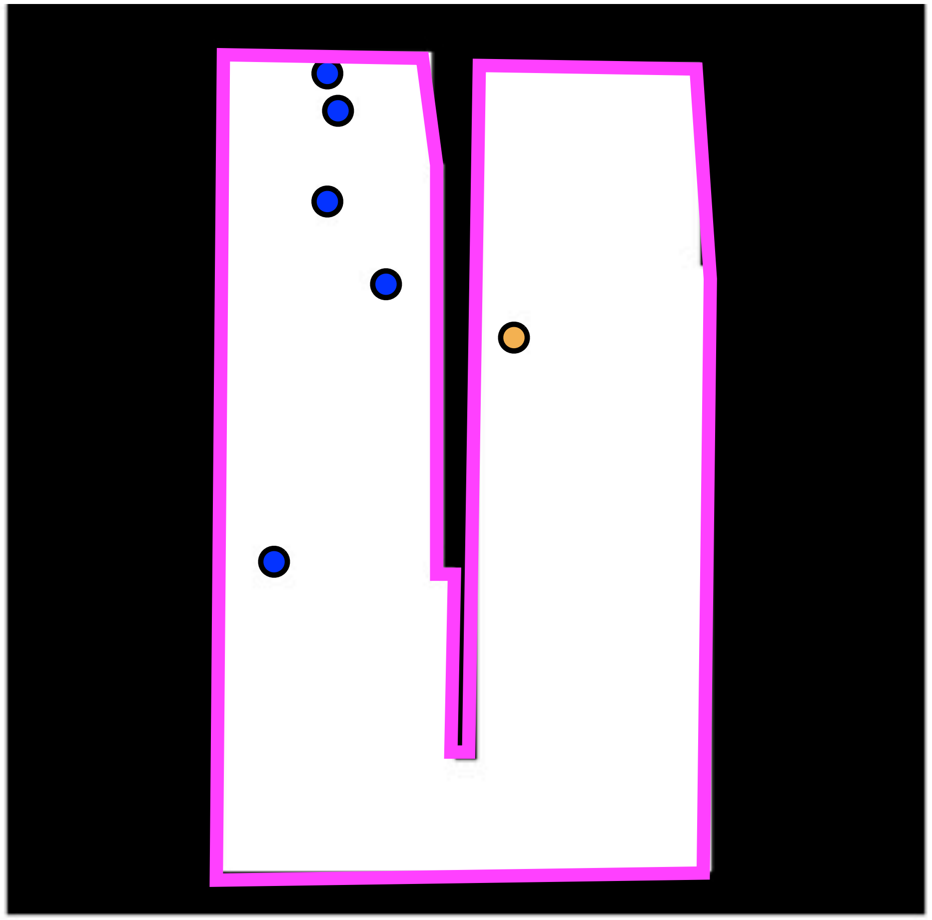

Reasons of success
- sparsity of sources
- *known* room shape
- *known* boundaries

