Sparse Signal Processing

*Parcimonie en Traitement du Signal*

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Two inverse problems in audio processing

- **Source localization**
  - S. Nam

- **Audio inpainting**
  - A. Adler, N. Bertin, V. Emiya, M. Elad, C. Guichaoua, M. Jafari, M. Plumbley
Source localization
with S. Nam
Localization with few microphones

- Possible goals
  - localize emitting sources
  - reconstruct emitted signals
  - extrapolate acoustic field

- Linear inverse problem
  \[ y = Mx \]
  - time-series recorded at sensors \( \in \mathbb{R}^m \)
  - (discretized) spatio-temporal acoustic field \( \in \mathbb{R}^N \)

- Need a model
Localization with few microphones

• Possible goals
  ✓ **localize** emitting sources
  ✓ **reconstruct** emitted signals
  ✓ **extrapolate** acoustic field

• Linear inverse problem
  \[ y = Mx \]
  time-series recorded at sensors \( \in \mathbb{R}^m \)
  (discretized) spatio-temporal acoustic field \( \in \mathbb{R}^N \)

• Need a model
Physics-driven design of model

- **Pressure field** \( p(\vec{r}, t) \)

- **Wave equation on a domain**

\[
(\Delta p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2})(\vec{r}, t) = s(\vec{r}, t), \quad \vec{r} \in \mathcal{D}
\]

- **Boundary + initial conditions**, e.g.

\[
\frac{\partial p}{\partial n}(\vec{r}, t) = 0, \quad \vec{r} \in \partial\mathcal{D}
\]
Physics-driven design of model

- **Pressure field** $p(\vec{r}, t)$

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\[(\Delta p - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} p)(\vec{r}, t) = s(\vec{r}, t), \ \vec{r} \in \mathcal{D}\]

- **Boundary + initial conditions**, e.g.

\[\frac{\partial p}{\partial n}(\vec{r}, t) = 0, \ \vec{r} \in \partial\mathcal{D}\]
Group sparse source model

- Few non-moving sources = spatially sparse
Group sparse regularization

- Inverse problem \( y = Mx \)

- Regularization with mixed norm

\[ \hat{x} = \arg \min_x \frac{1}{2} \| y - Mx \|_2^2 + \lambda \| \Omega x \|_{1,2} \]

- Convex optimization: efficient & provably convergent algorithms

Sparse Field Reconstruction

**Setting**
- 2D+t vibrating plate 77x77
- 2 sources, random location
- 6 microphones, random location
- known complex boundaries
- ground truth generated with naive discretization

**Results**

Ground truth

Sparse reconstruction

Sparse Field Reconstruction

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Ground truth

Sparse reconstruction

Localizing the source next door

- Domain, Source and Microphones
Localizing the source next door

- Domain, Source and Microphones
- Sparse source localization
Localizing the source next door

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Reasons of success
- sparsity of sources
- known room shape
- known boundaries
Localizing the source next door

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What if shape is unknown?
Audio inpainting
with A. Adler, V. Emiya, M. Elad, M. Jafari, M. Plumbley
Declipping as a linear inverse problem

- Original (unknown) samples $x$
- Clipped (observed) samples $y$
- Subset of reliable samples $y_{\text{reliable}}$

Linear inverse problem

$$y_{\text{reliable}} = M$$
Sparse audio models

- Time domain
- Time-frequency domain

\[ x \approx Dz \]
Audio Declipping

Model
✓ sparsity in time-frequency dictionary \( x = Dz \)

Algorithm:
✓ find sparse coefficients \( \hat{z} \) such that
✦ (Orthonormal) Matching Pursuit (Mallat & Zhang 93)

✓ estimate \( \hat{x} = D\hat{z} \)

Audio Declipping

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- **Algorithm:**
  - ✓ find sparse coefficients \( \hat{z} \) such that
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  - ✓ ensure compatibility with clipping constraint
    - ✦ Convex optimization
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Audio Declipping

• Model
  ✓ sparsity in time-frequency dictionary $\mathbf{x} = \mathbf{Dz}$

• Algorithm:
  ✓ find sparse coefficients $\hat{\mathbf{z}}$ such that $\mathbf{y} = \mathbf{MD}\hat{\mathbf{z}}$
  ✦ (Orthonormal) Matching Pursuit (Mallat & Zhang 93)
  ✓ + ensure compatibility with clipping constraint
  ✦ Convex optimization
  ✓ estimate $\hat{\mathbf{x}} = \mathbf{D}\hat{\mathbf{z}}$

Audio Declipping

- **Model**
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Summary & next challenges
Inverse problems ...
Inverse problems ... and sparse models
Choosing a model

• **Expert knowledge (Fourier / wavelets)**
  - Harmonic analysis / physics
  - *Evolution of species*

• **Training from corpus**
  - Dictionary learning
  - *Individual experience*

• «Online» *training / adaptivity* ?
  - Blind Calibration & Deconvolution
  - *Adaptation to new environment*
Data Jungle

• New data beyond signals and images

✓ Hyperspectral Satellite imaging
✓ Spherical geometry Cosmology, HRTF (3D audio)
✓ Graphs Social networks Brain connectivity
✓ Vector valued Diffusion tensor

Key problem
Versatile low-dimensional models
What’s next, please?

- **Unified efficient data processing**
  - Signal processing
  - *Machine Learning*

- **Ground-breaking advances**
  - Compressive acquisition and compressive learning
  - Sparse models beyond dictionaries

- **Upcoming applications**
  - Inpainting / super-resolution (image/video/audio)
  - Distributed video coding
  - Astronomical imaging (interferometry)
  - Low-dose biomedical imaging (CT & IRM)
  - Audio recording @ high spatial resolution
  - Low-power compressive-sensors
  - Dynamic high-resolution brain imaging
  - ...
P L E A S E
projection, learning and sparsity for efficient data processing
• Frédéric Bimbot
• Nancy Bertin, Emmanuel Vincent
• Current Docs & Postdocs:
  ✓ Alexis Benichoux, Anthony Bourrier, Srdjan Kitic, Lei Yu, Cagdas Bilen, ...
• Stéphanie Lemaile
• Jules Espiau