- **Sparse source localization**

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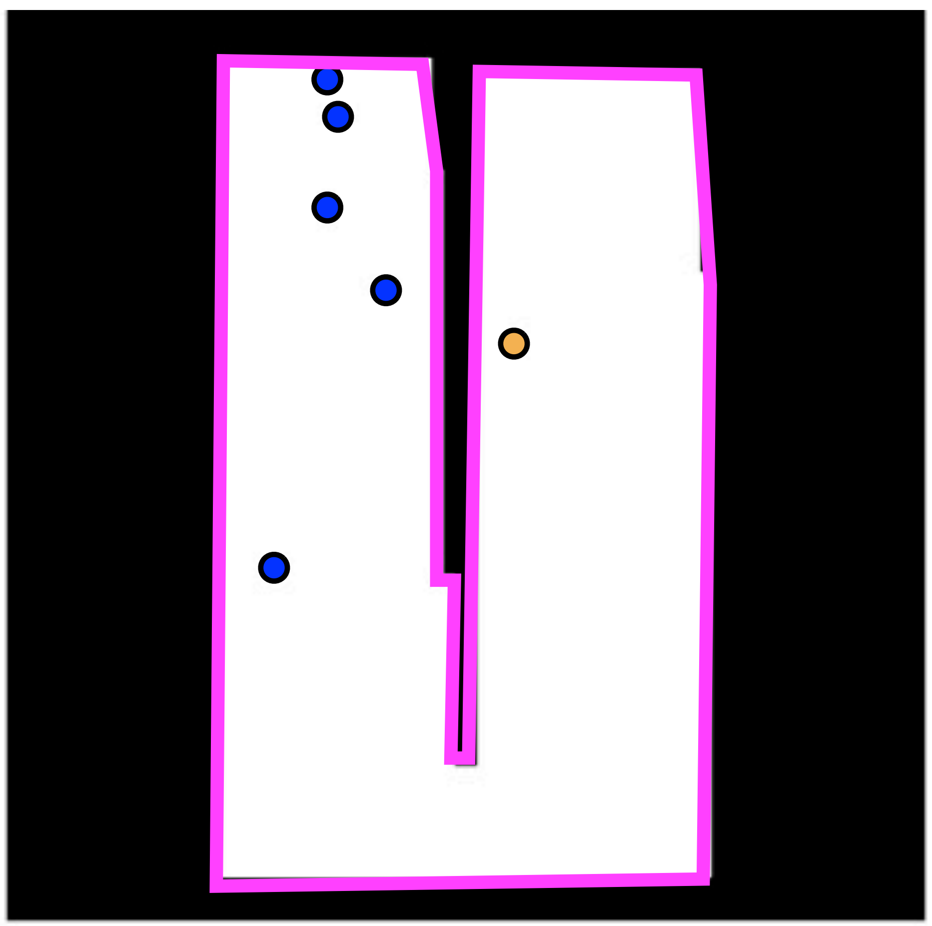

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What if shape is unknown ?

- **Sparse source localization**

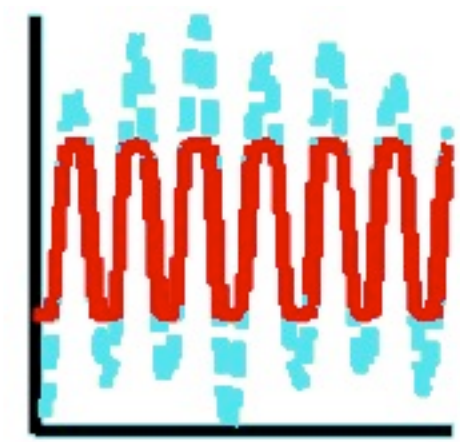
Audio inpainting

with A. Adler, V. Emiya, M. Elad, M. Jafari, M. Plumbley

Declipping as a linear inverse problem

- Original (unknown) samples $\mathbf{x}$
- Clipped (observed) samples $\mathbf{y}$
- Subset of reliable samples $\mathbf{y}_{\text{reliable}}$

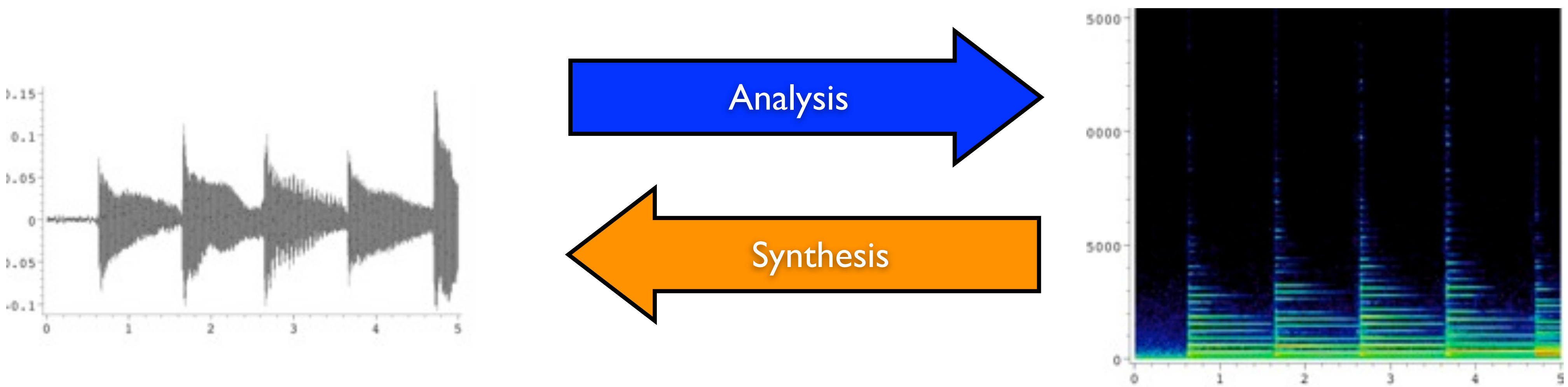


- **Linear inverse problem**

$$\mathbf{y}_{\text{reliable}} = \mathbf{M}\,\mathbf{x}$$

Sparse audio models

- Time domain
- Time-frequency domain

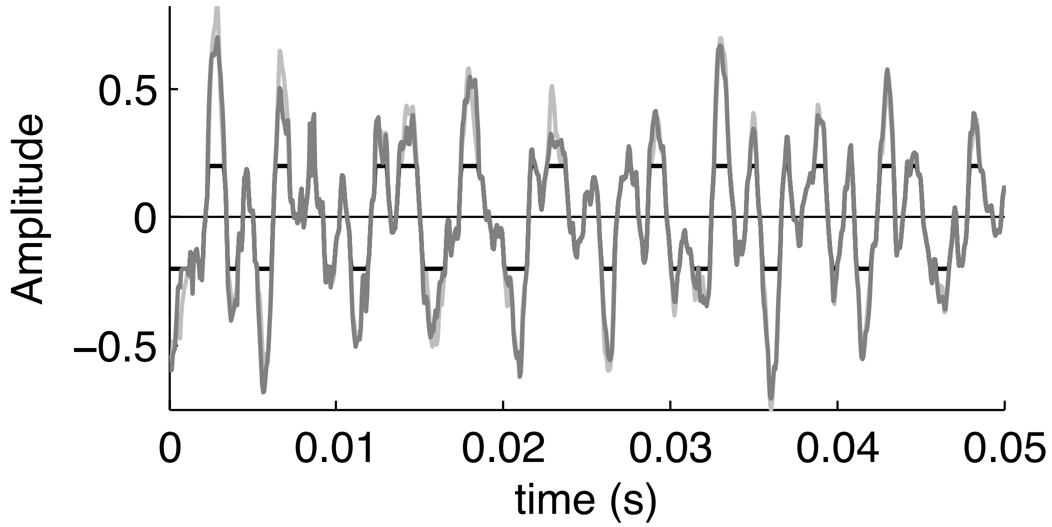


(Black = zero)

$$\mathbf{x} \approx \mathbf{D}\mathbf{z}$$

Audio Declipping

- **Model**
  - ✓ sparsity in time-frequency dictionary $\mathbf{x} = \mathbf{D}\mathbf{z}$
- **Algorithm:**
  - ✓ find sparse coefficients $\hat{\mathbf{z}}$ such that $\mathbf{y} = \mathbf{M}\mathbf{D}\hat{\mathbf{z}}$
    - ✦ (Orthonormal) Matching Pursuit (*Mallat & Zhang 93*)
  - ✓ estimate $\hat{\mathbf{x}} = \mathbf{D}\hat{\mathbf{z}}$



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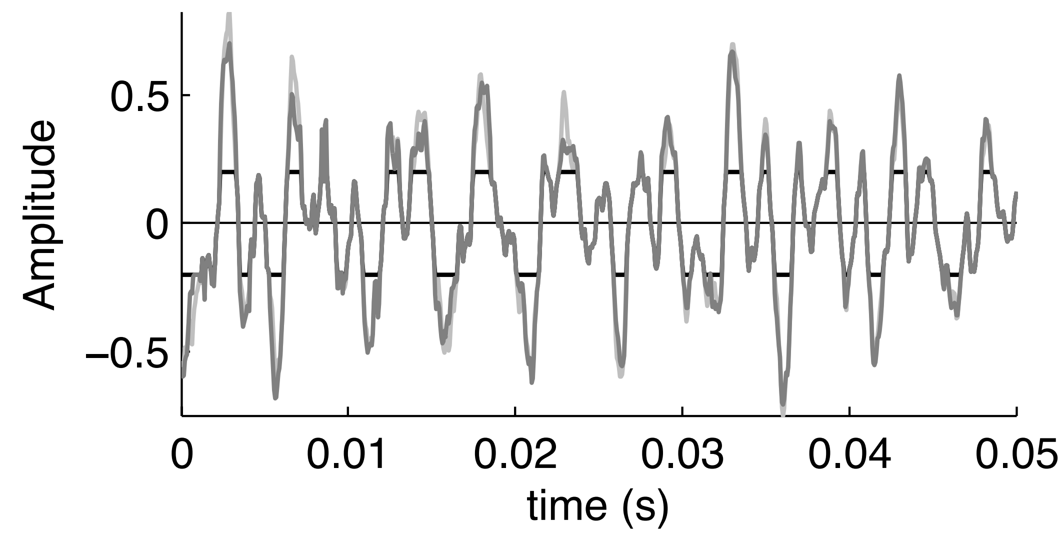

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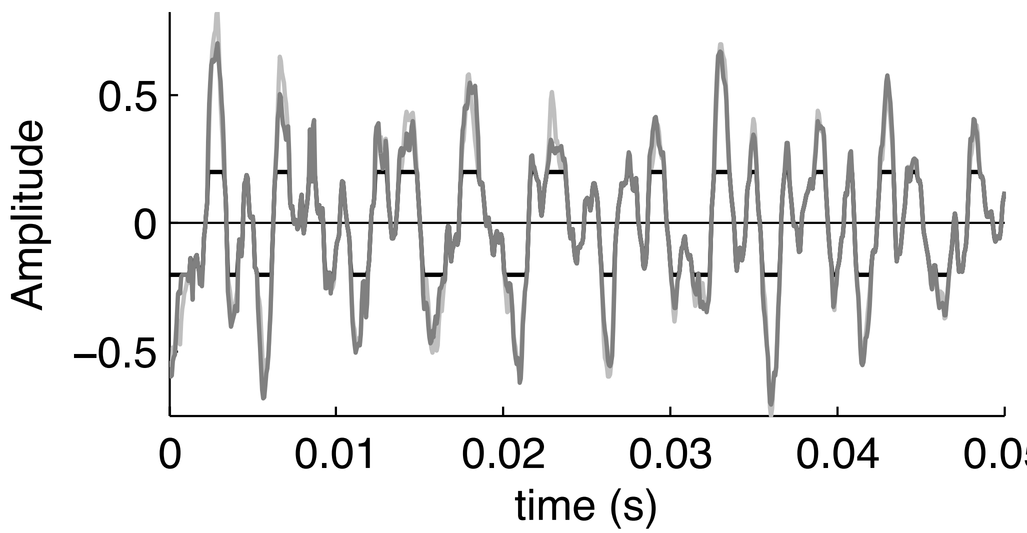
Clipped



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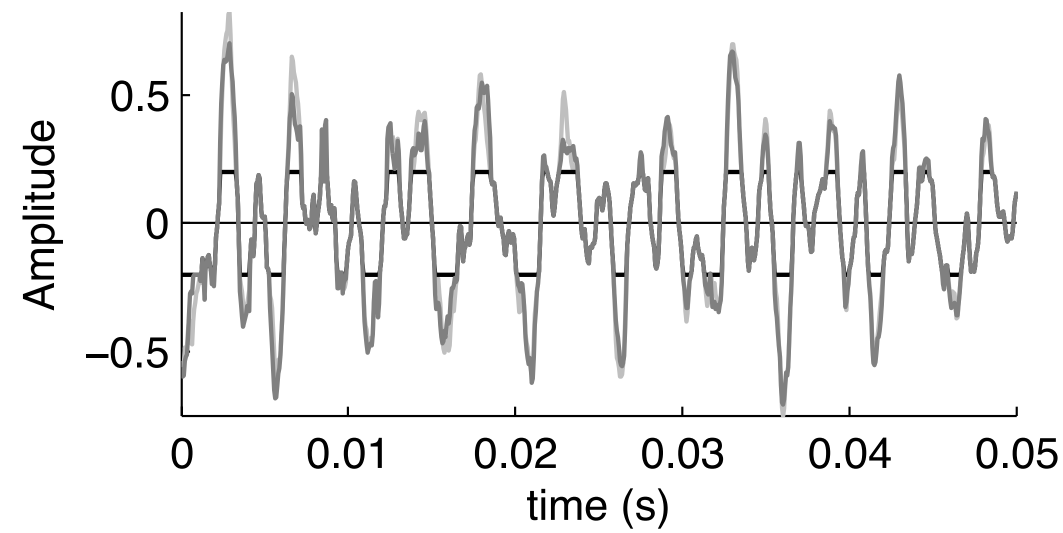

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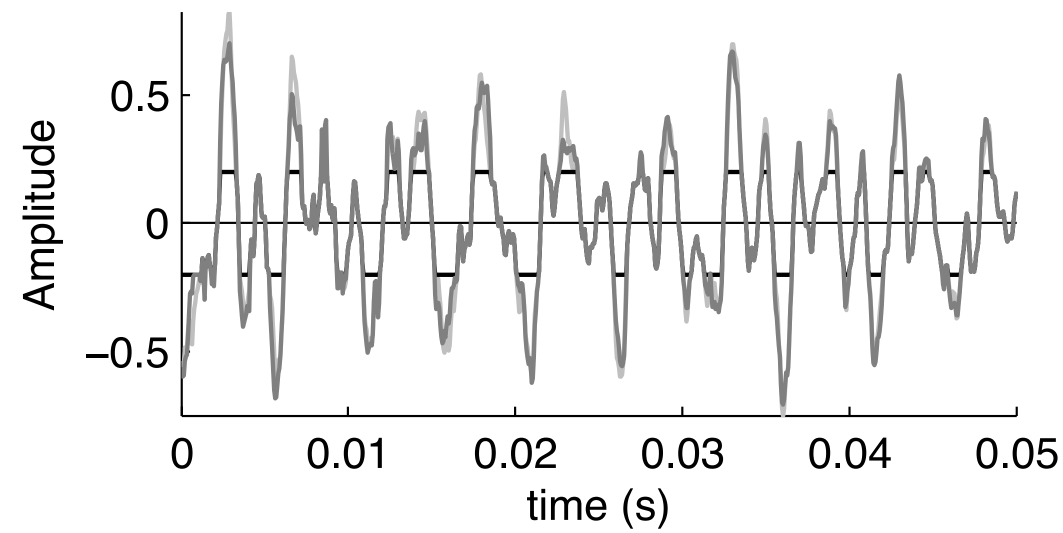
Clipped | Declipped | Original



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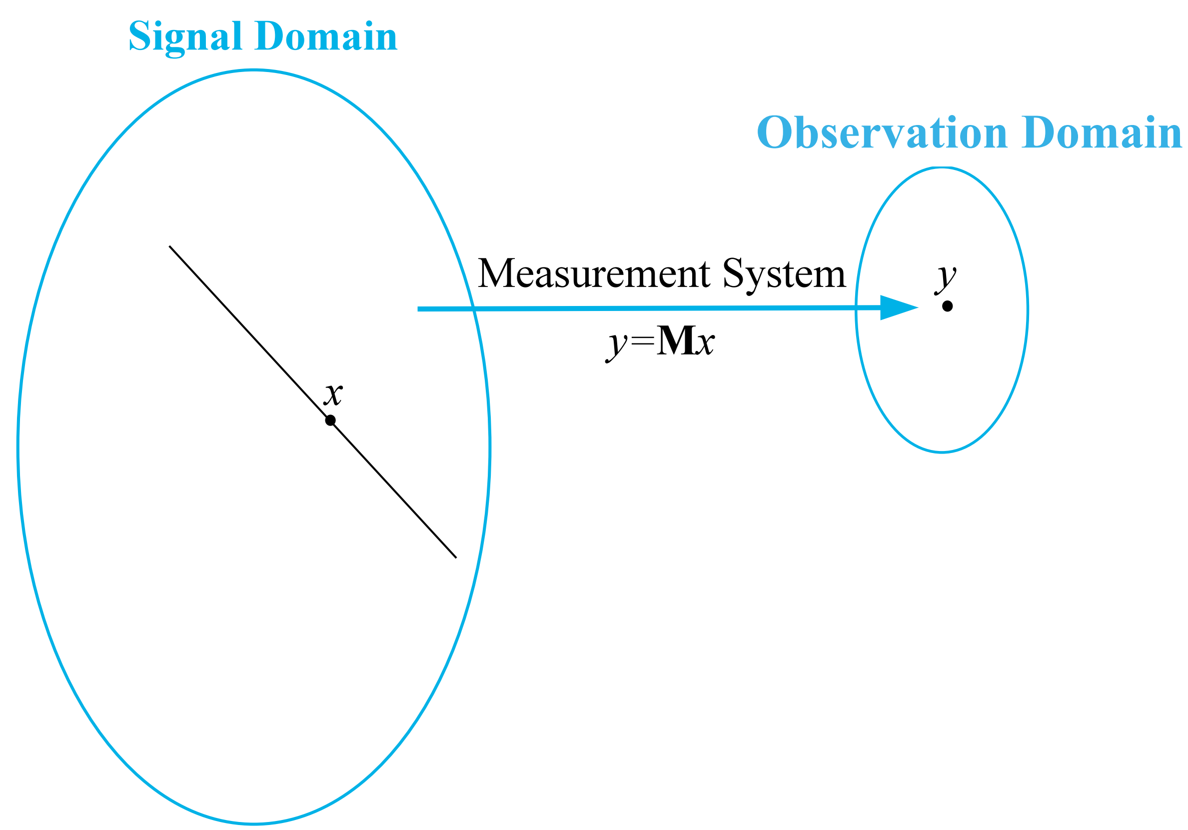
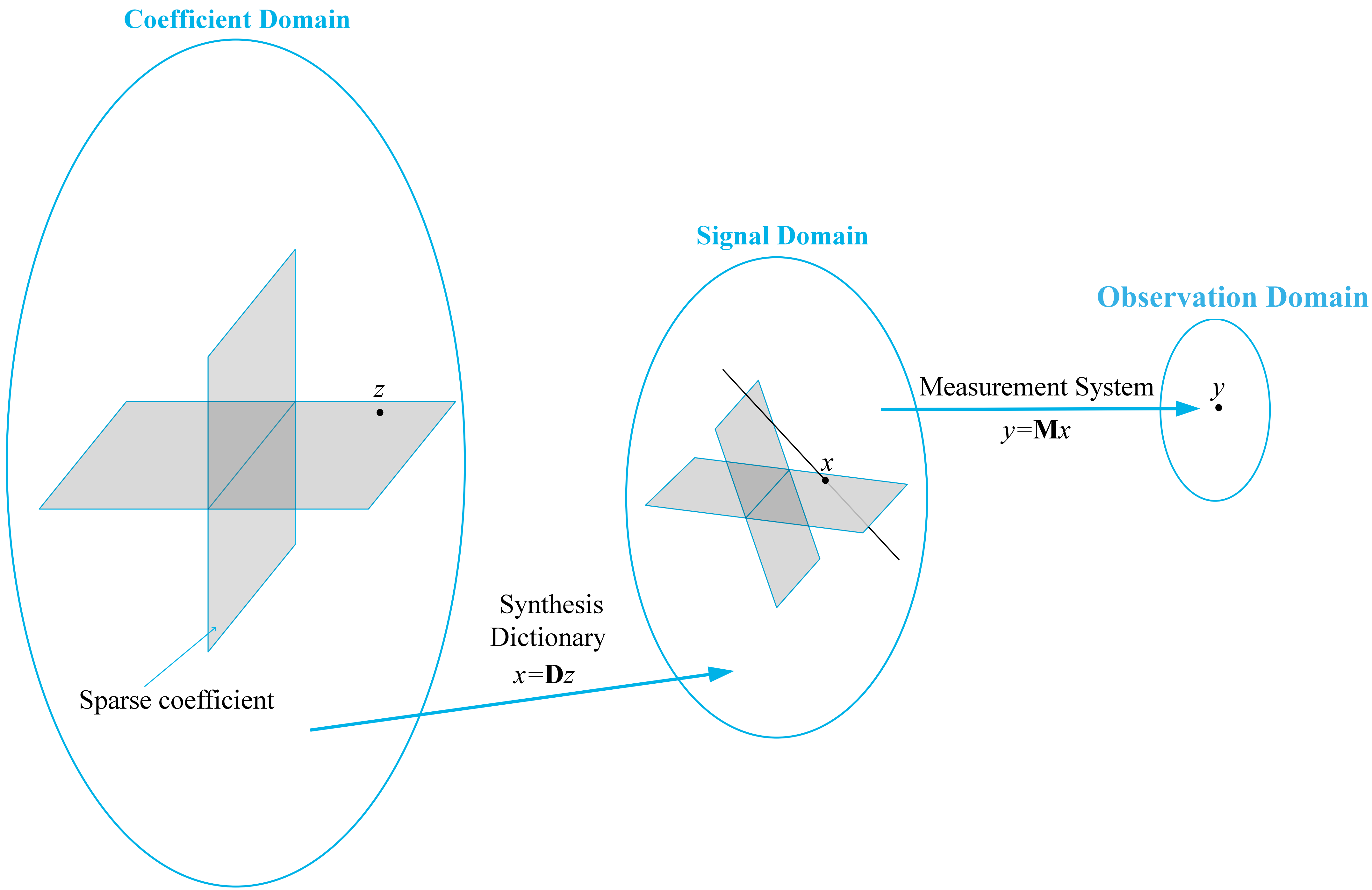
Summary & next challenges

Inverse problems ...



Inverse problems ... and sparse models



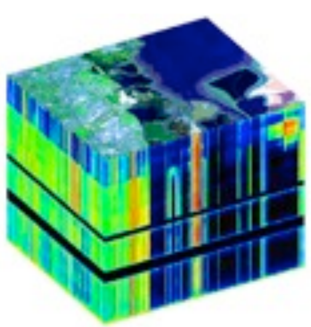
Choosing a model

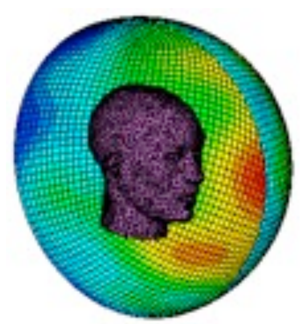
- **Expert knowledge (Fourier / wavelets)**
  - ✓ Harmonic analysis / physics
  - ✓ *Evolution of species*
- **Training from corpus**
  - ✓ Dictionary learning
  - ✓ *Individual experience*
- **«Online» training / adaptivity ?**
  - ✓ Blind Calibration & Deconvolution
  - ✓ *Adaptation to new environment*

Data Jungle

- New data **beyond signals and images**

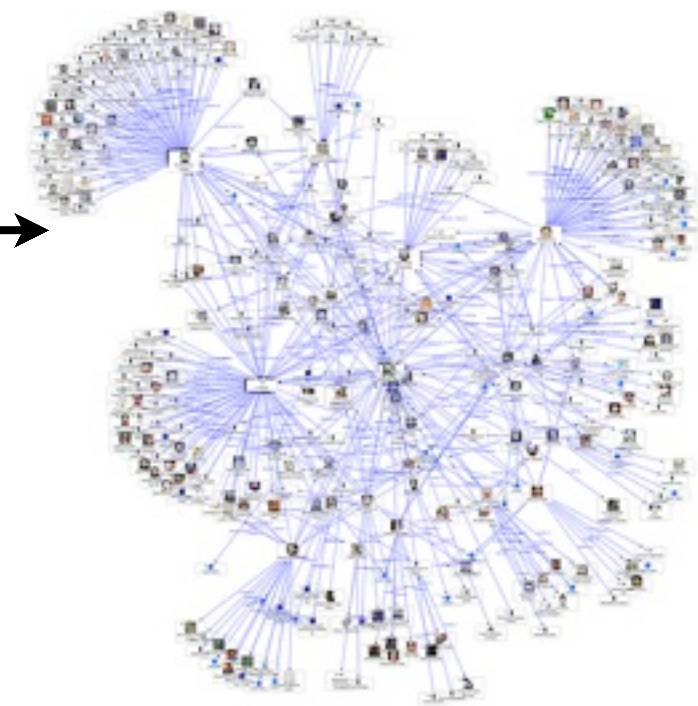
✓Hyperspectral
Satellite imaging

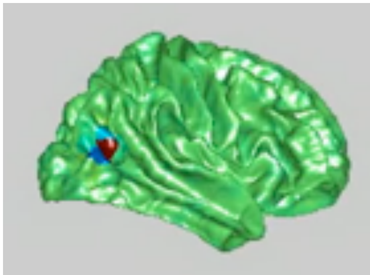




✓Spherical geometry
Cosmology, HRTF (3D audio)

✓Graphs
Social networks
Brain connectivity





✓Vector valued
Diffusion tensor

Key problem

Versatile low-dimensional **models**

What's next, please ?

- **Unified efficient data processing**
  - Signal processing
  - *Machine Learning*
- **Ground-breaking advances**
  - Compressive acquisition and compressive learning
  - Sparse models beyond dictionaries
- **Upcoming applications**
  - Inpainting / super-resolution (image/video/audio)
  - Distributed video coding
  - Astronomical imaging (interferometry)
  - Low-dose biomedical imaging (CT & IRM)
  - Audio recording @ high spatial resolution
  - Low-power compressive-sensors
  - Dynamic high-resolution brain imaging
  - ...

PLEASE

projection, learning and sparsity for efficient data processing

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PLEASE

projection, learning and sparsity for efficient data processing

THANKS

JEAN-PIERRE

SPECIAL THANKS

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